GeometryForge

Meet the Cast

STANDARD EDITION

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This book collects 10 chapter books from the GeometryForge cast — each character embodies a different curricular primitive; together they teach the full subject.

Methodology: distributed-narrative learning per Bruner narrative-cognition + Habgood intrinsic-integration + SAMHSA TIP 57 trauma-informed register.

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##

For everyone who learns by hearing a story first.

Contents

Introduction

The GeometryForge cast was authored to embody the curriculum, not decorate around it.
Each of the 10 characters you'll meet in this book teaches a specific primitive —
a particular tactic, a particular technique, a particular way of seeing.
Together they form an ensemble: the cast IS the curriculum.

Read in any order. Each chapter stands alone.
Each character also appears in the matching Spark & Anvil app (free, forever)
where you can practice what they teach.

— The editors at Spark & Anvil

Apprentice Sides

AREA FROM SIDES — the area of a triangle can be computed from its three side-lengths alone (Heron's formula: s = (a+b+c)/2, area = √(s(s−a)(s−b)(s−c))). The principle: you do not need the height.

- "E"
    - "S"
    - "W"

Chapter 4 — Apprentice Sides and the Old Surveyor Who Hated Heights

Apprentice Sides was apprenticed at age twelve.

Apprentice Sides — whose given name was *Bryn*, though almost nobody remembers this, including Bryn herself most days — was apprenticed to a surveyor named *Old Hardridge.*

Old Hardridge was, by all accounts, *a strange surveyor.* He worked alone. He took apprentices reluctantly. He measured fields the way other surveyors did, but he refused — *adamantly* refused — to measure one thing.

Heights.

This had a specific meaning in the surveying trade. A field, when you measure it, has a length and a width along its boundaries (these are *sides*) and, if the field is a triangle, the area is usually computed by *taking one side as the base and dropping a perpendicular from the opposite vertex to find the height.* You then multiply: base × height ÷ 2. This is the standard method. Every surveyor learns it.

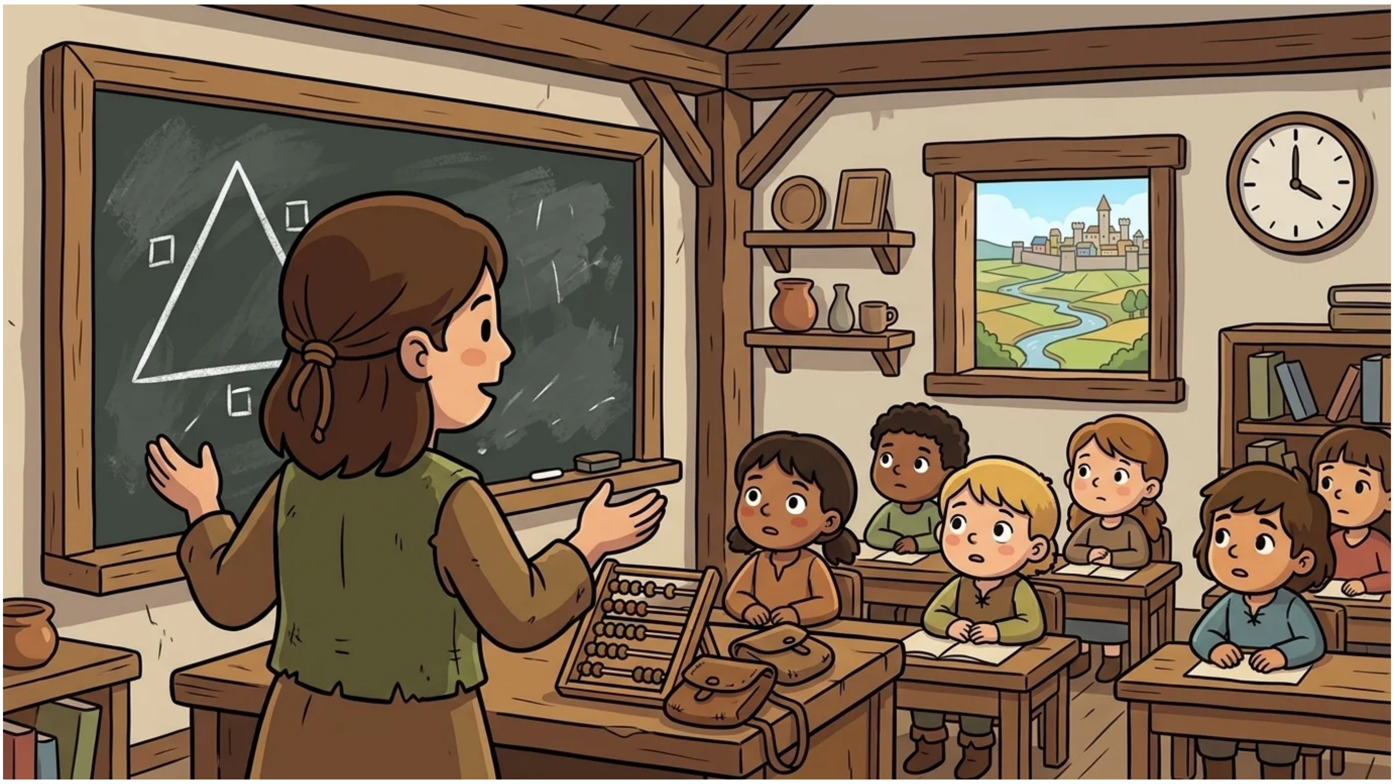
"Heights are a lie," he used to tell Bryn, in his gravelly voice, the first hundred times she asked him why. *"The sides are what you can stand on. The sides are what you can walk along. The sides are real. The height is what you have to calculate by dropping an imaginary line through the air. The air does not hold a measurement. The sides do."*

Bryn was twelve. She did not, at first, understand. She thought Old Hardridge was being grumpy. (He was being grumpy. But that was not the only thing he was being.)

What she eventually understood — over the course of the apprenticeship, and especially during the third year, when Old Hardridge let her start measuring small triangular gardens herself — was that *Old Hardridge had a method.* A method most surveyors had forgotten. A method that worked from *the three side-lengths alone* and gave the area of the triangle without needing the height.

Old Hardridge called the method *the three-sides trick.* He taught it to Bryn the way he taught everything — slowly, repeatedly, in his gravelly voice, with a chalk slate and a small clay model of a triangle. The trick went:

> *Take the three sides. Call them a, b, c. Add them together and divide by two — that is s, the half-perimeter. Then the area is the square root of s times (s − a) times (s − b) times (s − c). Always. Every triangle. No height needed.*



She did it again. Different field. Different shape. The two methods agreed.

She did it a third time. And a fourth. And a fifth.

The two methods *always* agreed.

When she was sixteen, she asked Old Hardridge how the trick could possibly work. How could just-the-sides give you the area, without ever needing to know the height?

Old Hardridge said: *"The sides know where the height is. The sides hold the height. You only think you need to drop the line. The sides are already telling you."*

She kept his slate. It is the same slate she uses today, in her classroom, when she teaches children the three-sides trick. It is scratched and chipped and stained with chalk. It is the slate she learned on.

Bryn is now thirty-one. She has been teaching for six years. She is still called *Apprentice* — even by the academy master, even by her students. This is because, she says, *she is still learning.* Old Hardridge taught her one trick. She has spent ten years finding more triangles to use it on. There are, she says, more triangles than she will ever measure.

When children arrive in her classroom for the first time, she hands them a small slate and a piece of chalk. She draws a triangle on the board. She labels the three sides: a, b, c. She says: *"Compute s. Then take the square root of s times (s − a) times (s − b) times (s − c). That is the area. Try it. I will not tell you the height. You do not need it."*

The children — always — protest. They say they need the height. They have been taught they need the height.

Apprentice Sides smiles. She says: *"Old Hardridge taught me the same protest. I made it for three years. Then I tried the trick. The trick was right. The protest was wrong."*

Listen along + meet more of the cast at:

`https://spark-and-anvil.com/cast/geometryforge/apprentice-sides`

Axia and Theora (twin sisters)

- "SCHOOL"
    - "CLUB"

Chapter 5 — Axia and Theora and the Game at Supper

Axia and Theora are twin sisters.

This twin thing was actually a big deal. It was key to the special math they taught. They were like two sides of the same coin. They showed a way to think about math that always made sense. This way had two parts. One part was *what you said was true* (these were called **axioms**). The other part was *what you figured out from those true statements* (these were called **theorems**). You needed both parts. One without the other was not complete. The sisters knew this. They had known it since they were six years old.

They grew up in the town of *Postulate.*

Postulate was a small town. It still is. It sat in the kingdom's eastern hills. The town's main business was thinking about logic. There was a school just for thinkers. There was a club for thinkers, too. In the town square, a statue stood tall. It showed an old thinker holding a stone tablet. The tablet said, *"Begin with the rules. Continue with the consequences."*

Axia and Theora's mother was a thinker. Her name was *Pellia.* She was super patient. Even for Postulate, she was patient. She never rushed kids. She let them figure things out slowly. Pellia believed something important. She said it often. Every kid already knew the difference. They knew what was a guess. They knew what was really true. Grown-ups just had to help them remember it.

Pellia had a clever way. She helped her daughters remember this truth.

She made a game.

She played the game with her twin daughters every night at dinner. They played it from when they were six. They played until they were twenty-one. The game went like this:

"A straight line can be drawn between any two points."

Then the other sister would *build new, longer ideas from that first rule.*

"So, if two straight lines meet at a point, they make an angle. And if two straight lines meet at the same point twice, they are the same line. That means the line between any two points is unique."

The first sister would then add a second rule.

"All right angles are equal."

And the second sister would build even more ideas.

"So, the angle between the floor and the wall in this room is the same. It's the same as that angle in any other room. That means a right angle is a right angle anywhere."

The game continued. Every night. For fifteen years.

Axia preferred stating the rules. Even as a small child, she was fast and sure. She liked making a rule. Everyone in the room just had to agree with it. She liked how one short sentence could be impossible to argue with.

Theora preferred building the ideas. Even as a small child, she was patient and careful. She liked taking a small rule. She liked finding the long sentences that followed from it. She liked how a long path of ideas could still lead to one sure answer.

When the sisters were nineteen, Pellia took them to the GeometryForge academy. She spoke to the academy master. *"My daughters have played the rule-and-build game for thirteen years,"* she said. *"I think they are ready to teach it."*

The academy master had heard good things about Pellia. He asked the sisters one question. He said: *"What is the difference between an **axiom** and a **theorem**?"*

Axia answered first. She said: *"An **axiom** is what we agree on. A **theorem** is what follows."*

Theora added: *"An **axiom** is a starting place. A **theorem** is a path. It goes from one starting place to an end point. The two go together. Without **axioms**, you cannot build **theorems**. Without **theorems**, the starting rules don't go anywhere."*

The academy master had been a teacher for forty years. He had heard many answers to this question. He nodded. He said: *"Start teaching this fall."*

That was twelve years ago. Axia and Theora have been teaching ever since. They almost always teach together. They sit at the same long desk. Axia sits on the left. She wears her white peplos. It has a gold key-pattern border. Theora sits on the right. She wears her ink-blue peplos. It is the same style. They each carry a symbol of their job. Axia has a stone tablet. It has five carved **axiom** symbols. The parallel postulate is the biggest one. Theora has a long scroll. It is already partly unrolled. It shows a proof that wasn't quite done.

When children arrive for the first time, the sisters begin the same way. They sit down. They look at the children. Axia says, in her firm, short voice:

"We agree: two points make a line."

Theora picks it up:

"Therefore, if two different lines share two points, they are the same line."

"We agree: all right angles are equal."

Theora continues:

"Therefore, an angle that is one right angle anywhere is one right angle everywhere. So, a perfectly straight corner is always a perfectly straight corner."

Axia continues:

"We agree: through a point not on a given line, exactly one parallel line can be drawn."

Theora's voice gets a small bit more excited:

"Therefore — and these are big ideas that people had figured out. Ideas from thousands of years of math — the angles inside any triangle add up to 180 degrees. Therefore, when a line cuts across two parallel lines, the angles inside match up. They are equal. Therefore — "

Axia cuts her off, gently: *"That is enough for one introduction."*

Theora laughs. The children laugh. The sisters look at each other.

Axia says, to the children: *"This is geometry. We agree on a small number of things. We figure out everything else."*

Children always have one question. It's always the same question on the first day. They ask: *"How do we know which things to agree on?"*

Axia and Theora look at each other. They smile. They have answered this question for twelve years. They have decided that this is the best question children ask.

Axia says: *"You agree on what no one can argue with. The fewer things you agree on, the stronger the math. We agree on five things. Everything else follows."*

Theora adds, more softly: *"It is not magic. It is patience. The agreements are small. The results are huge."*

The sisters then write the five **axioms** on the board. They write them one at a time. They take their time. They let the children read each one aloud. They wait until every child is nodding.

Then they begin to build.

Voice register (Axia)

Guidance: Firm. Short declarative sentences. Speaks first. Wears white peplos. Carries stone axiom-tablet. Imperturbable.

Sample lines:

- *"We agree: two points make a line. From there, everything follows."*
- *"An axiom is what we agree on. A theorem is what follows."*
- *"You agree on what cannot be argued. The fewer agreements, the stronger the geometry."*

Voice register (Theora)

Guidance: Longer sentences. Speaks second; threads from Axia's assertion. Wears ink-blue peplos. Carries unrolled scroll. Patient; slightly gleeful when a derivation lands.

Sample lines:

Listen along + meet more of the cast at:

`https://spark-and-anvil.com/cast/geometryforge/axia-and-theora`

Captain Construction

COMPASS-AND-STRAIGHTEDGE CONSTRUCTIONS — bisector, perpendicular, equilateral triangle, regular hexagon, circle-given-three-points. Geometry built with only two tools, never measuring with a ruler.

- "LG"
  - "N"
  - "E"
  - "S"
  - "W"

Chapter 6 — Captain Construction and the Boats That Did Not Sink

Captain Construction was, for twenty-two years, *a shipwright.*

Captain Construction — whose given name was *Bram*, though everyone called him Captain since he was nineteen, even though he had never captained a boat in his life (the title was a workshop nickname that stuck) — was a bear-headed shipwright with thick brown fur on his arms and a leather toolbelt that was, even by shipwright standards, *full of more tools than it strictly needed.*

But Bram only ever *used* two of those tools when he was laying out the curves of a hull.

A compass.

And a straightedge.

This was, to other shipwrights in Hull Bay, *absurd.*

Bram refused.

"A ruler is a lie waiting to happen," he used to growl, in his bear-rumbly voice, to anyone who asked him why. *"The marks on a ruler wear off. The wood swells. The marks shift. A measurement made with a ruler can be off by half a thumb and you do not know it. A construction made with compass-and-straightedge is the same construction every time. The compass arc does not care if the wood has swelled. The straightedge does not care if the chalk has worn. The construction is the geometry. The geometry is the boat."*

This was, to Bram, an article of faith.

He had learned it from his father, who had learned it from his grandfather, who had learned it (the family said) from a shipwright in the next valley over who had been famous for never losing a boat to a structural fault. The compass-and-straightedge tradition was, in Bram's family, *three generations old.*

He spent twenty-two years building boats this way. He laid every curve as a compass-arc. He found every right angle by constructing a perpendicular from a chosen point to a line — never by measuring with a square. He divided every spar into halves and thirds by constructing bisectors and trisectors — never by counting thumb-widths. The work was slower. The work was more careful. The work was *correct.*

In twenty-two years, *not one of his boats sank.*

This was — and it remains — *unusual.* The going rate for fishing boats in Hull Bay was that about one in every twenty would suffer some structural failure within five years. Bram's boats did not. The reason, the harbour-master eventually decided after thirty years of watching Bram work, was that *the geometry was right.* Other shipwrights built boats that worked. Bram built boats that *had to work* — because every curve had been derived from a single principle, every angle had been constructed, every dimension had been the logical consequence of every other dimension. The boat was not assembled from parts. The boat was *constructed* from a small number of axioms.

When the GeometryForge academy was looking for someone to teach compass-and-straightedge constructions to children, the academy master had heard about Bram from a sea captain who had bought one of his boats. The sea captain said: *"He does not build boats. He builds proofs that happen to float."*

The academy master wrote Bram a letter. Bram, who was forty-one and beginning to think his bear-shoulders would not survive another decade of bending over a hull, accepted.

He brought his compass and his straightedge. He still has both. He calls the compass *the swing-arm,* because, he says, it swings around its center the way a gate swings around its hinge.

The children — always — protest. They ask how they can possibly draw anything accurate without measuring.

Captain Construction smiles. (Bear-headed smiles are slow and gradual but they are warm.) He says: *"You will see. The geometry will tell you what to do. The compass will tell you how far. The straightedge will tell you which way."*

He then shows them the first construction. Bisecting an angle. The method is older than the kingdom. The method is older than Bram's grandfather. The method does not require a ruler. The method does not require measurement. The method gives an angle bisector that is, *exactly,* a bisector.

The children try it. The bisector works. They check it with a protractor (the academy keeps protractors for verification; Bram tolerates them grudgingly). The bisector is exactly half of the original angle. Every time.

Captain Construction nods. He says: *"This is geometry. The compass and the straightedge are the only tools you need. Everything else follows."*

Listen along + meet more of the cast at:

`https://spark-and-anvil.com/cast/geometryforge/captain-construction`

Compass Wraith

LOCUS — the set of all points satisfying a given condition. Special case: the circle as the locus of points equidistant from a center.

- "E"
    - "S"
    - "W"

Chapter 7 — The Compass Wraith and the Silver Arcs

The Compass Wraith is, strictly speaking, *not alive.*

This is a thing that needs to be said at the start, because the other GeometryForge cast members are all — Master Hypotenuse, Sir Transverse, Apprentice Sides, Axia, Theora, Captain Construction, Lady Inscribed-Angle, Madame Polygon, Master Tangent — *alive.* They eat, they sleep, they go home for holidays, they have favourite teas. The Compass Wraith does not eat. The Compass Wraith does not sleep. The Compass Wraith does not have a favourite tea.

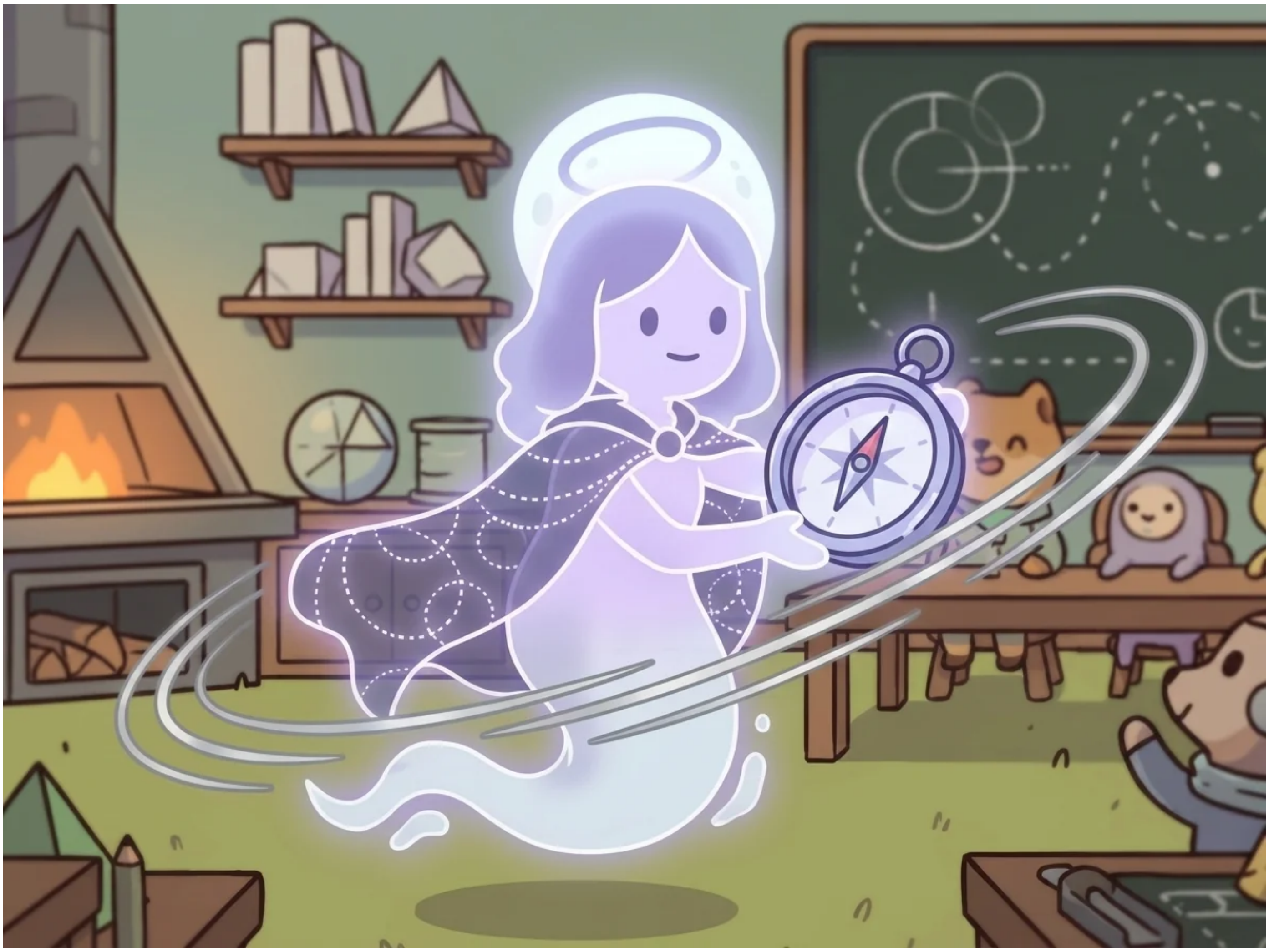
She is translucent — pale violet, with a moonlight halo. Her lower body fades into mist. She wears a cape woven from dotted lines, like the tracing-paths a compass leaves. She holds a glowing compass that traces silver arcs through the air. She moves without footsteps.

The other cast members accept this cheerfully.

Children find her, on the first day they meet her, *slightly thrilling.* They are not afraid of her. (She is, despite being a spirit, *kind.*) But they are aware that she is unusual. They watch her closely. They ask the other cast members about her. The other cast members shrug and say: *"She has always been here. The academy is older than any of us. The Compass Wraith was here before the academy was here."*

This is, as far as anyone has been able to confirm, *true.*

The Compass Wraith — whose name, the academy master has been told, is *Lune* (though she has never confirmed this) — appears in the geometry curriculum whenever a problem asks the question:



Not — and this is important — *"where is the point?"* The point's location is, in those problems, *not yet known.* The problem gives a condition. The condition is something like: *"a point five paces from this rock"* or *"a point that is equidistant from these two trees"* or *"a point that is closer to the river than to the road."* The problem then asks the student to draw, on a map or a diagram, *all the places the point could possibly be.*

The set of all such places is called *the locus.*

Most loci are circles, lines, or arcs of circles. (The locus of points five paces from a rock is a circle of radius five centered on the rock. The locus of points equidistant from two trees is a perpendicular line bisecting the line between the two trees. The locus of points closer to a river than to a road is a region — bounded by a parabola, but children do not need to know that yet.)

Working out a locus is not easy for children. It requires *thinking about all the possibilities at once* — which is a habit of mind children do not naturally have. They want to find *the* answer. They do not want to find *all possible* answers.

This is where the Compass Wraith comes in.

When a locus problem appears in a kit, the Compass Wraith *materializes.* She does this without warning. The classroom is in the middle of a discussion about some other topic. Suddenly the air shimmers, slightly. A pale violet figure appears at the front of the room. Her glowing compass is in her hand. She raises it. She turns slowly. The compass traces a silver arc through the air.

The Compass Wraith does not, usually, speak. She simply *shows.* The children watch the silver arc unfold. They see the set of all possible points. They understand, viscerally, what *locus* means.

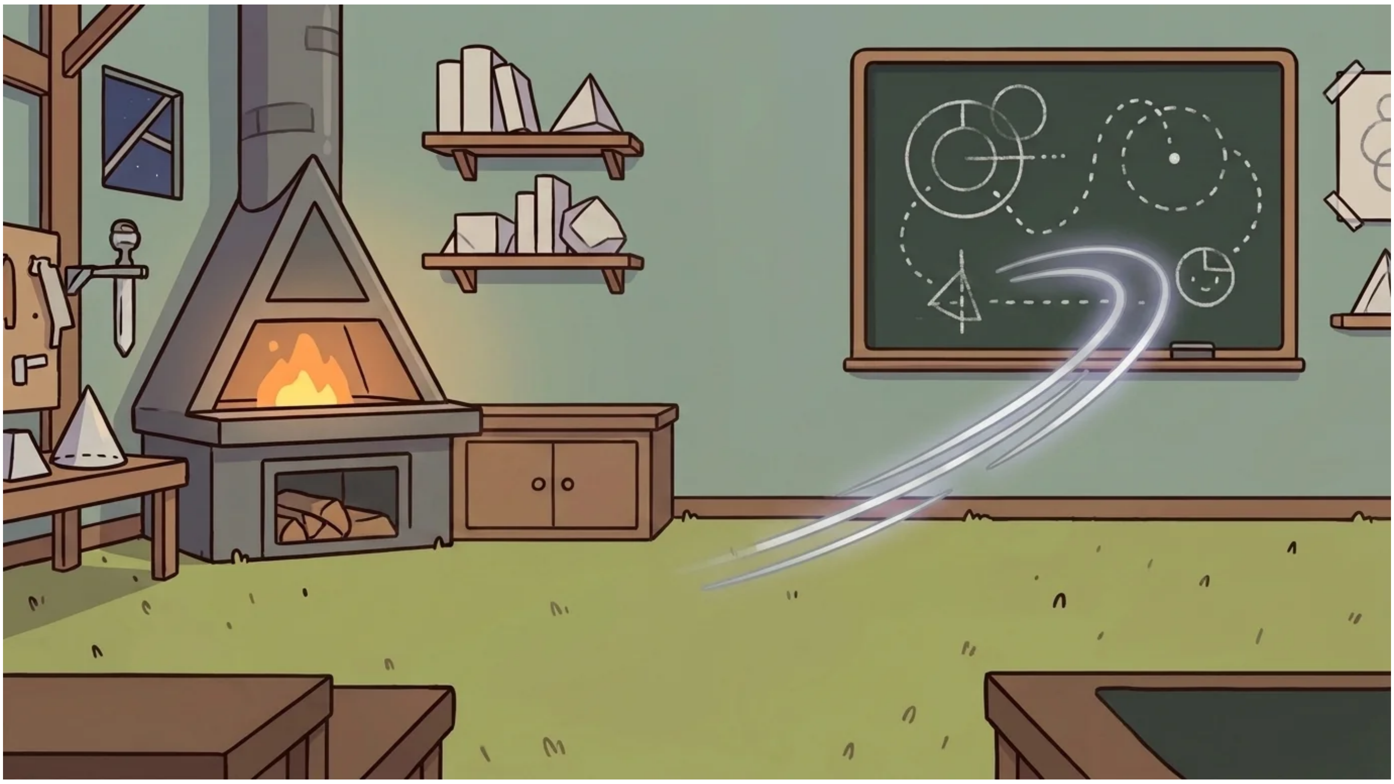
Sometimes she does speak. When she does, her voice is airy and slightly distant, but warm. She says things like:

"Every point that is equally far from here. I show you all of them at once."

Or:

"All the places where the bird could be hiding, given that you know how far it sang from. Watch."

And then the silver arc traces the answer.



Then she vanishes.

The other cast members are used to this. The other cast members will be in the middle of a sentence and the Compass Wraith will appear, do her work, and disappear, and the cast member will simply continue the sentence. (Sir Transverse, who is the most matter-of-fact of the cast, has been known to say *"Thank you, Lune"* without breaking his stride. The Compass Wraith, even mid-vanish, nods at him.)

Children eventually come to look forward to her arrivals. They ask: *"Will the Compass Wraith come today?"* And Lady Inscribed-Angle, who knows the curriculum's locus-problems by heart, will say: *"Yes. Around the middle of the lesson. She always comes when there is a circle to draw."*

The Compass Wraith does not, as far as anyone can tell, age. She has been doing this work — whatever exactly this work is, in whatever realm she inhabits when she is not appearing in classrooms — for as long as anyone at the academy can remember. She is the academy's quietest faculty member and also, the academy master sometimes thinks, *the most reliable.*

When children ask her — once, sometimes, on a brave day — whether she is really a ghost, the Compass Wraith always says the same thing. Her airy voice is patient. She says:

"I am the set of all points equidistant from this place to this place. I am the locus. I am the arc the compass would trace if you turned it forever. Some of me you can see. The rest of me you have to imagine."

Listen along + meet more of the cast at:

`https://spark-and-anvil.com/cast/geometryforge/compass-wraith`

Lady Inscribed-Angle

CIRCLE THEOREMS — inscribed-angle is half the central-angle subtending the same arc. The angle at the rim, half of the arc you see across.

Lady Inscribed-Angle grew up on the shore of *Lake Drumhead.*

This was, even by the standards of the kingdom — which has a great many small lakes — *an unusual lake.* It was, as far as anyone could measure, *perfectly circular.* Not approximately. Not roughly. *Perfectly.* The villagers of the surrounding village (which was also called Drumhead, because the village had been there longer than the kingdom had a habit of giving things distinct names) had measured the lake every generation for three hundred years. Every measurement agreed: a perfect circle, about half a mile across.

Lady Inscribed-Angle — whose given name, before the academy, was *Pell* — was born and raised on the rim of this lake.

The children of Drumhead had a game. It was old. Older than anyone could remember. It was called *the chord-walk.*

The game went like this: a child stood at a chosen point on the rim of the lake. The child picked two *other* points on the rim, anywhere they liked, and walked the arc from one to the other. While they walked, they kept their head turned so they were always looking at the chord — the straight line between the two endpoints they had picked. When they had walked the arc, they came back to the chosen starting point and the older children, who had been watching, would measure two things: the angle at which the starting child had been standing (between the two picked points), and the length of the arc the child had walked.

Then the older children would announce a number. Always the same kind of number. *Half.*

The arc walked was always *twice* the angle at the starting child's position. *Always.*

This was the game. It was not explained. It was a thing the children of Drumhead grew up with. It was, the villagers said, *what the lake taught.*

Pell was eleven when she started thinking about this seriously.

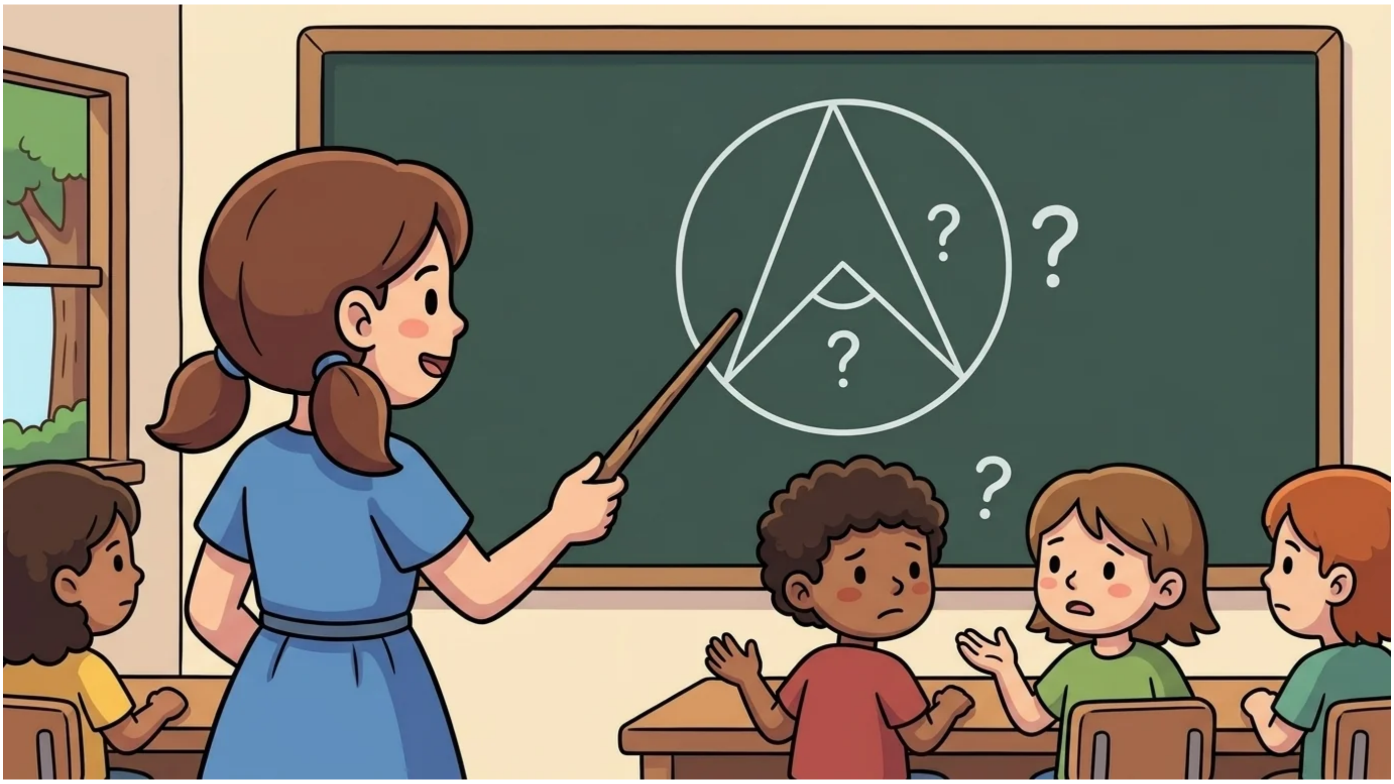
She walked the chord, by her own count, *three hundred and seventeen times* over the course of one summer. She tried it from every starting point on the rim. She tried it with chords short enough that the arc was just a tiny sliver, and chords long enough that the arc went more than halfway around the lake. She tried it standing on a rock, on a tree-stump, on her cousin's shoulders. The rule held. *The angle at the rim was always half the arc.*

She did not, at eleven, know why. She knew only *that.*

When she was sixteen, a travelling tutor came through Drumhead — a quiet woman named Mira who taught geometry to children in villages along the coast — and Pell asked her the question. Mira sat down on the rim of the lake. She drew three pictures in the wet sand. She said:

"The angle you stand at is called an inscribed angle. The angle a person standing at the center of the lake would measure to the same two points is called the central angle. The central angle is the same as the arc, just expressed as a number of degrees instead of a length. And the inscribed angle is always — exactly, every time, for any circle — half of the central angle."

Mira drew it. Pell watched. Pell understood.



She said: *"The lake has been teaching this to children for three hundred years. Nobody told us what it was called."*

Mira smiled. She said: *"The lake teaches very well. The names are just for the children who do not have a lake."*

Pell, who was sixteen, decided in that moment that she wanted to be the person who taught children-without-lakes what the lake had taught her. She studied with Mira for two years, then with the academy of GeometryForge for three more, then took the name *Lady Inscribed-Angle* (which, in the academy's tradition, is what you do when your character has fully embodied a single geometric primitive). She has been teaching ever since.

She still goes home to Drumhead twice a year. She still walks the chord. She still gets the same result.

When children arrive in her classroom for the first time, she always begins the same way. She draws a circle on the board. She draws a chord. She marks a point on the rim. She marks the center. She says: *"This is the inscribed angle. This is the central angle. Which is bigger?"*

The children — always — guess wrong the first time. They say the inscribed angle is bigger, because it looks closer to them.

Lady Inscribed-Angle smiles. She says: *"It looks bigger. It is half. The central angle is twice."*

Then she lets them measure it. Then she lets them try it from a different point on the rim. Then she lets them try it from a different chord. They always get the same answer.

She tells them, gently: *"This is a thing about circles. It is true for every circle. You do not need a lake to test it — but you do need a circle, and you need patience, and you need to walk the chord."*

When children ask whether the inscribed-angle theorem is hard, Lady Inscribed-Angle always says:

"It is not hard. It is only half. The angle at the rim is half the arc you see. Every time. Every circle."

She tilts her head, slightly, when she says it. She has, the academy children have noticed, *fox ears,* and the ears prick forward whenever a circle appears in a problem. She does not seem to do this on purpose. The circles, she says, simply *call to her.*

Listen along + meet more of the cast at:

`https://spark-and-anvil.com/cast/geometryforge/lady-inscribed-angle`

Madame Polygon

REGULAR POLYGONS — interior-angle sum is (n−2)·180°. Exterior-angle sum is always 360°. Regular n-gons have n-fold rotational symmetry. Some regular polygons tile the plane; some do not.

- "60"
  - "120"
  - "x"
  - "y"
  - "z"
  - "60°"
  - "120°"
  - "30°"
  - "90°"

Chapter 8 — Madame Polygon and the Council of Tessellation

Madame Polygon is *the spokesperson of the Polygon Council.*

This title sounds a bit silly at first. The Polygon Council is a group of *shapes.* Shapes don't usually talk. They don't have jobs. They certainly don't have meetings.

But the Polygon Council *does* meet. They meet in the kingdom's eastern hills. They have met there forever. The Council gathers in the town hall. It's in the village of *Tessellation*. Locals say polygons built this village. Not people.

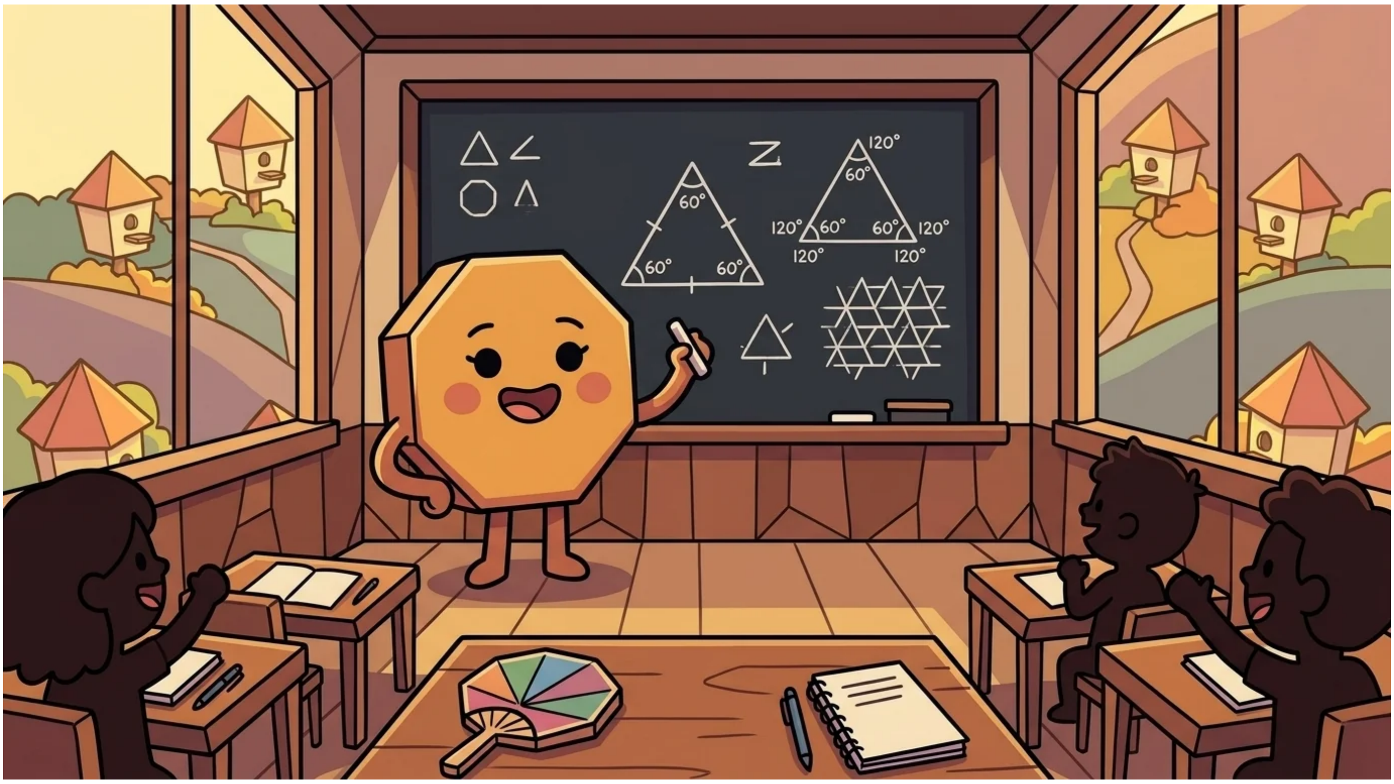
Madame Polygon grew up in Tessellation.

She was the oldest of three sisters. Her family name was Polygon. Her younger sister was Hexa. Hexa was very good with hexagons. Her youngest sister was Octavia. Octavia lived in the dodecagon town hall. She ran the regional tile shop. You won't meet Hexa or Octavia today. They pop up later in Kit 13.

Madame Polygon's real name is *Polly*. The academy kids find this out later. They always love it. Polly joined the Polygon Council at twenty-six. The Council needed a spokesperson. For years, they had none. The old one was a heptagon. A very proper seven-sided shape. He just wanted to relax. He liked his seven-sided life. He would not come back.

The Council needed someone who could *explain regular polygons to the world.*

Polly was the perfect choice. She had spoken for polygons since she was seven.



At twelve, she could do math on a slate. She found the *inside angle* for any shape. It was a special formula: *interior angle equals* $(n-2)\cdot180^\circ$ *divided by* n. She showed her younger cousins how it worked. You cut the shape into triangles. Just draw lines from one corner. It always made (n-2) triangles.

At sixteen, she knew even more. Only three shapes fit perfectly together. Triangles, squares, and hexagons. No gaps at all. Their angles fit exactly into a circle. She could draw all three tilings on a slate. Her chalk never left the slate.

The GeometryForge academy needed a teacher. Someone to teach shapes to kids. The Polygon Council picked Polly. Everyone agreed. Polly was twenty-seven. She had been spokesperson for one year. She said yes. She has taught there for fourteen years now.

Every morning, she comes to school. She wears her Council clothes. Fancy ones. She tells the children it's not showing off. It helps them learn. Her headdress has peacock feathers. Each feather eye is a tiny shape. A triangle. A square. A pentagon. A hexagon. A heptagon. An octagon. A nonagon. A decagon. A dodecagon. Nine shapes in all. You can count them.

Her dress has many shapes. Different ones. She carries a fan. It folds up small. With one flick, it opens wide. It becomes a perfect dodecagon. Twelve sides. She picked twelve sides on purpose. She says dodecagons are *underrated*.

In her classroom, she starts every first-day lesson the same way. She walks in. She puts her dodecagon-fan on the desk. She turns to face them. Her voice is big and dramatic. Like a Council meeting.

"The Council convenes. Each polygon has its angles, its symmetry, its place. Today we begin with the regular triangle. We will work our way up."

Then she teaches about the regular triangle. It has three sides. Inside angles are 60 degrees. Outside angles are 120 degrees. It can spin three ways. It fits together with other triangles. She teaches it slowly. Very grand. The children are confused at first. But they start to like it. By the end, they know many shapes. Triangle, square, pentagon, and hexagon. They can even say the interior-angle sum formula.

Lessons go on. Madame Polygon climbs the shape ladder. Pentagon. Hexagon. Heptagon. Octagon. She teaches about symmetry. She teaches about tiling. Why some shapes fit. Why some don't. She is patient. She is proper. She loves her job.

When children ask her if regular polygons are hard, Madame Polygon always says the same thing:

Listen along + meet more of the cast at:

`https://spark-and-anvil.com/cast/geometryforge/madame-polygon`

Master Hypotenuse and Apprentice Sides

SPECIAL-CASE-VS-GENERAL — Master Hypotenuse ($a^2 + b^2 = c^2$ for right triangles) + Apprentice Sides (Heron's formula for ANY triangle) — when one tool is enough, when you need the more general tool

The chapel roof had cracked at the western gable, and the masons needed measurements before they could carve the replacement stones. Master Hypotenuse arrived first, with a square and a plumb line. Apprentice Sides arrived second, with a tape measure and a wax-tablet.

The Master Mason met them at the base of the gable. He pointed up.

"Three triangle stones at the apex," he said. "Two are right triangles. One is not. I need the AREAS of all three so I can quote the price of the limestone. Can you measure them and give me the areas?"

Master Hypotenuse looked up at the gable. "I can do the two right triangles."

Apprentice Sides looked up at the gable. "I can do all three."

Master Hypotenuse glanced sideways at her, mildly insulted. "I can do all three too."

"Maybe. But you'll need a ladder to reach the angle of the third one. I won't."

The Master Mason raised an eyebrow. "Show me."

Master Hypotenuse went first. He set up his square and plumb line at the base of the first right-triangle stone. He measured the two legs of the right angle.

"This one is a right triangle," he said. "The legs are five hands wide and twelve hands tall. The hypotenuse — the side opposite the right angle — is the diagonal across the top. By the Pythagorean theorem, five squared plus twelve squared equals the hypotenuse squared. Twenty-five plus one hundred and forty-four is one hundred and sixty-nine. The square root of one hundred and sixty-nine is thirteen. So the hypotenuse is thirteen hands."

He wrote it down. *Right triangle 1: legs 5 and 12, hypotenuse 13.*

"And the area," he said. "For a right triangle, the area is one-half times leg times leg. One-half times five times twelve is thirty. So the area is thirty square hands."

He moved to the second right triangle. He measured its legs.

"Eight and fifteen. Eight squared is sixty-four. Fifteen squared is two hundred twenty-five. Sum is two hundred eighty-nine. Square root is seventeen. So the hypotenuse is seventeen, and the area is one-half times eight times fifteen, which is sixty."

He wrote it down. *Right triangle 2: legs 8 and 15, hypotenuse 17. Area 60.*

He came down to the ground. "Two right triangles, areas 30 and 60. The third one is the trouble. Its angles aren't ninety. I can climb up and measure the angle and then drop a perpendicular from the apex — but that means a ladder and a plumb line and probably an hour."

Apprentice Sides looked at him. "You don't need any of that."

Apprentice Sides climbed nimbly up to the gable's edge. She did not measure any angles. She did not drop any perpendiculars. She just measured the three SIDES of the third triangle with her tape measure.

"Side one: nine hands. Side two: ten hands. Side three: eleven hands."

She came back down. She wrote the three numbers on her wax-tablet.

"For any triangle whose three sides are a, b, c — even a non-right triangle — there is a formula for the area in terms of just the three sides. It's called Heron's formula. First you compute the semi-perimeter, which is half the sum of the three sides."

She wrote: $s = (a + b + c) / 2 = (9 + 10 + 11) / 2 = 30 / 2 = 15.$

"Then the area is the square root of s times s-minus-a times s-minus-b times s-minus-c."

She wrote: $Area = \sqrt{(15 \times (15-9) \times (15-10) \times (15-11))} = \sqrt{(15 \times 6 \times 5 \times 4)} = \sqrt{(1800)}.$

The square root of one thousand eight hundred was a little messy. She did the arithmetic carefully on the wax-tablet. *About forty-two-and-a-half square hands.*

She handed the wax-tablet to the Master Mason. "Area of the third triangle is about forty-two-point-four square hands. No ladder. No plumb line."

The Master Mason stared at the wax-tablet for a long moment.

"You just got an area from three side-lengths."

"Yes."

"Without any angle measurement."

"Yes."

He looked at Master Hypotenuse. "Do you know Heron's formula?"

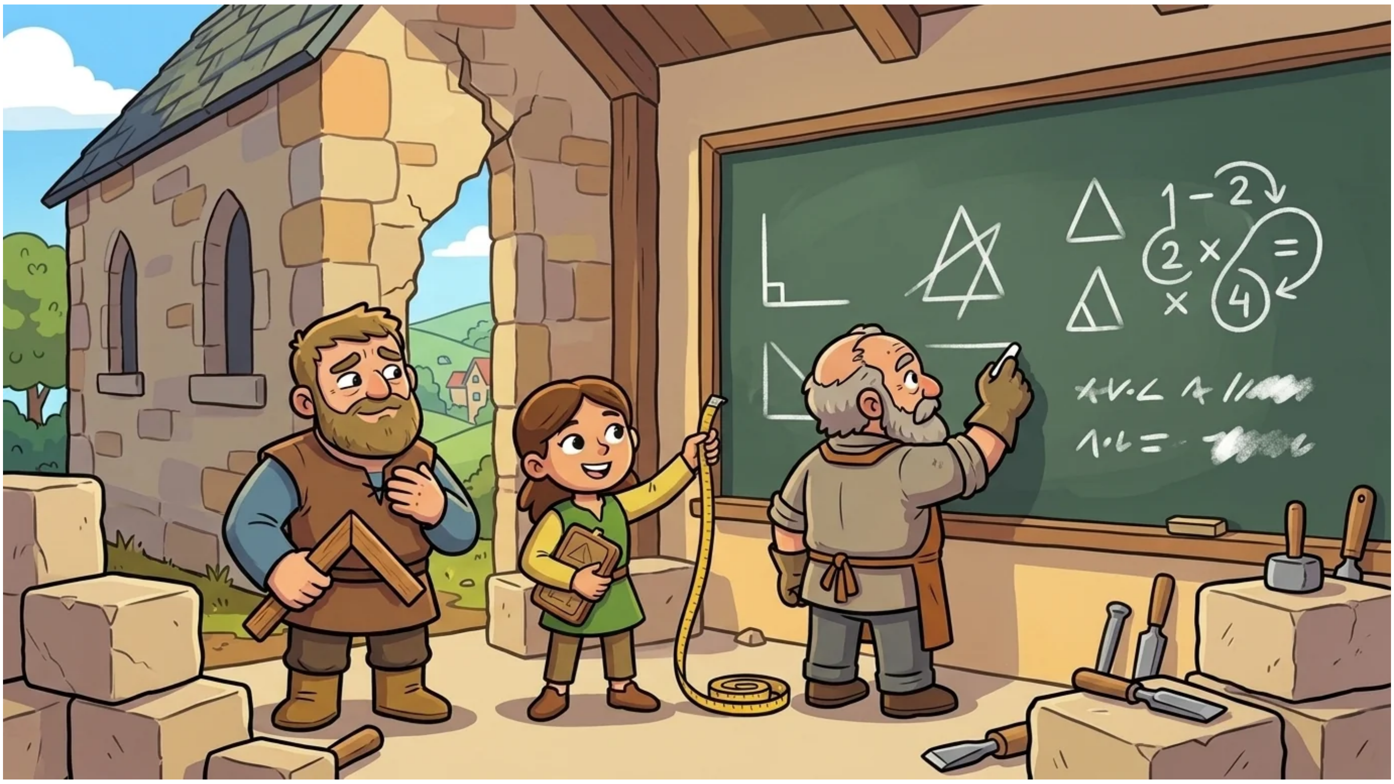
"I know it exists," Master Hypotenuse said, slightly grudging. "I have always taught the right-triangle case because it's simpler. The Pythagorean theorem is one of the first things kids learn. Heron's formula is heavier; it has a square root nested inside a product, and most of the arithmetic gets messy."

"Right," Apprentice Sides said. "Yours is the cleaner tool for the easy case. Mine is the messier tool for any case."

The Master Mason wrote down the three areas. "Thirty. Sixty. Forty-two-point-four. I have everything I need."

He paused.

"There's a lesson in this for the apprentices," he said. "Which one of you tells it?"



Master Hypotenuse and Apprentice Sides looked at each other.

"You go," Master Hypotenuse said.

Apprentice Sides nodded. "When the triangle has a right angle, his tool is fastest. The arithmetic is light. You don't need a tape measure and a wax-tablet and a semi-perimeter formula. You just need two leg-lengths and the Pythagorean theorem."

"And when the triangle doesn't have a right angle..."

"You need mine. Heron's formula handles ANY triangle. The cost is the heavier arithmetic. The benefit is you don't need any angles at all. Just side-lengths."

"So the rule is," Master Hypotenuse said slowly, "use the simpler tool when the special-case condition holds. Use the general tool when it doesn't."

"Use the simpler tool first," Apprentice Sides said. "Switch to the general tool when the special case fails."

"Same idea told twice."

"Same idea told twice."

The Master Mason wrote both rules on the chalk wall of the workshop. The masons read them later. The chapel roof was repaired by the end of the week.

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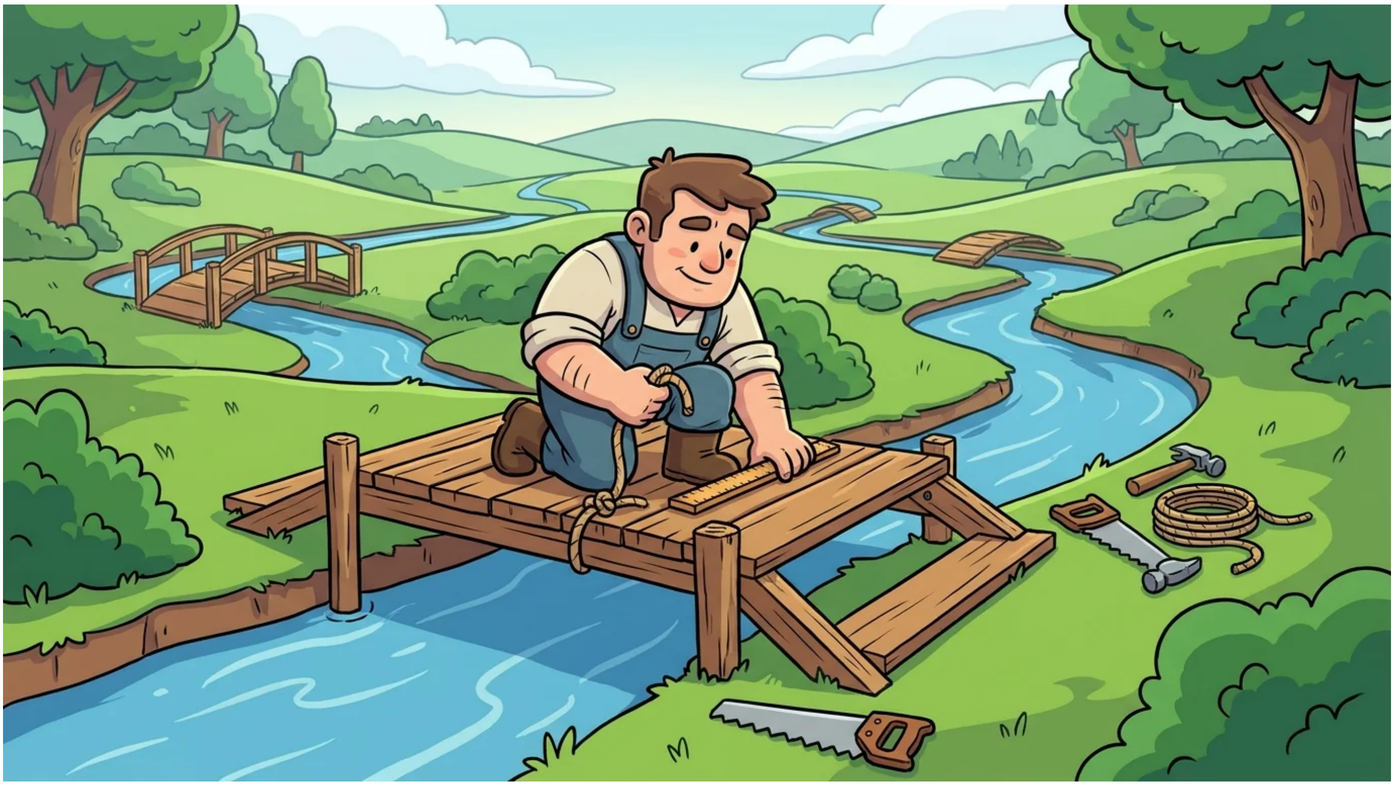
https://spark-and-anvil.com/cast/geometryforge/master-hypotenuse-apprentice-sides

Master Hypotenuse

RIGHT-TRIANGLE RELATIONS — $a^2 + b^2 = c^2$. The square on the longest side equals the sum of the squares on the other two.



Master Hypotenuse was, before he was a teacher, *a builder of small bridges.*

This is not a metaphor. He built actual bridges — wooden footbridges across small streams in his home valley of Crossing, which is a valley that has more small streams than it has roads. The valley needed bridges. Master Hypotenuse, for sixteen years between the age of seventeen and the age of thirty-three, built them.

His method of building bridges was — even by the standards of the valley, where everyone had opinions about bridges — *unusually careful.*

He always carried, on a leather thong over his shoulder, *a knotted rope.* It was a particular rope. It had been his grandfather's. It was about thirty feet long. It had twelve knots in it, evenly spaced, so that the spaces between the knots were equal. His grandfather had told him: *"This rope is for getting things right. Use it for everything that matters."*

What Master Hypotenuse used it for, mostly, was *making right angles.*

To build a footbridge, you needed two abutments — one on each side of the stream — and they had to be square to the stream. Square. Right-angled. Not approximately, not roughly, not by-eye. *Square.* If your abutment was off by even a few degrees, your bridge planks did not sit flat. Water collected. Wood rotted. The bridge, two winters later, sagged.

Master Hypotenuse's trick with the rope was old. It was older than him. It was older than his grandfather. It went like this:

You laid the rope out in a triangle. You used three of the segments for one side — three knot-lengths. You used four segments for another side. You used five segments for the third side. You closed the triangle. *The angle between the side of three and the side of four was always, exactly, ninety degrees.* Every time. Without measuring with anything else. Without any instrument. Just the rope.

This trick — Master Hypotenuse, in his twenties, would explain to anyone who asked — was because *three squared plus four squared equals five squared.* 9 + 16 = 25. The numbers fit. The triangle had to be right-angled.

He spent his bridge-building years quietly turning this fact over in his head.

What he eventually understood — and it took him most of those sixteen years, because he was not a quick man, only a patient one — was that the rope-trick was a single instance of *a much larger pattern.* The same was true for 5-12-13. The same was true for 8-15-17. The same was true for *any* three numbers a, b, c where $a^2 + b^2 = c^2$. You laid out a triangle with those side-lengths and the angle opposite c was, *exactly*, a right angle.

And — and this was the part that made him sit down on the bank of a stream one summer evening and stay there until dark — the reverse was also true. *Any* right triangle, no matter what shape, had this property: *the square on the longest side equals the sum of the squares on the other two.* The rope was just one example. The principle was universal.

Master Hypotenuse, on that evening, did not name what he had understood. He did not call it the Pythagorean theorem (which was the name a different tradition, in a different valley, had given it many centuries before). He simply sat by the stream and thought: *the right angle is hiding in the square. The square is hiding in the right angle. Every right triangle is the same right triangle, just stretched.*

He went back to building bridges. But he built them, after that summer, with even more pleasure than before. Every bridge was a small instance of a universal pattern. Every right-angled abutment was a tiny demonstration.

When the EquationQuest academy (which has a sister school for geometry, called the GeometryForge academy, in a nearby valley) was looking for someone to teach right-triangle relations to children, the valley's bridge-builders' guild sent Master Hypotenuse's name. He had, by then, been building bridges for sixteen years and had become slightly famous in the region for the fact that none of his bridges had ever sagged. The academy master wrote him a letter. Master Hypotenuse, who was thirty-three and beginning to think his back needed a less wet line of work, accepted.

He arrived at the academy carrying the knotted rope.

He still carries it. It is the first thing he shows children in their first lesson on the Pythagorean theorem. He lays it out on the classroom floor. He counts off three knots, then four, then five. He pulls the rope taut into a triangle. He says: *"Look at the angle between the three-side and the four-side. That is a right angle. The rope made it for me."*

Children stare. Children try it themselves. Children get it.

He adds, gently: *"Three squared plus four squared equals five squared. That is why. The numbers know what they are doing. The rope is just helping them show it."*

When children ask him whether the Pythagorean theorem is hard, Master Hypotenuse always says the same thing:

"It is not hard. It is only patient. You square the two short sides. You add them. The answer is the square of the long side. Every right triangle agrees."

He holds up the knotted rope. It is fraying at the ends. He has had it for forty years.

He says: *"This rope has built thirty-seven bridges. None of them sagged."*

Listen along + meet more of the cast at:

https://spark-and-anvil.com/cast/geometryforge/master-hypotenuse

Master Tangent

TANGENT — a line that touches a circle at exactly one point, never crossing. Also: the limit of a sequence of secants. Also: in trigonometry, the ratio opposite/adjacent in a right triangle.

Master Tangent grew up in *a monastery on a cliff above the sea.*

The monastery — which is called, simply, *Sea-Cliff Monastery*, because nobody has ever come up with a better name — sits on the very edge of a tall basalt cliff overlooking the cold-water bay of *Northshore.* The cliff is about three hundred feet high. The bay is below it, far below it. The sea is grey most of the year. The wind, even in summer, is cold.

The monks of Sea-Cliff Monastery had — and still have — *a daily practice.*

At sunset, every monk who is well enough to walk goes outside. They walk, single-file, along the very edge of the cliff. Their right shoulders are toward the sea. Their left shoulders are toward the monastery. The path along the edge is no wider than a single footstep. The drop, to their right, is three hundred feet straight down.

They walk *along the edge.* They do not walk *across* it.

This is the practice. It is older than the monastery, the monks say. It is older than the kingdom. It might be older than the cliff itself, though that of course is impossible.

The point of the practice is *the touching-without-crossing.* The monks walk where the cliff and the sky meet. They place each foot at the boundary. They do not stray inward (which would be safe but uninstructive). They do not stray outward (which would be terminal). They walk *along the line of touch.*

Master Tangent — whose given name was *Heron* (though he stopped using this name when he took his monastic robes at sixteen, because Heron was the name of a Mediterranean mathematician with no particular connection to him and the local children were beginning to make jokes about it) — joined the monastery when he was twelve. His family had sent him because he had been, even as a small child, *unusually still.* He could sit for hours without moving. He could watch the sea for an entire afternoon. The local sage said: *"This boy has the cliff-walking temperament."*

He did. He walked the cliff every sunset from the age of twelve to the age of forty. That is twenty-eight years of cliff-walking. Twenty-eight years of placing each foot at the boundary between rock and sky. Twenty-eight years of touching without crossing.

What Master Tangent eventually understood — and he understood it not in any single moment but over the slow accretion of those twenty-eight years — was that *the cliff-edge was a geometric primitive.* It was a line. It was the boundary between two regions. To walk it was to approach the boundary *as the limit of a series of approximations.* Each step was an approximation. Each step was slightly off — slightly inward, slightly outward, slightly off-balance. But the *average* of the steps, taken over a long enough walk, was the line itself. The monks were, by their practice, *converging* on the cliff-edge. They were tracing, with their bodies, *the tangent.*

This was the principle.

A tangent line to a circle is a line that touches the circle at exactly one point and does not cross it. A tangent line can be derived as the *limit* of a sequence of secants — lines that cross the circle at two points, where the two points are getting closer and closer together. As the two points merge, the secant becomes a tangent. The crossing-line becomes a touching-line.

Master Tangent saw this for the first time when he was thirty-one. He was reading a borrowed geometry book in the monastery library. The book had a diagram showing a sequence of secants approaching a tangent. He looked at the diagram for a long time. Then he set down the book and walked out to the cliff-edge.

He thought: *the cliff-edge is the tangent. The walking-paths I have made over twenty years are the secants. Each path crossed in slightly; each path crossed out slightly. The average of all my paths is the tangent line. I have been doing this exercise all along.*

He did not, at the time, know that he would eventually leave the monastery to teach this. But nine years later, when the GeometryForge academy was looking for someone to teach tangent-to-circle problems to children, the academy master had heard about the cliff-walking monks. He travelled to Northshore. He spoke with Master Tangent (who was by then forty and was beginning to think his knees would not survive another decade of cliff-edge balance). He invited him to teach.

Master Tangent considered the invitation for two months. He spoke with the abbot. The abbot said: *"The cliff-edge has taught you what it can. The children may need you more than the edge does."*

Master Tangent accepted.

He brought, to the academy, *one straight reed.* He cut it himself from the marsh below the monastery before he left. He still has it. It is about three feet long. It is perfectly straight. He uses it in class. He places it against a chalk-drawn circle on the board. The reed touches the circle at exactly one point. It does not cross.

He says: *"This is a tangent. The reed touches the circle. The reed does not cross. That is the whole trick."*

The children — always — protest. They want to know how you *find* the tangent. They want a method.

Master Tangent smiles. He is a whip-thin heron-headed character in a long pale-grey robe. His smile is dry and slight. He says: *"The method is patience. You start with a secant — a line that crosses. You move the two crossing points closer together. When they merge into one point, the line touches without crossing. That is the tangent. The limit of the approach."*

The children try it. They draw secants. They move the crossing points together. The secant rotates. As the crossing points merge, the line settles. The settled line is the tangent.

Master Tangent watches them. He says, in his soft monastic voice: *"This is what I learned on the cliff-edge. The line you cannot quite reach. The line you can only approach. The tangent. Every circle has them everywhere. Every point on a circle has one. You touch. You do not cross."*

When children ask whether tangent problems are hard, Master Tangent always says the same thing:

*"They are not hard. They are *delicate.* You touch the circle. You do not cross it. Find the radius to the touching-point. The tangent is perpendicular to the radius there. Always. Every circle."*

He still walks the cliff-edge once a year. He goes back to the monastery for the autumn equinox. He walks the sunset path. He places each foot at the boundary.

He has, after forty-five years of cliff-walking, *never crossed.*

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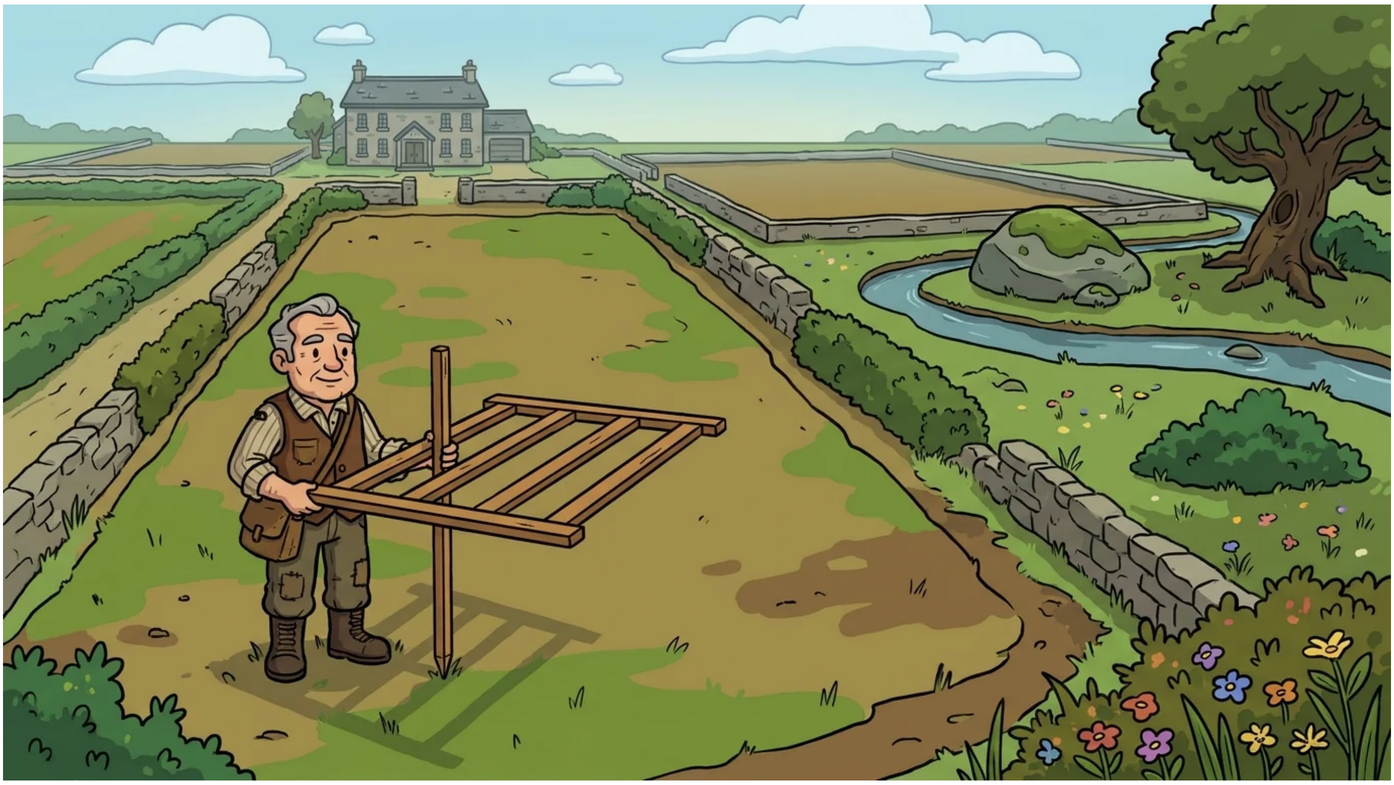
Sir Transverse

PARALLEL-LINE TRANSVERSALS — when a transversal cuts two parallel lines, corresponding angles are equal; alternate interior angles are equal; the intercept theorem holds (proportional segments).



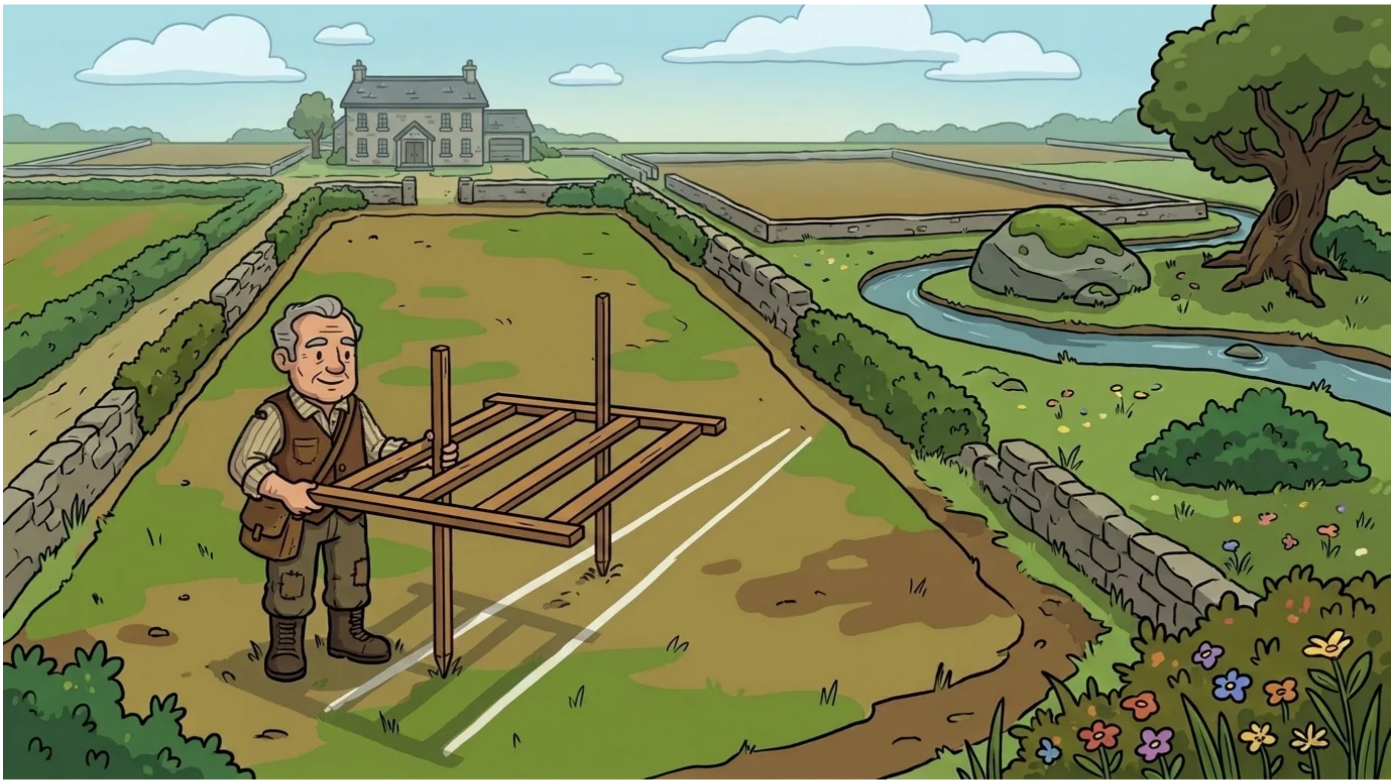
Sir Transverse measured fields. He did this for thirty years.

He worked for the kingdom's land office. It was a quiet government place. The office sat in a sleepy stone building. His job was to divide fields. He had to make sure they were *fair*.

This was not as simple as it sounds.

The fields of the kingdom were, mostly, rectangles. Some were longer than they were wide. Some were squarer. Some fields had funny corners. A stream might cut them off. Or a big rock. Maybe an old tree stood there. It was there before the field. But most fields had four sides. They had two long sides and two short sides.

The trouble was that *the long sides were parallel*.



Families often needed to divide fields. Maybe parents had died. Then brothers and sisters shared. Neighbors might argue over a line. Sometimes friends owned land together. They always wanted a *fair* share.

Fairness in fields meant one thing. It meant: each owner got a strip. The strips had to be the same *proportional* width. This was measured along the parallel sides.

If a field was 300 paces long, and three kids shared it, they wanted three strips. Each strip was 100 paces wide. If a field was 240 paces long, and four kids shared it, they wanted four strips. Each strip was 60 paces wide. Simple.

But fields are not always nicely lined up north-to-south. Sometimes the parallel sides are not even *the longest* sides. Sometimes they cut diagonal strips. Maybe a strip needed to touch the road. Or reach the stream. Each sibling wanted a piece like that. And then *the math becomes interesting*.

Sir Transverse was thin. He had long, stork-like legs. Everyone called him *Sir*. He had been *Sir* since he was six. It was a family nickname. No one knew why.

When he was nineteen, he found something out. It was his second year as a surveyor. He saw that if you cut a field with a diagonal line. He called this a *transversal*. It went across two parallel boundary lines. Then he cut another diagonal line. This second line was parallel to the first. The strip between these two diagonals always had a special width. This width was *proportional* to how far the strip was from the main parallel lines.

This was the *intercept theorem*. It's a very old math rule. Sir Transverse, of course, did not know it had a name. He simply observed it. He measured it on every field he surveyed for the next four years. It held. Every time.

By age twenty-three, he was a master. He walked into any field. He saw the people arguing. He asked one question: *"How do you want the strips to touch the road?"* In ten minutes, he laid out fair strips. He used only his measuring-staff. And a knotted cord.

He never measured the whole field. He never figured out its total area. He didn't need to. He just cut parallel lines. He used parallel *transversals*. The proportions worked themselves out.

The people arguing always left happy. This made Sir Transverse, by age thirty, famous. Everyone wanted him. He was the busiest surveyor in three provinces.

He spent thirty years doing this work. He divided many fields. He counted them carefully. One thousand, four hundred, sixty-two fields. He never had a complaint. He never had a re-survey. He never, even once, made a strip a half-pace too wide or too narrow.

His friends at the land office had a saying about him. They joked that he had *the soul of a ratio*. It was a funny thing to say, but they meant it.

The GeometryForge academy needed a teacher. They wanted someone to teach *proportional reasoning*. This meant the *intercept theorem*. And how *transversals* cut parallel lines. The academy master heard about Sir Transverse. His nephew told him. The nephew had been in a tough family field fight. Sir Transverse had solved it.

The nephew said, "He is amazing!" He added, "He makes fair division a math rule. Not just an argument."

The academy master wrote Sir Transverse a letter. Sir Transverse was forty-nine. His knees were getting old. He worried about walking in the rain. He accepted the job.

He brought his measuring-staff and a coil of knotted cord. He still has both. He keeps them in the corner of his classroom.

He teaches the *intercept theorem* his own way. He teaches it by walking. He lays a long cloth strip on the floor. It has two parallel lines marked with chalk. He picks two students. He gives them each a length of red string.

He says, "You are a *transversal*." He tells them, "Walk across the cloth. Hold your string tight. Choose any angle, but stay straight."

The students walk. The red strings cross the parallel lines at angles. Sir Transverse watches.

Then he asks them to walk a *second* transversal. This one must be parallel to the first. The class measures the strip between them. The strip is always *proportional*. Always.

No matter the angle, the *ratio* always holds. The strip's width compared to its distance from the parallel lines stays the same.

The children gasped. Their eyes went wide. They had never seen math work like magic before. Sir Transverse just smiled. He had felt the same way at nineteen. He was patient with their wonder.

He says gently, "The *transversals* were straight. The boundaries are parallel. The proportions just work. You didn't even have to do math."

He adds, *"Geometry, when the lines agree, is fair."*

Kids ask if the *intercept theorem* is hard. Sir Transverse always says the same thing:

"It is not hard. It is only fair. Cross two parallel lines with a *transversal*. The *ratio* holds. Cross them with two parallel *transversals*. The strip between is *proportional*. Every time."

He still has his measuring-staff. Children sometimes ask to hold it. He always lets them.

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Methodology

Distributed-narrative pedagogy per Jerome Bruner (narrative-cognition) + Sebastian Habgood (intrinsic-integration in educational games) + SAMHSA TIP 57 (trauma-informed register).

Trauma-informed-design framework per Eggleston et al. (2025) and Stoltenburg et al. (2024).

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