# RatioRealm — Flashcards

*A Spark & Anvil printable flashcard deck. Quiz yourself on the RatioRealm cast's curriculum — one card at a time.*

---

## How to use these cards

**Fold** each sheet down the middle line so the answers are hidden. Read the **question** on the left, say your answer out loud, then **unfold** to check the right side. Testing yourself — then checking — is one of the most powerful ways to remember. Get one wrong? That's the best kind of practice: try it again tomorrow.

*Want real flashcards? Cut along the fold line and the rows into cards — question on one side, answer on the other.*



---

# 1. Ratios

| ✂ Question (fold →) | Answer |
|---|---|
| A recipe calls for 3 cups of water for every 1 cup of rice. If you use 4 cups of rice, how many cups of water do you need? | **12** — With a 3:1 ratio, multiply both by 4: rice = 1×4 = 4 cups and water = 3×4 = 12 cups. The answer 7 incorrectly adds 3+4. Always multiply both parts by the same scale factor. |
| Evaluate whether the ratio 6:9 or 8:12 better represents 'two-thirds'. Explain your reasoning. | **Both represent two-thirds because 6:9 = 2:3 and 8:12 = 2:3** — Both ratios simplify to 2:3, so both represent two-thirds equally well. 6÷3=2 and 9÷3=3; 8÷4=2 and 12÷4=3. Equivalent ratios represent the same relationship regardless of the actual numbers used. |
| A bag has 4 red marbles and 6 blue marbles. What is the ratio of red to total marbles? | **4:10** — The total is 4 + 6 = 10 marbles. The ratio of red to total is 4:10. The ratio 4:6 is part-to-part, not part-to-whole. |
| A jar has 7 red and 3 green candies. What is the part-to-whole ratio of green candies to all candies? | **3:10** — The total is 7 + 3 = 10 candies. Green to total = 3:10. The ratio 3:7 is part-to-part (green to red). The ratio 7:10 is the part-to-whole ratio for red, not green. |
| In a class of 30 students, 12 wear glasses. What fraction of the class wears glasses? | **12/30 or 2/5** — The part-to-whole ratio is 12/30, which simplifies to 2/5 by dividing both by 6. The fraction 12/18 compares glasses-wearers to non-glasses-wearers, which is part-to-part. |
| In a class of 12 boys and 18 girls, what is the ratio of boys to girls in simplest form? | **2:3** — Dividing both 12 and 18 by their GCF of 6 gives 2:3. The ratio 3:2 reverses the order, 12:18 is not simplified, and 6:9 is only partially simplified. |
| Write the ratio of 15 to 25 as a fraction in simplest form. | **3/5** — Dividing numerator and denominator by 5 gives 15/25 = 3/5. The fraction 5/3 inverts the ratio. 15/25 is correct but not simplified. 5/25 equals 1/5, which is wrong. |
| A garden has 8 rose bushes and 12 tulip plants. Express the ratio of roses to tulips using a colon. | **8:12** — The ratio of roses to tulips is written as 8:12 using a colon (or 2:3 in simplest form). However, the question asks for the direct colon notation, which is 8:12. 12:8 reverses the order. |
| Why is the order important in a ratio? | **Changing the order changes which quantity is being compared to which** — The order in a ratio matters because it specifies which quantity is compared to which. A ratio of 3:5 means something different from 5:3 — the first represents 3 of one thing per 5 of another. |
| A lemonade recipe uses a 4:1 ratio of water to lemon juice. Which combination follows this ratio? | **8 cups water and 2 cups lemon juice** — 8:2 simplifies to 4:1 (dividing both by 2). The option 4:2 = 2:1, 5:1 stays as 5:1, and 8:4 = 2:1. Only 8 cups water and 2 cups juice maintains the required 4:1 ratio. |
| In a parking lot with 20 cars and 5 trucks, what is the part-to-part ratio of trucks to cars? | **1:4** — Trucks to cars = 5:20 = 1:4 after dividing by 5. The ratio 4:1 reverses the order. The ratio 5:25 incorrectly uses the total. Always match the order stated in the question. |
| A team has 9 boys and 6 girls. | **No, it should be 3:2** — Dividing both by 3: 9:6 = 3:2, not 2:3. Sarah reversed the |

| | |
|---|---|
| Sarah says the ratio of boys to girls is 2:3. Is she correct? | order. Boys are listed first, so the correct simplified ratio is 3:2, meaning 3 boys for every 2 girls. |
| A necklace uses red and blue beads in a 2:5 ratio. If there are 10 red beads, how many blue beads are there? | **25** — Since 10 ÷ 2 = 5, multiply both parts by 5: red = 2×5 = 10 and blue = 5×5 = 25. This maintains the 2:5 ratio with 10 red and 25 blue beads. |
| A school has a student-to-teacher ratio of 20:1. If there are 5 teachers, how many students attend the school? | **100** — With a 20:1 ratio, 5 teachers means 20 × 5 = 100 students. The answer 25 incorrectly adds 20+5. The answer 105 incorrectly adds the teachers to the students. |
| A basket has 3 apples and 5 oranges. What is the ratio of apples to oranges? | **3 to 5** — A ratio compares two quantities in order. Since apples are mentioned first, the ratio is 3 to 5. The ratio 5 to 3 reverses the order, while 3 to 8 and 8 to 3 mix in the total. |
| In ratio notation, what does the colon (:) represent? | **The word 'to'** — The colon in ratio notation stands for the word 'to', so 3:5 is read as '3 to 5'. While ratios relate to division, the colon specifically represents the comparison relationship 'to'. |
| A recipe uses 2 cups of flour for every 1 cup of sugar. What is the ratio of flour to sugar? | **2:1** — The ratio of flour to sugar is 2:1 because for every 2 cups of flour there is 1 cup of sugar. The order matters: flour is listed first. |
| Design a real-world scenario where the part-to-whole ratio is 1:4. What could the quantities represent? | **1 out of every 4 students chose pizza as their favorite lunch** — A 1:4 part-to-whole ratio means 1 out of every 4 items belongs to the subgroup, like 1 in 4 students choosing pizza. The option '4 cats and 1 dog' is part-to-part (1:4 dog-to-cat), not part-to-whole. |
| How can you tell the difference between a part-to-part ratio and a part-to-whole ratio? | **Part-to-whole compares a subset to the entire group; part-to-part compares two subsets** — A part-to-part ratio compares two subgroups (like boys to girls), while a part-to-whole ratio compares one subgroup to the entire group (like boys to all students). They convey different information about the same data. |
| A map shows that 1 inch represents 5 miles. Two towns are 3 inches apart on the map. How far apart are they in real life? | **15 miles** — Using the scale ratio 1 inch : 5 miles, multiply: 3 inches × 5 miles/inch = 15 miles. The answer 8 incorrectly adds 5+3. The answer 30 incorrectly multiplies 5 by an extra factor. |
| A pizza has 8 slices: 3 pepperoni, 2 mushroom, and 3 cheese. What is the part-to-whole ratio of mushroom slices? | **2:8** — There are 8 total slices. Mushroom to total = 2:8 (or 1:4 simplified). The ratio 2:6 and 2:3 incorrectly use only some slices as the whole. Always use the complete total for part-to-whole. |
| A fruit bowl has 6 bananas and 4 grapes. What is the part-to-part ratio of bananas to grapes in simplest form? | **3:2** — The ratio 6:4 simplifies to 3:2 by dividing both by the GCF of 2. This is part-to-part because it compares bananas directly to grapes, not to the total of 10 fruits. |
| Which of these is NOT a valid way to write the ratio '3 to 7'? | **3/7 or 7\3** — The three standard ways to write a ratio are 'a to b', 'a:b', and 'a/b'. Writing '7\3' reverses the order and uses an invalid notation, so it is not a correct representation of 3 to 7. |
| | **Two quantities** — A ratio is a comparison of two quantities that shows the relative |

| | |
|---|---|
| What does a ratio compare? | size of one quantity to another. It does not compare operations, equations, or shapes. |
| If the ratio of cats to dogs is 5:3, which fraction represents this ratio? | **5/3** — The ratio 5:3 means 5 cats for every 3 dogs, which as a fraction is 5/3. The fraction 3/5 reverses the comparison. 5/8 and 3/8 use the total as denominator. |

## 2. Ratios

| ✂ Question (fold →) | Answer |
|---|---|
| A student simplifies 14:21 as 7:10. Identify the error and provide the correct simplification. | **The student subtracted 7 from each instead of dividing; the correct answer is 2:3** — The student subtracted 7 from each (14-7=7, 21-7=10), which is wrong. Correct method: divide by GCF=7. 14÷7=2 and 21÷7=3, giving 2:3. Ratios are simplified by division, never subtraction. |
| Generate three ratios equivalent to 5:2. | **10:4, 15:6, 20:8** — Multiplying both parts: ×2 gives 10:4, ×3 gives 15:6, ×4 gives 20:8. All simplify back to 5:2. The option 15:8 does not simplify to 5:2 (it would need 15/5=3 and 8/5=1.6, which is not whole). |
| If 6:? = ?:15, and both blanks use the same ratio multiplier, what are the two missing numbers? | **6:10 = 9:15** — If the base ratio is 3:5, then 6:10 (×2) and 9:15 (×3) are both equivalent to 3:5. Check: 6/10=3/5 and 9/15=3/5. Both ratios express the same relationship. |
| Are the ratios 12:16 and 9:12 equivalent? Explain. | **Yes, both simplify to 3:4** — 12:16 ÷ 4 = 3:4 and 9:12 ÷ 3 = 3:4. Since both simplify to the same ratio (3:4), they are equivalent. Two ratios are equivalent when they reduce to the same simplest form. |
| Explain why 5:10 and 1:2 represent the same ratio but 5:10 and 10:5 do not. | **5:10 simplifies to 1:2 by dividing by 5, but 10:5 reverses the comparison direction** — Dividing 5:10 by 5 gives 1:2, so they are equivalent. However, 10:5 = 2:1, which reverses the relationship — it means 2 of the first per 1 of the second, the opposite of 1:2. |
| A ratio table starts with 1:6. What is the fifth entry in the table? | **5:30** — Each entry multiplies both parts by the row number. The fifth entry: 1×5 = 5 and 6×5 = 30, giving 5:30. A ratio table generates equivalent ratios systematically. |
| Build a ratio table for 7:3 with four entries. What is the sum of all second-column values? | **30** — The four entries are 7:3, 14:6, 21:9, 28:12. The second-column values are 3, 6, 9, 12. Their sum is 3+6+9+12 = 30. Each value is a multiple of 3. |
| Which of these ratios is already in simplest form? | **7:9** — 7 and 9 share no common factor other than 1 (7 is prime, $9=3^2$), so 7:9 is in simplest form. 4:8=1:2, 6:15=2:5, and 10:25=2:5 can all be simplified further. |
| The ratio of fiction to nonfiction books on a shelf is 5:3. There are 40 books total. How many are fiction? | **25** — Total parts = 5+3 = 8. Each part = 40÷8 = 5 books. Fiction = 5 parts × 5 books = 25. Nonfiction = 3 × 5 = 15. Check: 25+15 = 40 total books. |
| Simplify the ratio 18:24. | **3:4** — Dividing both by 6: 18÷6=3 and 24÷6=4, giving 3:4. While 9:12 and 6:8 are also equivalent, only 3:4 is fully simplified because 3 and 4 share no common factors other than 1. |
| Which ratio is equivalent to 4:6? | **2:3** — Dividing both parts by 2: 4÷2=2 and 6÷2=3, giving 2:3. The ratio 3:2 reverses the order, 4:3 changes the relationship, and 8:18 = 4:9 which is a different ratio. |
| How does a ratio table help you find equivalent ratios? | **It shows a pattern of multiplying both quantities by the same number** — A ratio table works by multiplying both quantities by the same whole number (1, 2, 3, ...), producing equivalent ratios. Adding the same number to both sides would NOT produce equivalent ratios. |

| | |
|---|---|
| Complete the ratio table: if 2:5, then 4:? | **10** — In a ratio table, both numbers are multiplied by the same factor. Since 2 × 2 = 4, then 5 × 2 = 10. The ratio 4:10 is equivalent to 2:5 because both simplify to the same relationship. |
| Purple paint is made by mixing red and blue in a 3:5 ratio. To make 24 liters of purple paint, how much blue paint is needed? | **15 liters** — Total parts = 3+5 = 8. Each part = 24÷8 = 3 liters. Blue = 5 parts × 3 liters = 15 liters. Red = 3×3 = 9 liters. Check: 9+15=24 liters total. |
| A recipe calls for 250 mL of oil and 750 mL of vinegar. What is the oil-to-vinegar ratio in simplest form? | **1:3** — 250÷250=1 and 750÷250=3, giving 1:3. This means for every 1 part oil, you need 3 parts vinegar. The ratio 1:4 is incorrect because 250×4=1000, not 750. |
| A store sells 3 notebooks for $6. At this rate, how much would 9 notebooks cost? | **$18** — The ratio is 3 notebooks:$6. Since 9 ÷ 3 = 3, multiply both: 3×3=9 notebooks and $6×3=$18. This means 9 notebooks cost $18, maintaining the same rate. |
| Evaluate this claim: 'If two ratios have the same difference between their terms, they must be equivalent.' Is this true? Give an example. | **False. 3:5 and 1:3 both have a difference of 2, but 3:5 ≠ 1:3** — This is false. Example: 3:5 has difference 2 and 1:3 has difference 2, but 3/5 ≠ 1/3. Ratios are about multiplicative relationships (division), not additive relationships (subtraction). Equal differences do not guarantee equivalence. |
| Maria needs to mix paint in a 2:7 ratio (red to blue). She has 14 mL of red paint. How much blue paint does she need? | **49 mL** — The multiplier is 14÷2=7. So blue = 7×7 = 49 mL. The equivalent ratio is 14:49, which simplifies back to 2:7. Maria needs 49 mL of blue paint. |
| A school's boy-to-girl ratio is 7:8. If 56 students are boys, how many are girls? | **64** — Each part = 56÷7 = 8 students. Girls = 8 parts × 8 students = 64. Total students = 56+64 = 120. The ratio 56:64 simplifies to 7:8, confirming the answer. |
| Compare two methods for checking if 8:14 and 20:35 are equivalent. | **Method 1: Simplify both (4:7 = 4:7). Method 2: Cross-multiply (8×35=280 = 14×20=280).** — Two valid methods: (1) Simplify both: 8:14÷2=4:7 and 20:35÷5=4:7, so they match. (2) Cross-multiply: 8×35=280 and 14×20=280, which are equal. Both methods confirm equivalence. |
| Create a word problem where 3:8 is the answer. Explain why your scenario uses that ratio. | **A trail mix has 3 cups of raisins for every 8 cups of nuts, because the recipe portions compare raisins to nuts in a 3:8 relationship** — A valid word problem must compare two quantities in a 3:8 relationship. For example, 3 cups raisins to 8 cups nuts creates a 3:8 ratio. The scenario must clearly define what each number represents and why they are compared. |
| What is 45:60 in simplest form? | **3:4** — 45÷15=3 and 60÷15=4, so the simplest form is 3:4. The ratio 5:6 is wrong because 45/5=9 and 60/5=12, not 5 and 6. |
| Three friends split profits in the ratio 2:3:5. If total profit is $200, what does the person with the largest share receive? | **$100** — Total parts = 2+3+5 = 10. Each part = $200÷10 = $20. Largest share (5 parts) = 5×$20 = $100. The other shares are $40 and $60. Check: $40+$60+$100=$200. |
| | |

| | |
|---|---|
| In a ratio table for 3:4, which pair does NOT belong? (6:8, 9:12, 12:15, 15:20) | **12:15** — 6:8 = 3:4, 9:12 = 3:4, 15:20 = 3:4, but 12:15 = 4:5 (dividing by 3). The pair 12:15 does not simplify to 3:4 and does not belong in the table. |
| Find the missing value: 5:8 = 15:? | **24** — The multiplier is 15 ÷ 5 = 3. So 8 × 3 = 24. The equivalent ratio is 15:24. Both parts must be multiplied by the same factor to maintain equivalence. |

## 3. Rates

| ✂ Question (fold →) | Answer |
|---|---|
| Two trains leave at the same time. Train A goes 55 mph and Train B goes 70 mph. After 3 hours, how much farther has Train B traveled? | **45 miles** — Train A: 55×3=165 miles. Train B: 70×3=210 miles. Difference: 210-165=45 miles. Or: (70-55)×3 = 15×3 = 45 miles. The speed difference of 15 mph compounds over 3 hours. |
| Create a scenario where a higher total cost actually represents a better value. Explain using unit rates. | **A 2-pound bag of rice for $3 vs a 5-pound bag for $6: the larger bag costs more but is $1.20/lb vs $1.50/lb** — Example: 2 lbs for $3 ($1.50/lb) vs 5 lbs for $6 ($1.20/lb). The $6 option costs more total but saves $0.30/lb. This shows why unit rates, not total prices, determine true value. Bulk buying often offers lower unit costs. |
| A store sells 6 cans of soup for $9. What is the unit price per can? | **$1.50** — Unit price = $9 ÷ 6 = $1.50 per can. A unit rate has 1 as the denominator. $0.67 incorrectly divides 6÷9. $3.00 divides 9÷3 (wrong divisor). $15 incorrectly adds. |
| A car travels 150 miles in 3 hours. What is the rate? | **150 miles per 3 hours** — The rate is 150 miles per 3 hours (or equivalently, 50 miles per hour as a unit rate). The answer 450 incorrectly multiplies instead of dividing. Rates include both the quantities and their units. |
| A factory makes 840 widgets in 7 hours. Another makes 1,080 in 9 hours. Which factory is more efficient? | **Both produce 120 widgets per hour, so they are equally efficient** — Factory 1: 840÷7 = 120 widgets/hr. Factory 2: 1080÷9 = 120 widgets/hr. Both produce at the same rate of 120 widgets per hour, making them equally efficient despite different totals. |
| A printer produces 120 pages in 4 minutes. Identify the two quantities and their units in this rate. | **120 pages (output) and 4 minutes (time)** — The two quantities are 120 pages (measuring printed output) and 4 minutes (measuring time). A rate always involves two measurements with different units, describing how one quantity relates to another. |
| Juice boxes come in packs of 6 for $4.50 or packs of 10 for $7.00. Which pack has the lower unit cost? | **Pack of 10 at $0.70 each** — Pack of 6: $4.50÷6 = $0.75 each. Pack of 10: $7.00÷10 = $0.70 each. The 10-pack saves $0.05 per box. Larger quantities often (but not always) have lower unit costs. |
| A cyclist rides at 12 miles per hour. How far will they ride in 2.5 hours? | **30 miles** — Distance = 12 × 2.5 = 30 miles. When you know speed and time, multiply them to find distance. 14.5 incorrectly adds 12+2.5. 24 only multiplies by 2, forgetting the 0.5. |
| Brand A: 16 oz for $3.20. Brand B: 24 oz for $4.32. Which is the better buy? | **Brand B at $0.18/oz** — Brand A: $3.20÷16 = $0.20/oz. Brand B: $4.32÷24 = $0.18/oz. Brand B is cheaper per ounce ($0.18 < $0.20). Even though Brand B costs more total, it gives more value per ounce. |
| Which of these is an example of a rate? | **60 miles per hour** — A rate compares two quantities measured in different units. '60 miles per hour' compares distance (miles) to time (hours). The ratio 3:5 compares the same type of thing (fruits), making it a ratio, not a rate. |
| Gas Station A charges $3.60 for 1.2 gallons. Gas Station B charges $4.50 for 1.5 gallons. Which has the lower price per gallon? | **Both charge $3.00 per gallon** — Station A: $3.60÷1.2 = $3.00/gal. Station B: $4.50÷1.5 = $3.00/gal. Both charge exactly $3.00 per gallon. The different totals reflect different amounts purchased, not different rates. |
| | |

| | |
|---|---|
| Which has a lower unit price: 8 oz for $2.40 or 12 oz for $3.00? | **12 oz for $3.00** — $2.40 ÷ 8 = $0.30/oz and $3.00 ÷ 12 = $0.25/oz. Since $0.25 < $0.30, the 12 oz option is cheaper per ounce. Always compare unit rates to find the better deal. |
| Why is '12 dollars for 4 pounds of apples' considered a rate? | **It compares two different units: dollars and pounds** — A rate compares quantities with different units. Here, dollars (money) and pounds (weight) are different measurement types. Not all ratios are rates — a ratio of 3 boys to 5 girls uses the same unit (students). |
| Is it possible for a car to have an average speed of 60 mph for a trip but never actually travel at exactly 60 mph during the trip? Explain. | **Yes, average speed is total distance divided by total time, so the car could vary between speeds above and below 60 mph throughout** — Yes, average speed = total distance ÷ total time. A car could go 40 mph then 80 mph and average 60 mph without ever traveling exactly 60 mph. Average speed summarizes the entire trip, not any single moment. |
| A runner covers 10 kilometers in 50 minutes. What is the unit rate in minutes per kilometer? | **5 minutes per kilometer** — Unit rate = 50 min ÷ 10 km = 5 min/km. This means the runner takes 5 minutes for each kilometer. The rate 0.2 would be km per minute (the inverse), not minutes per km. |
| Faucet A fills a bucket at 2 gallons per 3 minutes. Faucet B fills at 3 gallons per 5 minutes. Which fills faster? | **Faucet A at 0.67 gal/min vs Faucet B at 0.6 gal/min** — Faucet A: 2÷3 ≈ 0.667 gal/min. Faucet B: 3÷5 = 0.6 gal/min. Faucet A fills faster because 0.667 > 0.6. Without converting to the same unit rate, the comparison would be misleading. |
| Analyze why two runners with speeds of 8 km/h and 480 km/min are actually the same speed despite looking very different. | **480 km/min is not the same — 8 km/h = 0.133 km/min, so they are very different. 480 km/min would be absurdly fast** — 8 km/h = 8÷60 ≈ 0.133 km/min, which is vastly different from 480 km/min. The premise is flawed — 480 km/min equals 28,800 km/h. This highlights why unit conversion is critical when comparing rates. |
| A store advertises 'Buy 3, get 1 free' on items that normally cost $2 each. What is the effective unit price with this deal? | **$1.50 per item** — You pay 3 × $2 = $6 for 4 items total. Unit price = $6 ÷ 4 = $1.50 per item. The free item reduces the effective price by 25% from $2.00 to $1.50. This is a common real-world unit rate application. |
| Worker A assembles 15 toys in 3 hours. Worker B assembles 24 toys in 4 hours. Who works faster? | **Worker B at 6 toys/hour vs 5 toys/hour** — Worker A: 15÷3 = 5 toys/hr. Worker B: 24÷4 = 6 toys/hr. Worker B produces 1 more toy per hour. Comparing unit rates reveals who is faster, regardless of total output. |
| A shopper sees: Small bag (5 lb) for $8.75, Large bag (12 lb) for $19.80. Determine which is the better value and explain your reasoning. | **Small bag at $1.75/lb vs Large bag at $1.65/lb** — Small: $8.75÷5=$1.75/lb. Large: $19.80÷12=$1.65/lb. The large bag is $0.10/lb cheaper. When comparing value, always use unit rates rather than total prices or total quantities. |
| A recipe uses 2 teaspoons of salt for every 3 cups of flour. Is this a rate or a ratio? Justify your answer. | **It is a rate because teaspoons and cups are different units of measurement being compared** — This is a rate because it compares quantities in different units (teaspoons vs cups). Even though both measure volume, they are different units. A ratio would compare items in the same unit, like 2 cups of salt to 3 cups of flour. |
| A car travels 240 miles in 4 hours. What is its average speed? | **60 mph** — Speed = 240 ÷ 4 = 60 miles per hour. Speed is a unit rate that compares distance to time. 960 incorrectly multiplies 240×4. Always divide distance by time to find speed. |
| A plane flies 2,400 miles in 5 hours with a tailwind, then returns the | **The outbound leg was faster by 80 mph** — Outbound: 2400÷5 = 480 mph. |

| | |
|---|---|
| same distance in 6 hours against the wind. Which leg had a higher average speed, and by how much? | Return: 2400÷6 = 400 mph. Difference: 480-400 = 80 mph faster on the outbound leg. The tailwind increased speed while the headwind decreased it. |
| Internet Plan A offers 100 Mbps for $50/month. Plan B offers 200 Mbps for $90/month. Which offers more speed per dollar? | **Plan A at 2 Mbps per dollar vs Plan B at 2.22 Mbps per dollar — Plan B wins** — Plan A: 100÷50 = 2.0 Mbps/$. Plan B: 200÷90 ≈ 2.22 Mbps/$. Plan B gives more speed per dollar. However, Plan A has a lower total cost — the 'better' choice depends on budget and needs. |
| Water flows at 3/4 gallon per 1/2 minute. What is the unit rate in gallons per minute? | **1.5 gallons per minute** — (3/4) ÷ (1/2) = (3/4) × (2/1) = 6/4 = 1.5 gallons per minute. When dividing fractions, multiply by the reciprocal. This is a common middle school skill combining fractions with rates. |

## 4. Proportions

| ✂ Question (fold →) | Answer |
|---|---|
| Solve: x/8 = 15/24 | **x = 5** — Cross-multiplying: 24x = 8×15 → 24x = 120 → x = 5. Check: 5/8 = 0.625 and 15/24 = 0.625. Both ratios equal 0.625, confirming x = 5. |
| A trail mix recipe uses nuts and dried fruit in a 5:3 ratio. If you want 40 ounces total, how many ounces of nuts do you need? | **25 ounces** — Total parts = 5+3 = 8. Each part = 40÷8 = 5 oz. Nuts = 5×5 = 25 oz. Fruit = 3×5 = 15 oz. Check: 25+15=40 oz and 25:15 = 5:3. |
| A car uses 3 gallons of gas to travel 90 miles. Set up a proportion to find how far it can travel on 7 gallons. | **3/90 = 7/x** — The proportion 3/90 = 7/x keeps gallons on top and miles on the bottom in both ratios. Alternatively, 90/3 = x/7 also works. The key is consistency: same units in the same positions. |
| If 5 notebooks cost $12, which proportion correctly finds the cost of 8 notebooks? | **5/12 = 8/x** — 5/12 = 8/x sets up corresponding ratios: notebooks/dollars = notebooks/dollars. Solving: x = (8×12)/5 = 96/5 = $19.20. Each fraction must compare the same quantities in the same order. |
| If 4 identical erasers weigh 60 grams, how much do 7 erasers weigh? | **105 grams** — Unit rate: 60÷4 = 15 g/eraser. 7 erasers: 15×7 = 105 g. Or: 4/60 = 7/x → 4x = 420 → x = 105 g. The answer 420 is the cross-product, not the final answer. |
| A model airplane is built at a 1:48 scale. If the real plane's wingspan is 96 feet, what is the model's wingspan? | **2 feet** — Model wingspan = 96 ÷ 48 = 2 feet. A 1:48 scale means every 1 unit on the model equals 48 units in reality. So the real measurement is divided by 48 to find the model dimension. |
| If 3 pounds of bananas cost $1.80, how much do 10 pounds cost? | **$6.00** — 3/1.80 = 10/x → 3x = 18 → x = $6.00. Or find the unit rate first: $1.80÷3 = $0.60/lb, then $0.60×10 = $6.00. Both methods give the same answer. |
| A recipe serves 4 people and uses 6 eggs. Write a proportion to find eggs needed for 10 people. | **4/6 = 10/x** — 4/6 = 10/x maintains people/eggs in both ratios. Solving: 4x = 60, so x = 15 eggs. You can also write 6/4 = x/10 (eggs/people = eggs/people), which gives the same answer. |
| A machine fills 45 bottles in 9 minutes. How many bottles does it fill in 20 minutes? | **100 bottles** — 45/9 = x/20 → 9x = 900 → x = 100 bottles. Unit rate: 45÷9 = 5 bottles/min, then 5×20 = 100. The answer 225 incorrectly multiplies 45×5 then doesn't adjust. |
| Solve: 7/10 = x/30 | **x = 21** — Cross-multiplying: 7×30 = 10x → 210 = 10x → x = 21. Check: 7/10 = 0.7 and 21/30 = 0.7. The answer 210 forgot to divide by 10. |
| Evaluate: 'If you double both terms of a ratio, the ratio stays the same. But if you double only one term, the ratio doubles.' Is the second claim correct? | **Not exactly — doubling the first term doubles the ratio value, but doubling the second term halves it** — Starting with 3:4 (=0.75): doubling the first gives 6:4 (=1.5, doubled). Doubling the second gives 3:8 (=0.375, halved). Doubling the numerator doubles the ratio; doubling the denominator halves it. They have opposite effects. |
| A recipe uses milk and flour in a 3:5 ratio. You have 2 cups of milk but want to know if you have enough flour (you have 3 cups). Determine using proportional reasoning. | **You need 3⅓ cups of flour, so 3 cups is not enough** — 3/5 = 2/x → 3x = 10 → x = 10/3 ≈ 3.33 cups. Since you only have 3 cups of flour, you are short by about 1/3 cup. Proportional reasoning reveals whether your ingredients match the required ratio. |

| | |
|---|---|
| A car's fuel efficiency is 30 miles per gallon. If the tank holds 12 gallons, what is the maximum range? | **360 miles** — Range = 30 mpg × 12 gallons = 360 miles. The units cancel: (miles/gallon) × gallons = miles. The answer 42 incorrectly adds 30+12 instead of multiplying. |
| Is cross-multiplication valid for inequalities? Test with 2/3 and 5/8. | **Yes for comparing: 2×8=16 and 3×5=15, so 16>15 means 2/3 > 5/8** — Cross-multiplication extends to comparisons: 2×8=16 vs 3×5=15. Since 16>15, we know 2/3 > 5/8. This works because multiplying both sides of a/b vs c/d by bd preserves the inequality when b,d > 0. |
| A school surveys 50 students and finds 30 prefer basketball. If the school has 800 students, predict how many prefer basketball. | **480** — 30/50 = x/800 → 50x = 24,000 → x = 480. Or: 30/50 = 0.6 = 60%, and 0.6 × 800 = 480. This assumes the survey sample is representative of the whole school. |
| On a map, 2 cm represents 50 km. If two cities are 7 cm apart on the map, what is the actual distance? | **175 km** — 2/50 = 7/x → 2x = 350 → x = 175 km. The scale ratio is 1:25 (each cm = 25 km), so 7 cm × 25 km/cm = 175 km. The answer 350 forgot to divide by 2. |
| Design a proportion problem where the answer is 42. Write the problem, set up the proportion, and solve it. | **If 6 tickets cost $36, how much do 7 tickets cost? 6/36 = 7/x → 6x = 252 → x = $42** — Example: 6 tickets/$36 = 7 tickets/x → 6x = 252 → x = $42. The proportion works because 36/6 = 6 (unit rate), and 6×7 = 42. A well-designed problem has realistic context and a clear proportional relationship. |
| If 2 workers can paint a room in 6 hours, how long would 3 workers take (assuming they work at the same rate)? | **4 hours** — Total work = 2 workers × 6 hours = 12 worker-hours. With 3 workers: 12 ÷ 3 = 4 hours. This is inverse proportionality — as workers increase, time decreases, but total work stays constant. |
| A student solves 3/x = 9/12 and gets x = 36. What error did they make? | **They multiplied 3×12 instead of cross-multiplying correctly; the answer should be x = 4** — Correct cross-multiplication: 3×12 = 9x → 36 = 9x → x = 36/9 = 4. The student got 36 by multiplying 3×12 but forgot to divide by 9. Check: 3/4 = 0.75 and 9/12 = 0.75. |
| A baker uses 2.5 cups of sugar for 30 cookies. She wants to make 48 cookies. How much sugar does she need? | **4 cups** — 2.5/30 = x/48 → 30x = 120 → x = 4 cups. Or: 2.5÷30 = 1/12 cup per cookie, then (1/12)×48 = 4 cups. The proportion scales the recipe up by a factor of 48/30 = 1.6. |
| A photograph is 4 inches wide and 6 inches tall. If it is enlarged so the width is 10 inches, what will the height be? | **15 inches** — 4/6 = 10/x → 4x = 60 → x = 15 inches. The photo's aspect ratio (2:3) is maintained: 10:15 = 2:3. If the height were 12, the ratio would be 10:12 = 5:6, which distorts the image. |
| Explain why 3/4 = 6/8 is a proportion but 3/4 = 6/7 is not. | **In 3/4 = 6/8, cross-products are equal (3×8=24=4×6), but in 3/4 = 6/7, they are not (3×7=21≠4×6=24)** — A proportion is an equation stating two ratios are equal. 3×8=24 and 4×6=24 (equal, so proportion). 3×7=21 and 4×6=24 (not equal, so not a proportion). Cross-multiplication is the definitive test for proportionality. |
| Solve for x: 4/x = 12/15 | **x = 5** — Cross-multiplying: 4×15 = 12x → 60 = 12x → x = 5. Check: 4/5 = 0.8 and 12/15 = 0.8. Both ratios are equal, confirming x = 5. |
| A shadow problem: a 6-ft person casts a 4-ft shadow. A tree casts a 20-ft shadow. Set up a proportion for the tree's height. | **6/4 = x/20** — 6/4 = x/20 compares height/shadow for both objects. Solving: 4x = 120, x = 30 feet. Shadow problems rely on similar triangles where corresponding sides are proportional. |

| | |
|---|---|
| The ratio of boys to girls in School A is 3:4. In School B it is 5:7. School A has 210 students and School B has 360. Which school has more boys? | **School B with 150 boys vs School A with 90 boys** — School A: 210 × 3/7 = 90 boys. School B: 360 × 5/12 = 150 boys. School B has 60 more boys. Even though School A's ratio has fewer parts, the actual count depends on both the ratio and total students. |

## 5. Percentages

| ✂ Question (fold →) | Answer |
|---|---|
| In a survey, 72 people out of 300 preferred chocolate ice cream. What percentage is this? | **24%** — (72 ÷ 300) × 100 = 0.24 × 100 = 24%. This means about 1 in 4 people surveyed preferred chocolate. The answer 72% confuses the count with the percentage. |
| A sale sign reads '33⅓% off.' What fraction of the original price do you save? | **1/3** — 33⅓% = 100/3 ÷ 100 = 100/300 = 1/3. This is the exact fraction, unlike 33/100 which only approximates it. Stores often use 33⅓% because it exactly equals the clean fraction 1/3. |
| Convert 3/4 to a percentage. | **75%** — 3 ÷ 4 = 0.75, and 0.75 × 100 = 75%. Alternatively, 3/4 = 75/100 by multiplying numerator and denominator by 25. The answer 34% misreads the fraction digits. |
| Which fraction converts to exactly 60%? | **3/5** — 60% = 60/100 = 3/5. Check: 3÷5 = 0.60 = 60%. The fraction 6/8 = 75%, 2/3 ≈ 66.7%, and 4/6 ≈ 66.7%. Only 3/5 equals exactly 60%. |
| What is 120% of 50? | **60** — 120% of 50 = 1.20 × 50 = 60. Since 120% is greater than 100%, the result (60) is greater than the original number (50). This represents the original plus 20% more. |
| What is 25% of 80? | **20** — 25% of 80 = 0.25 × 80 = 20. You can also think: 25% = 1/4, and 80 ÷ 4 = 20. The answer 25 confuses the percentage with the result. |
| Convert 1.25 to a percentage. | **125%** — 1.25 × 100 = 125%. A percentage greater than 100% is valid — it means more than the whole. For example, if sales grew by 125%, they more than doubled. |
| Convert 45% to a fraction in simplest form. | **9/20** — 45% = 45/100. Dividing both by 5: 45÷5=9 and 100÷5=20, giving 9/20. The fraction 4/5 = 80% and 9/10 = 90%, both much larger than 45%. |
| What is 2/5 as a percentage? | **40%** — 2 ÷ 5 = 0.40, and 0.40 × 100 = 40%. You can also see that 2/5 = 40/100 by multiplying both by 20. The answer 25% confuses 2/5 with 1/4. |
| Which of these percentages converts to a fraction that cannot be simplified? | **37%** — 37% = 37/100. Since 37 is prime and does not divide 100, this fraction is already in simplest form. 25/100=1/4, 50/100=1/2, 75/100=3/4 all simplify. |
| Convert 0.65 to a percentage. | **65%** — 0.65 × 100 = 65%. Moving the decimal point two places right converts any decimal to a percent. The answer 6.5% only moves one place, and 650% moves three places. |
| A student says 1/3 = 33%. Is this exact or approximate? Explain the difference. | **Approximate — 1/3 = 33.333...% (repeating), not exactly 33%** — 1 ÷ 3 = 0.3333... (repeating forever), so 1/3 = 33.333...%, not exactly 33%. The difference is 0.333...%, which matters in precise calculations. Some fractions (like 1/3, 1/6, 1/7) produce repeating decimals. |
| What fraction in simplest form equals 12.5%? | **1/8** — 12.5% = 12.5/100 = 125/1000 = 1/8 (dividing by 125). Check: 1÷8 = 0.125 = 12.5%. The fraction 1/12 ≈ 8.3% and 1/5 = 20%. |
| Convert 7/8 to a percentage. | **87.5%** — 7 ÷ 8 = 0.875, and 0.875 × 100 = 87.5%. Not all fractions convert to whole-number percentages. The answer 78% just concatenates the digits 7 and 8. |
| A shirt originally costs $45. It is 30% off. What is | **$13.50** — 30% of $45 = 0.30 × 45 = $13.50 discount. The sale price would be $45 - $13.50 = $31.50. Note: $31.50 is the sale price, not the discount amount — the question |

| | |
|---|---|
| the discount amount? | asks for the discount. |
| A phone battery is at 35%. If the full battery holds 4,000 mAh, how much charge remains? | **1,400 mAh** — 35% of 4,000 = 0.35 × 4,000 = 1,400 mAh remaining. The value 2,600 represents the charge already used (65%), not what remains. |
| Which decimal is equivalent to 0.5%? | **0.005** — 0.5% ÷ 100 = 0.005. Note: 0.5% is less than 1% — it is a very small percentage. The answer 0.05 = 5% and 0.5 = 50%, both much larger than 0.5%. |
| A school has 600 students. 55% are girls. How many boys attend the school? | **270** — Boys = 100% - 55% = 45%. 0.45 × 600 = 270 boys. Girls = 0.55 × 600 = 330. Check: 270 + 330 = 600. The answer 330 gives the number of girls, not boys. |
| A jar is 80% full with 240 mL of water. What is the jar's total capacity? | **300 mL** — 240 = 0.80 × capacity → capacity = 240 ÷ 0.80 = 300 mL. This is a 'reverse percentage' problem where you know the part and percent but need the whole. 192 incorrectly multiplies 240 × 0.80. |
| A test has 40 questions. A student answers 85% correctly. How many questions did they get right? | **34** — 0.85 × 40 = 34 questions correct. The student got 34 out of 40 right. The answer 85 confuses the percentage with the count. |
| Create a real-world scenario where knowing that '60% of 250 = 150' would be useful. Explain why the calculation matters. | **A teacher needs 60% of 250 students to pass for the school to meet its target — so 150 students must pass to reach the goal** — Example: If a school needs 60% attendance (of 250 students) for a field trip, they need 150 students. Percentage calculations help set targets, measure progress, and make decisions in business, education, health, and everyday planning. |
| Convert 150% to a mixed number. | **1 1/2** — 150% = 150/100 = 3/2 = 1 1/2. Percentages over 100% convert to numbers greater than 1. While 3/2 is also correct as an improper fraction, 1 1/2 is the mixed number form. |
| Explain two different methods to calculate 15% of 200, and show they give the same answer. | **Method 1: 0.15 × 200 = 30. Method 2: 10% of 200 = 20, plus 5% of 200 = 10, total = 30** — Method 1 (decimal): 0.15 × 200 = 30. Method 2 (benchmark decomposition): 10% of 200 = 20, and 5% = half of 10% = 10, so 20+10 = 30. Breaking percentages into benchmarks (10%, 5%, 1%) is a powerful mental math strategy. |
| Arrange these from least to greatest: 0.35, 30%, 1/4, 0.28. | **1/4, 0.28, 30%, 0.35** — Converting all to decimals: 1/4 = 0.25, 0.28 = 0.28, 30% = 0.30, 0.35 = 0.35. Order: 0.25 < 0.28 < 0.30 < 0.35, so 1/4, 0.28, 30%, 0.35. |
| What is 0.08 as a percentage? | **8%** — 0.08 × 100 = 8%. Students often confuse 0.08 with 80%, but 80% = 0.80, not 0.08. Pay attention to the position of the decimal point. |

## 6. Percentage Applications

| ✂ Question (fold →) | Answer |
|---|---|
| Your total bill is $54 including 8% tax. What was the pre-tax price? | **$50** — $54 = 1.08 × pre-tax → pre-tax = $54 ÷ 1.08 = $50. Check: $50 + 8% tax ($4) = $54. The answer $49.68 incorrectly subtracts 8% of $54 instead of dividing. |
| A lake's water level dropped from 50 feet to 42 feet. What is the percent decrease? | **16%** — Change = 8 ft. Percent decrease = (8/50)×100 = 16%. The answer 8% uses the raw change as a percentage. The answer 19% divides by 42 instead of 50. Always use the original as the denominator. |
| If a quantity increases by 50% and then increases by 50% again, is the total increase 100%? Prove your answer with a specific example. | **No. Starting at 100: +50% gives 150, then +50% of 150 gives 225. Total increase is 125%, not 100%** — Start with 100. First +50%: 100×1.5=150. Second +50%: 150×1.5=225. Total change: 225-100=125, so 125% increase. Percentages compound multiplicatively (1.5×1.5=2.25), not additively (1.5+0.5=2.0). |
| Your test score improved from 72 to 90. What is the percent increase? | **25%** — Change = 18. Percent increase = (18/72) × 100 = 25%. The answer 18% uses the point change, not the percentage change. The answer 20% incorrectly divides 18/90. |
| A stock price rises from $40 to $52. What is the percent increase? | **30%** — Change = $12. Percent increase = (12/40) × 100 = 30%. The answer 23.1% divides by 52 (the new value) instead of 40 (the original). 130% would be the new value as a percentage of the old. |
| Construct a scenario where a 10% decrease followed by a 10% increase does NOT return to the original value. Explain why. | **Start at $100: -10% = $90, then +10% of $90 = $99. You lose $1 because the 10% increase applies to a smaller base** — Starting at $100: 10% decrease gives $90. Then 10% increase of $90 = $9, giving $99. You do not return to $100 because the increase applies to the reduced amount ($90), not the original ($100). This asymmetry is a fundamental property of percentage changes. |
| Your restaurant bill is $40. You want to leave a 20% tip. How much is the tip? | **$8.00** — Tip = 0.20 × $40 = $8.00. Total with tip = $40 + $8 = $48. A quick mental math trick: 10% = $4, so 20% = $4 × 2 = $8. The answer $48 is the total bill, not just the tip. |
| You buy a $60 item with 7.5% tax. What is the tax amount? | **$4.50** — Tax = 0.075 × $60 = $4.50. The total would be $60 + $4.50 = $64.50. The answer $7.50 confuses the percentage rate with the dollar amount. |
| A store reduces prices by 35%. If the original price was $80, what is the new price? | **$52** — Discount = 0.35 × $80 = $28. New price = $80 - $28 = $52. Or: 0.65 × $80 = $52. The answer $28 is the amount saved, not the new price. |
| A meal costs $65. You want to tip 18%. What is the total bill including tip? | **$76.70** — Tip = 0.18 × $65 = $11.70. Total = $65 + $11.70 = $76.70. Or: $65 × 1.18 = $76.70. The answer $11.70 is just the tip, not the total. |
| A phone's value depreciates from $1,000 to $600 in one year, then from $600 to $420 the next. Compare the percent decrease each year. | **Year 1: 40% decrease. Year 2: 30% decrease. Depreciation slowed.** — Year 1: (400/1000)×100 = 40%. Year 2: (180/600)×100 = 30%. Although dollar amounts differ ($400 vs $180), the percentage rate of decrease slowed from 40% to 30%, meaning the phone lost value more slowly in Year 2. |
| You want to leave exactly $6 tip on a meal. If this | **$40** — $6 = 0.15 × bill → bill = $6 ÷ 0.15 = $40. Check: 15% of $40 = 0.15 × $40 = $6. |

| | |
|---|---|
| represents a 15% tip, what was the bill? | The answer $46 incorrectly adds $6 to $40 (that would be the total with tip). |
| An item is marked 15% off, bringing the price to $51. What was the original price? | **$60** — If $51 = 85% of original, then original = $51 ÷ 0.85 = $60. Check: 15% of $60 = $9, and $60 - $9 = $51. The answer $58.65 incorrectly adds 15% to $51. |
| Compare two tipping methods: (1) Calculate 15% of the pre-tax bill of $80. (2) Calculate 15% of the post-tax bill of $86.40 (8% tax). What is the difference in tip amount? | **$0.96 difference ($12.00 vs $12.96)** — Pre-tax tip: 0.15 × $80 = $12.00. Post-tax tip: 0.15 × $86.40 = $12.96. Difference: $0.96. Tipping on the post-tax amount gives a slightly larger tip because you are also tipping on the tax amount. |
| A store offers 30% off, then an additional 10% off the sale price. Is this the same as 40% off? Calculate for a $200 item. | **No. Sequential discounts give $126 (37% total), not $120 (40% off)** — First discount: $200 × 0.70 = $140. Second: $140 × 0.90 = $126. But 40% off: $200 × 0.60 = $120. Sequential discounts ($126) save less than a single 40% discount ($120) because the second discount applies to a smaller amount. |
| State A has 6% tax and State B has 9.5% tax. How much more tax do you pay on a $200 item in State B? | **$7.00 more** — State A tax: 0.06 × $200 = $12. State B tax: 0.095 × $200 = $19. Difference: $19 - $12 = $7. You pay $7 more in State B. The tax rate difference is 3.5%, and 3.5% of $200 = $7. |
| A jacket costs $80 and is 25% off. What is the sale price? | **$60** — Discount = 0.25 × $80 = $20. Sale price = $80 - $20 = $60. Alternatively, paying 75% of original: 0.75 × $80 = $60. The answer $20 is the discount amount, not the sale price. |
| A company's revenue went from $2 million to $2.5 million. Another went from $500,000 to $650,000. Which had a larger percent increase? | **The second company (30% vs 25%)** — Company 1: (500K/2M)×100 = 25%. Company 2: (150K/500K)×100 = 30%. Despite a smaller dollar increase ($150K vs $500K), Company 2 grew faster in percentage terms. Percent increase measures relative growth. |
| A town's population grew from 5,000 to 6,000. What is the percent increase? | **20%** — Change = 1,000. Percent increase = (1,000 ÷ 5,000) × 100 = 20%. The answer 16.7% incorrectly divides by the new value (1000/6000). Always divide by the original amount. |
| A bike originally costs $300 but drops to $240. What is the percent decrease? | **20%** — Change = $60. Percent decrease = (60/300)×100 = 20%. The answer 25% divides by 240 (new value) instead of 300 (original). The answer 80% represents what fraction of the original the new price is. |
| An item costs $25 before 8% sales tax. What is the total cost? | **$27.00** — Tax = 0.08 × $25 = $2.00. Total = $25 + $2 = $27.00. Or multiply by 1.08: $25 × 1.08 = $27.00. The answer $33 incorrectly calculates 8 × $25 ÷ 6. |
| Explain why calculating tax as a percentage is fairer than charging everyone a flat tax amount of the same dollar value. | **A percentage tax scales with the purchase** — Percentage-based tax is proportional: 8% of $10 = $0.80, while 8% of $1000 = $80. This scales fairly with the purchase amount. A flat tax would disproportionately burden small purchases and be negligible on expensive ones, violating proportional reasoning principles. — expensive items contribute more tax, while small purchases are taxed less, making it proportional. |
| Evaluate: Is it reasonable to always use 20% as a tip? | **20% is a generous standard, but tips should vary: exceptional service may warrant more, while takeout or counter service often uses 10-15%** — 20% is a |

| | |
|---|---|
| Discuss when a different percentage might be appropriate. | common generous standard for sit-down dining, but context matters. Counter service or takeout typically warrants 10-15%. Exceptional service may justify 25%+. Group auto-gratuities are often 18%. Understanding percentages helps navigate these social norms appropriately. |
| A $120 pair of shoes is marked 40% off. What do you pay? | **$72** — 40% off means you pay 60%: 0.60 × $120 = $72. The discount is $48 (0.40 × 120), so $120 - $48 = $72. The answer $48 is the discount, not what you pay. |
| Which is a better deal: 20% off a $50 item, or $8 off the same item? | **20% off saves $10 vs $8 off, so 20% off is better** — 20% of $50 = $10 off, which saves $2 more than the flat $8 discount. Percentage discounts depend on the original price — 20% of a $20 item would only be $4, making $8 off better. Always convert to dollar amounts to compare. |

# 7. Scale & Measurement

| ✂ Question (fold →) | Answer |
| --- | --- |
| Critique this statement: 'A 1:100 scale is more detailed than a 1:1000 scale.' Is this correct? Explain using map examples. | **Correct — 1:100 means 1 cm represents 1 m (more detail), while 1:1000 means 1 cm represents 10 m (less detail, covers more area)** — Correct. At 1:100, 1 cm represents just 1 m — small features like furniture are visible. At 1:1000, 1 cm represents 10 m — you can see buildings but not individual rooms. Smaller scale denominators mean more detail but less area coverage. City plans use ~1:1000; country maps use ~1:1,000,000. |
| On a blueprint at 1:100 scale, which is larger: a room drawn as a 3 cm × 4 cm rectangle or a room drawn as a 2 cm × 7 cm rectangle? By how much? | **The 2 cm × 7 cm room is larger: at 1:100 it is 2 m × 7 m = 14 sq m, versus the 3 cm × 4 cm room's 3 m × 4 m = 12 sq m — larger by 2 sq m** — At 1:100, multiply each drawn length by 100. Room 1: 3 cm × 4 cm → 3 m × 4 m = 12 sq m. Room 2: 2 cm × 7 cm → 2 m × 7 m = 14 sq m. Room 2 is larger by 2 sq m. |
| A dollhouse is built at 1:12 scale. If a real dining table is 3 feet long and 1.5 feet wide, what are the dollhouse table's dimensions in inches? | **3 inches by 1.5 inches** — Real: 36 in × 18 in. Dollhouse: 36/12 = 3 in × 18/12 = 1.5 in. The 1:12 scale is standard for dollhouses — 1 inch represents 1 foot, making conversions intuitive. |
| On a map where 1 inch = 20 miles, two cities are 3.5 inches apart. What is the actual distance? | **70 miles** — 3.5 inches × 20 miles/inch = 70 miles. Map distance problems multiply the measured distance by the miles-per-unit scale. The answer 23.5 incorrectly adds 20+3.5. |
| A model car is built at 1:24 scale. If the model is 7.5 inches long, how long is the real car? | **180 inches (15 feet)** — Real length = 7.5 × 24 = 180 inches = 15 feet. A 1:24 scale means the model is 1/24th the size of the real car. Divide if going from real to model; multiply if going from model to real. |
| Compare these two maps of the same park. Map A (1:10,000) shows the park as 15 cm wide. Map B (1:25,000) shows it as ? cm wide. Calculate the missing width. | **6 cm** — Real width: 15×10,000 = 150,000 cm. Map B: 150,000÷25,000 = 6 cm. The larger scale factor (25,000 vs 10,000) produces a smaller drawing because each cm represents more real distance. |
| A photo is 4 inches by 6 inches. It is enlarged by a scale factor of 3. What are the new dimensions? | **12 inches by 18 inches** — Both dimensions × 3: 4×3=12 and 6×3=18, giving 12 by 18 inches. The answer 7×9 adds 3 instead of multiplying. Both dimensions must be scaled equally to maintain proportions. |
| A map has a scale of 1 cm : 5 km. What does this mean? | **Every 1 centimeter on the map represents 5 kilometers in real life** — A scale of 1 cm : 5 km means 1 cm on the map equals 5 km in real life. The map is a reduced version of reality. This is a ratio comparing map distance to actual distance. |
| A blueprint uses a scale of 1 inch = 3 feet. A wall measures 4 inches on the blueprint. How long is the actual wall? | **12 feet** — 4 inches × 3 ft/inch = 12 feet. Each blueprint inch represents 3 real feet, so you multiply the drawing measurement by the scale factor. The answer 7 incorrectly adds 4+3. |
| The actual distance between two towns is 150 km. If the map scale | **6 cm** — Map distance = 150 ÷ 25 = 6 cm. When going from real distance to map |

| | |
|---|---|
| is 1 cm : 25 km, how far apart are they on the map? | distance, divide by the 'real' part of the scale. The answer 3,750 incorrectly multiplies 150 × 25. |
| A hiking trail map uses a scale of 1:50,000. A trail measures 8.4 cm on the map. How long is the actual trail in kilometers? | **4.2 km** — 8.4 × 50,000 = 420,000 cm = 4,200 m = 4.2 km. Converting cm to km: divide by 100,000. The answer 42 misses one factor of 10 in the conversion. |
| An architect draws a 50-foot hallway using a 1/4 inch = 1 foot scale. How long is the hallway on the drawing? | **12.5 inches** — 50 × 0.25 = 12.5 inches on the drawing. At this scale, 4 inches on the blueprint represents 16 feet. The answer 200 incorrectly multiplies 50×4 instead of 50×0.25. |
| Why must both measurements in a scale factor use the same units? Explain with an example of what goes wrong if they do not. | **Without same units, the ratio is meaningless. If 1 cm : 50 feet, you cannot simplify to 1:50 because cm and feet are different sizes** — If you write 1 cm : 50 ft and simplify to 1:50, this is incorrect because 1 cm ≠ 1 foot. Converting: 50 ft = 1,524 cm, so the actual scale is 1:1,524. Using mismatched units produces a wildly inaccurate scale factor. |
| Design a set of instructions to create a scale drawing of your classroom. What scale would you choose and why? | **Use 1 cm = 1 meter (1:100) for a room about 10m × 8m, producing a drawing about 10cm × 8cm that fits on standard paper** — Choosing 1 cm = 1 m (1:100) means a 10m × 8m room draws as 10cm × 8cm, fitting easily on paper. The scale should produce a drawing that fits the available paper while being large enough to show detail. Too large a scale and the drawing overflows; too small and details are lost. |
| A flag is 5 feet by 8 feet. A miniature version uses a scale factor of 1/4. What are the miniature's dimensions? | **1.25 feet by 2 feet** — 5 × 0.25 = 1.25 ft and 8 × 0.25 = 2 ft. The miniature is 1.25 by 2 feet. A scale factor less than 1 reduces the size. The ratio 1.25:2 = 5:8, maintaining the original proportions. |
| A park model uses a 1:500 scale. A pond in the model is 6 cm long. What is the real pond's length in meters? | **30 meters** — 6 × 500 = 3,000 cm = 30 m. Converting: 3,000 cm ÷ 100 = 30 m. The answer 3,000 forgets to convert centimeters to meters. Always check your final units match the question. |
| Two routes on a map: Route A measures 5.2 cm and Route B measures 4.8 cm. Scale: 1 cm = 15 km. How much longer is Route A in actual distance? | **6 km** — Map difference: 5.2 - 4.8 = 0.4 cm. Actual difference: 0.4 × 15 = 6 km. Route A is 6 km longer. You can scale the difference directly or calculate each distance separately: 78 km - 72 km = 6 km. |
| A blueprint room measures 2.5 inches wide. At 1/8 inch = 1 foot scale, what is the actual width? | **20 feet** — 2.5 ÷ 0.125 = 20 feet. Each 1/8 inch represents 1 foot, so divide the measurement by 1/8 (or multiply by 8). The answer 10 incorrectly uses 1/4 inch scale. |
| What is the scale factor if a 200-meter building is drawn as 10 cm on a blueprint? | **1:2,000** — Convert: 200 m = 20,000 cm. Scale = 10 cm : 20,000 cm = 1:2,000. The answer 1:20 forgets to convert meters to centimeters. Always use the same units when finding a scale factor. |
| A triangle has sides 3, 4, and 5 cm. After enlargement, the longest side is 15 cm. What are the other two sides? | **9 cm and 12 cm** — Scale factor = 15/5 = 3. Other sides: 3×3=9 and 4×3=12. The triangle 9-12-15 is a scaled version of 3-4-5. Check proportionality: 9/3=12/4=15/5=3. |
| A student measures a map | |

| | |
|---|---|
| distance as 7 cm, but the actual distance is known to be 280 km. Determine the map's scale and express it as 1 cm : ? km. | **1 cm : 40 km** — Scale = 280 ÷ 7 = 40 km per cm, so the scale is 1 cm : 40 km. This means each centimeter on the map represents 40 kilometers in reality. The answer 273 incorrectly subtracts 280-7. |
| A room on a blueprint is 3 inches by 4 inches. Scale: 1 inch = 4 feet. What is the actual room area? | **192 square feet** — Actual dimensions: 3×4=12 ft by 4×4=16 ft. Area = 12×16 = 192 sq ft. The answer 48 only scales one dimension. The answer 12 gives the blueprint area. Always scale both dimensions before finding area. |
| An image is reduced to 60% of its original size, then that result is enlarged by 150%. What percentage of the original size is the final image? | **90%** — 0.60 × 1.50 = 0.90 = 90% of original. The reductions and enlargements multiply together. 210% incorrectly adds 60%+150%. Sequential percentage changes always compound multiplicatively. |
| A map uses two different scales: 1:25,000 for urban areas and 1:100,000 for rural areas. A road measures 4 cm in the urban section and 3 cm in the rural section. What is the total real distance? | **4 km (1 km + 3 km)** — Urban: 4 cm × 25,000 = 100,000 cm = 1 km. Rural: 3 cm × 100,000 = 300,000 cm = 3 km. Total: 1+3 = 4 km. Different map sections may use different scales — always check which scale applies to each measurement. |
| Explain why doubling the scale of a blueprint quadruples the area of each room on the drawing, not just doubles it. | **Area depends on length × width, and both dimensions double, so the area multiplies by 2 × 2 = 4** — When scale doubles, both length and width double: 2 cm → 4 cm and 3 cm → 6 cm. Original area = 6 sq cm. New area = 24 sq cm = 4× original. Area scales by the square of the linear scale factor because area = length × width. |

## 8. Proportional Reasoning

| ✂ Question (fold →) | Answer |
| --- | --- |
| You are halving a bread recipe that calls for 3¾ cups of flour. How much flour do you need? Express as a mixed number. | **1⅞ cups** — 3¾ = 15/4. Half: 15/4 ÷ 2 = 15/8 = 1⅞ cups. This requires fraction division skill. The answer 1¾ incorrectly halves only the whole number part (3→1.5) and keeps ¾. |
| A recipe uses a 2:3:5 ratio of oil, vinegar, and herbs for dressing. If you need 200 mL total, how much of each ingredient? | **40 mL oil, 60 mL vinegar, 100 mL herbs** — Total parts: 2+3+5=10. Each part: 200/10=20 mL. Oil: 2×20=40, Vinegar: 3×20=60, Herbs: 5×20=100. Check: 40+60+100=200 mL. Three-part ratios work the same as two-part. |
| Two similar rectangles have areas of 18 sq cm and 50 sq cm. What is the ratio of their corresponding sides? | **3:5** — Area ratio: 18:50 = 9:25. Side ratio: √(9:25) = 3:5. Since area scales by the square of the linear scale factor, reverse by taking the square root. The ratio 9:25 is the area ratio, not the side ratio. |
| You need to dilute a cleaning solution from 80% concentration to 20% concentration. For every 1 cup of concentrate, how many cups of water must you add? | **3 cups of water** — 0.80 × 1 = 0.20 × V → V = 4 cups total. Since 1 cup is concentrate, water = 4-1 = 3 cups. Check: 0.80/4 = 0.20 = 20%. The answer 4 gives total volume, not water added. |
| An exchange rate is $1 USD = €0.85 EUR. A jacket costs €119 in Paris. What is the price in USD? | **$140** — USD = €119 ÷ 0.85 = $140. Since €0.85 = $1, you need more dollars than euros to represent the same value. The answer $101.15 incorrectly multiplies 119×0.85 (that converts dollars to euros). |
| A recipe for 4 servings uses 3 cups of flour. How much flour is needed for 10 servings? | **7.5 cups** — Per serving: 3/4 = 0.75 cups. For 10: 0.75 × 10 = 7.5 cups. Or: 3/4 = x/10 → x = 30/4 = 7.5. The answer 30 incorrectly multiplies 3×10 without adjusting for the original serving count. |
| Evaluate whether all circles are similar to each other. If so, explain why. If not, provide a counterexample. | **Yes, all circles are similar because any circle can be scaled to match any other circle — the scale factor equals the ratio of their radii** — All circles are similar because they share the same shape — a perfect round curve. Any circle with radius $r_1$ can be scaled by factor $r_2/r_1$ to match a circle with radius $r_2$. This is unique to circles (and other regular shapes). Rectangles, by contrast, are only similar if they have the same aspect ratio. |
| A photocopier enlarges at 150%. If the original is 8 inches wide, what is the width after TWO consecutive 150% enlargements? | **18 inches** — First enlargement: 8 × 1.5 = 12 inches. Second: 12 × 1.5 = 18 inches. Or: 8 × 1.5² = 8 × 2.25 = 18. Two 150% enlargements equal one 225% enlargement, not 200% (compound, not additive). |
| Convert 90 km/h to m/s. (1 | **25 m/s** — 90 km/h × (1,000 m/km) × (1 h/3,600 s) = 90,000/3,600 = 25 m/s. Both |

| | |
|---|---|
| km = 1,000 m, 1 hour = 3,600 seconds) | conversions happen simultaneously. The answer 324,000 incorrectly multiplies by 3,600 instead of dividing. |
| A car gets 30 miles per gallon. Gas costs $3.50 per gallon. How much does it cost to drive 210 miles? | **$24.50** — Gallons: 210/30 = 7 gallons. Cost: 7 × $3.50 = $24.50. Dimensional analysis: 210 miles × (1 gal/30 miles) × ($3.50/gal) = $24.50. Units cancel correctly: miles cancel, gallons cancel, leaving dollars. |
| Convert 5 miles to feet using dimensional analysis. (1 mile = 5,280 feet) | **26,400 feet** — 5 miles × (5,280 ft/1 mile) = 26,400 feet. The miles unit cancels, leaving feet. Dimensional analysis ensures units cancel correctly: miles × (feet/miles) = feet. |
| A juice concentrate is mixed with water in a 1:4 ratio. If you have 3 cups of concentrate, how much total juice will you make? | **15 cups** — Water: 3 × 4 = 12 cups. Total: 3 + 12 = 15 cups. The ratio 1:4 means concentrate-to-water, not concentrate-to-total. Common error: treating 1:4 as 1/4 of total instead of 1/5. |
| Two rectangles are similar. The first is 6 cm by 10 cm. If the second has a width of 9 cm, what is its length? | **15 cm** — Scale factor: 9/6=1.5. Length: 10×1.5=15 cm. Similar figures have all corresponding dimensions in the same ratio: 9/6 = 15/10 = 1.5. The answer 13 incorrectly adds 3. |
| Evaluate: A recipe is scaled up by 1/3 (making 4/3 of the original). Then the result is scaled up by 1/4 (making 5/4 of that). What is the total scale factor from the original? | **5/3 (the recipe makes 5/3 of the original)** — Total factor = (4/3) × (5/4) = 20/12 = 5/3 ≈ 1.667. The 4s cancel: (4/3)×(5/4) = 5/3. So the final recipe is 5/3 of the original — about 67% more. Sequential scaling multiplies; it does not add fractions. |
| Construct a dimensional analysis chain to convert 45 mph to feet per second. Show each conversion factor. | **45 mi/h × (5,280 ft/mi) × (1 h/3,600 s) = 66 ft/s** — 45 × 5,280/3,600 = 237,600/3,600 = 66 ft/s. The chain: (mi/h) × (ft/mi) × (h/s) = ft/s. Miles cancel with miles, hours cancel with hours, leaving feet per second. This multi-step conversion is the core skill of dimensional analysis. |
| A paint mixture uses 3 parts blue to 2 parts white. How many liters of blue paint are needed to make 25 liters of the mixture? | **15 liters** — Total parts: 3+2=5. Each part: 25/5=5 L. Blue: 3×5=15 L. White: 2×5=10 L. Check: 15+10=25 L and 15:10=3:2. |
| A soup recipe for 6 people uses 4 cups broth, 2 cups vegetables, and 1 cup cream. Scale it for 9 people. | **6 cups broth, 3 cups vegetables, 1.5 cups cream** — Scale factor: 9/6 = 1.5. Broth: 4×1.5=6, Vegetables: 2×1.5=3, Cream: 1×1.5=1.5. Every ingredient must be scaled by the same factor to maintain the recipe's proportions. |
| Analyze: A recipe mixes 2 solutions — one is 10% salt and another is 30% salt. Can you make a 50% salt solution by mixing only these two? Justify your answer. | **No, because the maximum concentration possible from mixing is 30% (using only the stronger solution). You cannot exceed the higher concentration by mixing.** — Impossible. When mixing two solutions, the result always falls between the two concentrations (10% to 30%). No combination of mixing can produce 50%. You would need a solution stronger than 50% to achieve this target. This is the weighted average principle applied to concentrations. |

| | |
|---|---|
| A cookie recipe for 24 cookies uses 2 eggs. You want to make 60 cookies. How many eggs do you need? | **5 eggs** — Scale factor: 60/24 = 2.5. Eggs: 2 × 2.5 = 5. The proportion 2/24 = x/60 gives 24x=120, x=5. Since you cannot use half an egg easily, 5 whole eggs works perfectly. |
| A faucet drips 3 mL per minute. How many liters does it waste in a day? | **4.32 liters** — 3 × 60 × 24 = 4,320 mL/day = 4.32 L/day. That is over 4 liters wasted daily from a single dripping faucet. The answer 4,320 forgets to convert mL to L (divide by 1,000). |
| A triangle with sides 5, 12, and 13 is similar to another triangle whose shortest side is 15. What are the other two sides? | **36 and 39** — Scale factor: 15/5=3. Sides: 12×3=36 and 13×3=39. The triangle 15-36-39 is similar to 5-12-13. Check: $15^2+36^2=225+1296=1521=39^2$ (Pythagorean theorem still holds). |
| If 8 workers can complete a job in 6 days, how many days would it take 12 workers? | **4 days** — Total work = 8 × 6 = 48 worker-days. With 12 workers: 48/12 = 4 days. Inverse proportion: when one quantity doubles, the other halves. The answer 9 incorrectly uses direct proportion. |
| Solution A is 40% acid and Solution B is 70% acid. If you mix 6 liters of A with 4 liters of B, what is the acid concentration of the mixture? | **52%** — Acid from A: 0.40×6=2.4 L. Acid from B: 0.70×4=2.8 L. Total acid: 5.2 L in 10 L total. Concentration: 5.2/10=52%. The answer 55% incorrectly averages the two percentages. |
| A gear with 20 teeth meshes with a gear with 30 teeth. If the small gear turns 90 times, how many times does the large gear turn? | **60 times** — 20×90 = 30×x → 1,800 = 30x → x = 60 turns. Gears with more teeth turn fewer times (inverse proportion). Each time a tooth passes the mesh point, it must match — so turns × teeth is constant. |
| Two similar pentagons have a scale factor of 2:5. If the smaller one has a perimeter of 20 cm, what is the larger one's perimeter? | **50 cm** — Perimeter scale = side scale = 2:5. So 20/P = 2/5 → P = 50 cm. Perimeters scale linearly with the scale factor. The answer 100 incorrectly squares the ratio (that would be for area). |

## 9. Foundational Concepts Review

| ✂ Question (fold →) | Answer |
|---|---|
| Which of these is NOT a valid way to write the ratio '3 to 7'? | **3/7 or 7\3** — The three standard ways to write a ratio are 'a to b', 'a:b', and 'a/b'. Writing '7\3' reverses the order and uses an invalid notation, so it is not a correct representation of 3 to 7. |
| A car gets 30 miles per gallon. Gas costs $3.50 per gallon. How much does it cost to drive 210 miles? | **$24.50** — Gallons: 210/30 = 7 gallons. Cost: 7 × $3.50 = $24.50. Dimensional analysis: 210 miles × (1 gal/30 miles) × ($3.50/gal) = $24.50. Units cancel correctly: miles cancel, gallons cancel, leaving dollars. |
| A photo is 4 inches by 6 inches. It is enlarged by a scale factor of 3. What are the new dimensions? | **12 inches by 18 inches** — Both dimensions × 3: 4×3=12 and 6×3=18, giving 12 by 18 inches. The answer 7×9 adds 3 instead of multiplying. Both dimensions must be scaled equally to maintain proportions. |
| Convert 90 km/h to m/s. (1 km = 1,000 m, 1 hour = 3,600 seconds) | **25 m/s** — 90 km/h × (1,000 m/km) × (1 h/3,600 s) = 90,000/3,600 = 25 m/s. Both conversions happen simultaneously. The answer 324,000 incorrectly multiplies by 3,600 instead of dividing. |
| A test has 40 questions. A student answers 85% correctly. How many questions did they get right? | **34** — 0.85 × 40 = 34 questions correct. The student got 34 out of 40 right. The answer 85 confuses the percentage with the count. |
| Is cross-multiplication valid for inequalities? Test with 2/3 and 5/8. | **Yes for comparing: 2×8=16 and 3×5=15, so 16>15 means 2/3 > 5/8** — Cross-multiplication extends to comparisons: 2×8=16 vs 3×5=15. Since 16>15, we know 2/3 > 5/8. This works because multiplying both sides of a/b vs c/d by bd preserves the inequality when b,d > 0. |
| Why must both measurements in a scale factor use the same units? Explain with an example of what goes wrong if they do not. | **Without same units, the ratio is meaningless. If 1 cm : 50 feet, you cannot simplify to 1:50 because cm and feet are different sizes** — If you write 1 cm : 50 ft and simplify to 1:50, this is incorrect because 1 cm ≠ 1 foot. Converting: 50 ft = 1,524 cm, so the actual scale is 1:1,524. Using mismatched units produces a wildly inaccurate scale factor. |
| A cyclist rides at 12 miles per hour. How far will they ride in 2.5 hours? | **30 miles** — Distance = 12 × 2.5 = 30 miles. When you know speed and time, multiply them to find distance. 14.5 incorrectly adds 12+2.5. 24 only multiplies by 2, forgetting the 0.5. |
| A necklace uses red and blue beads in a 2:5 ratio. If there are 10 red beads, how many blue beads are there? | **25** — Since 10 ÷ 2 = 5, multiply both parts by 5: red = 2×5 = 10 and blue = 5×5 = 25. This maintains the 2:5 ratio with 10 red and 25 blue beads. |
| An item is marked 15% off, bringing the price to $51. What was the original price? | **$60** — If $51 = 85% of original, then original = $51 ÷ 0.85 = $60. Check: 15% of $60 = $9, and $60 - $9 = $51. The answer $58.65 incorrectly adds 15% to $51. |
| Internet Plan A offers 100 Mbps for $50/month. Plan B offers 200 Mbps for | **Plan A at 2 Mbps per dollar vs Plan B at 2.22 Mbps per dollar — Plan B wins** — Plan A: 100÷50 = 2.0 Mbps/$. Plan B: 200÷90 ≈ 2.22 Mbps/$. Plan B gives more |

| | |
|---|---|
| $90/month. Which offers more speed per dollar? | speed per dollar. However, Plan A has a lower total cost — the 'better' choice depends on budget and needs. |
| Which of these is an example of a rate? | **60 miles per hour** — A rate compares two quantities measured in different units. '60 miles per hour' compares distance (miles) to time (hours). The ratio 3:5 compares the same type of thing (fruits), making it a ratio, not a rate. |
| A lemonade recipe uses a 4:1 ratio of water to lemon juice. Which combination follows this ratio? | **8 cups water and 2 cups lemon juice** — 8:2 simplifies to 4:1 (dividing both by 2). The option 4:2 = 2:1, 5:1 stays as 5:1, and 8:4 = 2:1. Only 8 cups water and 2 cups juice maintains the required 4:1 ratio. |
| A store sells 3 notebooks for $6. At this rate, how much would 9 notebooks cost? | **$18** — The ratio is 3 notebooks:$6. Since 9 ÷ 3 = 3, multiply both: 3×3=9 notebooks and $6×3=$18. This means 9 notebooks cost $18, maintaining the same rate. |
| If 4 identical erasers weigh 60 grams, how much do 7 erasers weigh? | **105 grams** — Unit rate: 60÷4 = 15 g/eraser. 7 erasers: 15×7 = 105 g. Or: 4/60 = 7/x → 4x = 420 → x = 105 g. The answer 420 is the cross-product, not the final answer. |
| In a survey, 72 people out of 300 preferred chocolate ice cream. What percentage is this? | **24%** — (72 ÷ 300) × 100 = 0.24 × 100 = 24%. This means about 1 in 4 people surveyed preferred chocolate. The answer 72% confuses the count with the percentage. |
| Construct a dimensional analysis chain to convert 45 mph to feet per second. Show each conversion factor. | **45 mi/h × (5,280 ft/mi) × (1 h/3,600 s) = 66 ft/s** — 45 × 5,280/3,600 = 237,600/3,600 = 66 ft/s. The chain: (mi/h) × (ft/mi) × (h/s) = ft/s. Miles cancel with miles, hours cancel with hours, leaving feet per second. This multi-step conversion is the core skill of dimensional analysis. |
| If a quantity increases by 50% and then increases by 50% again, is the total increase 100%? Prove your answer with a specific example. | **No. Starting at 100: +50% gives 150, then +50% of 150 gives 225. Total increase is 125%, not 100%** — Start with 100. First +50%: 100×1.5=150. Second +50%: 150×1.5=225. Total change: 225-100=125, so 125% increase. Percentages compound multiplicatively (1.5×1.5=2.25), not additively (1.5+0.5=2.0). |
| Convert 150% to a mixed number. | **1 1/2** — 150% = 150/100 = 3/2 = 1 1/2. Percentages over 100% convert to numbers greater than 1. While 3/2 is also correct as an improper fraction, 1 1/2 is the mixed number form. |
| A baker uses 2.5 cups of sugar for 30 cookies. She wants to make 48 cookies. How much sugar does she need? | **4 cups** — 2.5/30 = x/48 → 30x = 120 → x = 4 cups. Or: 2.5÷30 = 1/12 cup per cookie, then (1/12)×48 = 4 cups. The proportion scales the recipe up by a factor of 48/30 = 1.6. |
| A trail mix recipe uses nuts and dried fruit in a 5:3 ratio. If you want 40 ounces total, how many ounces of nuts do you need? | **25 ounces** — Total parts = 5+3 = 8. Each part = 40÷8 = 5 oz. Nuts = 5×5 = 25 oz. Fruit = 3×5 = 15 oz. Check: 25+15=40 oz and 25:15 = 5:3. |
| A model car is built at 1:24 scale. If the model is 7.5 inches long, how long is the real car? | **180 inches (15 feet)** — Real length = 7.5 × 24 = 180 inches = 15 feet. A 1:24 scale means the model is 1/24th the size of the real car. Divide if going from real to model; multiply if going from model to real. |

| | |
|---|---|
| A runner covers 10 kilometers in 50 minutes. What is the unit rate in minutes per kilometer? | **5 minutes per kilometer** — Unit rate = 50 min ÷ 10 km = 5 min/km. This means the runner takes 5 minutes for each kilometer. The rate 0.2 would be km per minute (the inverse), not minutes per km. |
| A shirt originally costs $45. It is 30% off. What is the discount amount? | **$13.50** — 30% of $45 = 0.30 × 45 = $13.50 discount. The sale price would be $45 - $13.50 = $31.50. Note: $31.50 is the sale price, not the discount amount — the question asks for the discount. |
| Your total bill is $54 including 8% tax. What was the pre-tax price? | **$50** — $54 = 1.08 × pre-tax → pre-tax = $54 ÷ 1.08 = $50. Check: $50 + 8% tax ($4) = $54. The answer $49.68 incorrectly subtracts 8% of $54 instead of dividing. |

## 10. Applied Problem Solving

| ✂ Question (fold →) | Answer |
|---|---|
| Create a real-world scenario where knowing that '60% of 250 = 150' would be useful. Explain why the calculation matters. | **A teacher needs 60% of 250 students to pass for the school to meet its target — so 150 students must pass to reach the goal** — Example: If a school needs 60% attendance (of 250 students) for a field trip, they need 150 students. Percentage calculations help set targets, measure progress, and make decisions in business, education, health, and everyday planning. |
| Is it possible for a car to have an average speed of 60 mph for a trip but never actually travel at exactly 60 mph during the trip? Explain. | **Yes, average speed is total distance divided by total time, so the car could vary between speeds above and below 60 mph throughout** — Yes, average speed = total distance ÷ total time. A car could go 40 mph then 80 mph and average 60 mph without ever traveling exactly 60 mph. Average speed summarizes the entire trip, not any single moment. |
| Explain why 3/4 = 6/8 is a proportion but 3/4 = 6/7 is not. | **In 3/4 = 6/8, cross-products are equal (3×8=24=4×6), but in 3/4 = 6/7, they are not (3×7=21≠4×6=24)** — A proportion is an equation stating two ratios are equal. 3×8=24 and 4×6=24 (equal, so proportion). 3×7=21 and 4×6=24 (not equal, so not a proportion). Cross-multiplication is the definitive test for proportionality. |
| A plane flies 2,400 miles in 5 hours with a tailwind, then returns the same distance in 6 hours against the wind. Which leg had a higher average speed, and by how much? | **The outbound leg was faster by 80 mph** — Outbound: 2400÷5 = 480 mph. Return: 2400÷6 = 400 mph. Difference: 480-400 = 80 mph faster on the outbound leg. The tailwind increased speed while the headwind decreased it. |
| Your test score improved from 72 to 90. What is the percent increase? | **25%** — Change = 18. Percent increase = (18/72) × 100 = 25%. The answer 18% uses the point change, not the percentage change. The answer 20% incorrectly divides 18/90. |
| What is 25% of 80? | **20** — 25% of 80 = 0.25 × 80 = 20. You can also think: 25% = 1/4, and 80 ÷ 4 = 20. The answer 25 confuses the percentage with the result. |
| A faucet drips 3 mL per minute. How many liters does it waste in a day? | **4.32 liters** — 3 × 60 × 24 = 4,320 mL/day = 4.32 L/day. That is over 4 liters wasted daily from a single dripping faucet. The answer 4,320 forgets to convert mL to L (divide by 1,000). |
| State A has 6% tax and State B has 9.5% tax. How much more tax do you pay on a $200 item in State B? | **$7.00 more** — State A tax: 0.06 × $200 = $12. State B tax: 0.095 × $200 = $19. Difference: $19 - $12 = $7. You pay $7 more in State B. The tax rate difference is 3.5%, and 3.5% of $200 = $7. |
| Solve for x: 4/x = 12/15 | **x = 5** — Cross-multiplying: 4×15 = 12x → 60 = 12x → x = 5. Check: 4/5 = 0.8 and 12/15 = 0.8. Both ratios are equal, confirming x = 5. |
| Which decimal is equivalent to 0.5%? | **0.005** — 0.5% ÷ 100 = 0.005. Note: 0.5% is less than 1% — it is a very small percentage. The answer 0.05 = 5% and 0.5 = 50%, both much larger than 0.5%. |
| You buy a $60 item with 7.5% tax. What is the tax amount? | **$4.50** — Tax = 0.075 × $60 = $4.50. The total would be $60 + $4.50 = $64.50. The answer $7.50 confuses the percentage rate with the dollar amount. |
| Convert 5 miles to feet using dimensional analysis. (1 mile = | **26,400 feet** — 5 miles × (5,280 ft/1 mile) = 26,400 feet. The miles unit cancels, leaving feet. Dimensional analysis ensures units cancel correctly: miles × |

| | |
|---|---|
| 5,280 feet) | (feet/miles) = feet. |
| A store advertises 'Buy 3, get 1 free' on items that normally cost \$2 each. What is the effective unit price with this deal? | **\$1.50 per item** — You pay 3 × \$2 = \$6 for 4 items total. Unit price = \$6 ÷ 4 = \$1.50 per item. The free item reduces the effective price by 25% from \$2.00 to \$1.50. This is a common real-world unit rate application. |
| A hiking trail map uses a scale of 1:50,000. A trail measures 8.4 cm on the map. How long is the actual trail in kilometers? | **4.2 km** — 8.4 × 50,000 = 420,000 cm = 4,200 m = 4.2 km. Converting cm to km: divide by 100,000. The answer 42 misses one factor of 10 in the conversion. |
| A car's fuel efficiency is 30 miles per gallon. If the tank holds 12 gallons, what is the maximum range? | **360 miles** — Range = 30 mpg × 12 gallons = 360 miles. The units cancel: (miles/gallon) × gallons = miles. The answer 42 incorrectly adds 30+12 instead of multiplying. |
| Are the ratios 12:16 and 9:12 equivalent? Explain. | **Yes, both simplify to 3:4** — 12:16 ÷ 4 = 3:4 and 9:12 ÷ 3 = 3:4. Since both simplify to the same ratio (3:4), they are equivalent. Two ratios are equivalent when they reduce to the same simplest form. |
| In a parking lot with 20 cars and 5 trucks, what is the part-to-part ratio of trucks to cars? | **1:4** — Trucks to cars = 5:20 = 1:4 after dividing by 5. The ratio 4:1 reverses the order. The ratio 5:25 incorrectly uses the total. Always match the order stated in the question. |
| Compare these two maps of the same park. Map A (1:10,000) shows the park as 15 cm wide. Map B (1:25,000) shows it as ? cm wide. Calculate the missing width. | **6 cm** — Real width: 15×10,000 = 150,000 cm. Map B: 150,000÷25,000 = 6 cm. The larger scale factor (25,000 vs 10,000) produces a smaller drawing because each cm represents more real distance. |
| Design a proportion problem where the answer is 42. Write the problem, set up the proportion, and solve it. | **If 6 tickets cost \$36, how much do 7 tickets cost? 6/36 = 7/x → 6x = 252 → x = \$42** — Example: 6 tickets/\$36 = 7 tickets/x → 6x = 252 → x = \$42. The proportion works because 36/6 = 6 (unit rate), and 6×7 = 42. A well-designed problem has realistic context and a clear proportional relationship. |
| Analyze why two runners with speeds of 8 km/h and 480 km/min are actually the same speed despite looking very different. | **480 km/min is not the same — 8 km/h = 0.133 km/min, so they are very different. 480 km/min would be absurdly fast** — 8 km/h = 8÷60 ≈ 0.133 km/min, which is vastly different from 480 km/min. The premise is flawed — 480 km/min equals 28,800 km/h. This highlights why unit conversion is critical when comparing rates. |
| Evaluate this claim: 'If two ratios have the same difference between their terms, they must be equivalent.' Is this true? Give an example. | **False. 3:5 and 1:3 both have a difference of 2, but 3:5 ≠ 1:3** — This is false. Example: 3:5 has difference 2 and 1:3 has difference 2, but 3/5 ≠ 1/3. Ratios are about multiplicative relationships (division), not additive relationships (subtraction). Equal differences do not guarantee equivalence. |
| A recipe uses 2 cups of flour for every 1 cup of sugar. What is the ratio of flour to sugar? | **2:1** — The ratio of flour to sugar is 2:1 because for every 2 cups of flour there is 1 cup of sugar. The order matters: flour is listed first. |
| A recipe uses a 2:3:5 ratio of oil, vinegar, and herbs for dressing. If you need 200 mL total, how much of each ingredient? | **40 mL oil, 60 mL vinegar, 100 mL herbs** — Total parts: 2+3+5=10. Each part: 200/10=20 mL. Oil: 2×20=40, Vinegar: 3×20=60, Herbs: 5×20=100. Check: 40+60+100=200 mL. Three-part ratios work the same as two-part. |

| | |
|---|---|
| A triangle has sides 3, 4, and 5 cm. After enlargement, the longest side is 15 cm. What are the other two sides? | **9 cm and 12 cm** — Scale factor = 15/5 = 3. Other sides: 3×3=9 and 4×3=12. The triangle 9-12-15 is a scaled version of 3-4-5. Check proportionality: 9/3=12/4=15/5=3. |
| Explain two different methods to calculate 15% of 200, and show they give the same answer. | **Method 1: 0.15 × 200 = 30. Method 2: 10% of 200 = 20, plus 5% of 200 = 10, total = 30** — Method 1 (decimal): 0.15 × 200 = 30. Method 2 (benchmark decomposition): 10% of 200 = 20, and 5% = half of 10% = 10, so 20+10 = 30. Breaking percentages into benchmarks (10%, 5%, 1%) is a powerful mental math strategy. |

## 11. Critical Analysis & Evaluation

| ✂ Question (fold →) | Answer |
|---|---|
| Create a word problem where 3:8 is the answer. Explain why your scenario uses that ratio. | **A trail mix has 3 cups of raisins for every 8 cups of nuts, because the recipe portions compare raisins to nuts in a 3:8 relationship** — A valid word problem must compare two quantities in a 3:8 relationship. For example, 3 cups raisins to 8 cups nuts creates a 3:8 ratio. The scenario must clearly define what each number represents and why they are compared. |
| Which of these percentages converts to a fraction that cannot be simplified? | **37%** — 37% = 37/100. Since 37 is prime and does not divide 100, this fraction is already in simplest form. 25/100=1/4, 50/100=1/2, 75/100=3/4 all simplify. |
| A school has 600 students. 55% are girls. How many boys attend the school? | **270** — Boys = 100% - 55% = 45%. 0.45 × 600 = 270 boys. Girls = 0.55 × 600 = 330. Check: 270 + 330 = 600. The answer 330 gives the number of girls, not boys. |
| What fraction in simplest form equals 12.5%? | **1/8** — 12.5% = 12.5/100 = 125/1000 = 1/8 (dividing by 125). Check: 1÷8 = 0.125 = 12.5%. The fraction 1/12 ≈ 8.3% and 1/5 = 20%. |
| What is 0.08 as a percentage? | **8%** — 0.08 × 100 = 8%. Students often confuse 0.08 with 80%, but 80% = 0.80, not 0.08. Pay attention to the position of the decimal point. |
| Why is '12 dollars for 4 pounds of apples' considered a rate? | **It compares two different units: dollars and pounds** — A rate compares quantities with different units. Here, dollars (money) and pounds (weight) are different measurement types. Not all ratios are rates — a ratio of 3 boys to 5 girls uses the same unit (students). |
| A map shows that 1 inch represents 5 miles. Two towns are 3 inches apart on the map. How far apart are they in real life? | **15 miles** — Using the scale ratio 1 inch : 5 miles, multiply: 3 inches × 5 miles/inch = 15 miles. The answer 8 incorrectly adds 5+3. The answer 30 incorrectly multiplies 5 by an extra factor. |
| A park model uses a 1:500 scale. A pond in the model is 6 cm long. What is the real pond's length in meters? | **30 meters** — 6 × 500 = 3,000 cm = 30 m. Converting: 3,000 cm ÷ 100 = 30 m. The answer 3,000 forgets to convert centimeters to meters. Always check your final units match the question. |
| A juice concentrate is mixed with water in a 1:4 ratio. If you have 3 cups of concentrate, how much total juice will you make? | **15 cups** — Water: 3 × 4 = 12 cups. Total: 3 + 12 = 15 cups. The ratio 1:4 means concentrate-to-water, not concentrate-to-total. Common error: treating 1:4 as 1/4 of total instead of 1/5. |
| A blueprint uses a scale of 1 inch = 3 feet. A wall measures 4 inches on the blueprint. How long is the actual wall? | **12 feet** — 4 inches × 3 ft/inch = 12 feet. Each blueprint inch represents 3 real feet, so you multiply the drawing measurement by the scale factor. The answer 7 incorrectly adds 4+3. |
| Evaluate whether all circles are similar to each other. If so, explain why. If not, provide a counterexample. | **Yes, all circles are similar because any circle can be scaled to match any other circle — the scale factor equals the ratio of their radii** — All circles are similar because they share the same shape — a perfect round curve. Any circle with radius $r_1$ can be scaled by factor $r_2/r_1$ to match a circle with radius $r_2$. This is unique to circles (and other regular shapes). Rectangles, by contrast, are only |

| | |
|---|---|
| | similar if they have the same aspect ratio. |
| Solve: x/8 = 15/24 | **x = 5** — Cross-multiplying: 24x = 8×15 → 24x = 120 → x = 5. Check: 5/8 = 0.625 and 15/24 = 0.625. Both ratios equal 0.625, confirming x = 5. |
| A recipe uses milk and flour in a 3:5 ratio. You have 2 cups of milk but want to know if you have enough flour (you have 3 cups). Determine using proportional reasoning. | **You need 3⅓ cups of flour, so 3 cups is not enough** — 3/5 = 2/x → 3x = 10 → x = 10/3 ≈ 3.33 cups. Since you only have 3 cups of flour, you are short by about 1/3 cup. Proportional reasoning reveals whether your ingredients match the required ratio. |
| Explain why calculating tax as a percentage is fairer than charging everyone a flat tax amount of the same dollar value. | **A percentage tax scales with the purchase** — Percentage-based tax is proportional: 8% of $10 = $0.80, while 8% of $1000 = $80. This scales fairly with the purchase amount. A flat tax would disproportionately burden small purchases and be negligible on expensive ones, violating proportional reasoning principles. — expensive items contribute more tax, while small purchases are taxed less, making it proportional. |
| Create a scenario where a higher total cost actually represents a better value. Explain using unit rates. | **A 2-pound bag of rice for $3 vs a 5-pound bag for $6: the larger bag costs more but is $1.20/lb vs $1.50/lb** — Example: 2 lbs for $3 ($1.50/lb) vs 5 lbs for $6 ($1.20/lb). The $6 option costs more total but saves $0.30/lb. This shows why unit rates, not total prices, determine true value. Bulk buying often offers lower unit costs. |
| A student solves 3/x = 9/12 and gets x = 36. What error did they make? | **They multiplied 3×12 instead of cross-multiplying correctly; the answer should be x = 4** — Correct cross-multiplication: 3×12 = 9x → 36 = 9x → x = 36/9 = 4. The student got 36 by multiplying 3×12 but forgot to divide by 9. Check: 3/4 = 0.75 and 9/12 = 0.75. |
| A model airplane is built at a 1:48 scale. If the real plane's wingspan is 96 feet, what is the model's wingspan? | **2 feet** — Model wingspan = 96 ÷ 48 = 2 feet. A 1:48 scale means every 1 unit on the model equals 48 units in reality. So the real measurement is divided by 48 to find the model dimension. |
| You want to leave exactly $6 tip on a meal. If this represents a 15% tip, what was the bill? | **$40** — $6 = 0.15 × bill → bill = $6 ÷ 0.15 = $40. Check: 15% of $40 = 0.15 × $40 = $6. The answer $46 incorrectly adds $6 to $40 (that would be the total with tip). |
| A recipe calls for 3 cups of water for every 1 cup of rice. If you use 4 cups of rice, how many cups of water do you need? | **12** — With a 3:1 ratio, multiply both by 4: rice = 1×4 = 4 cups and water = 3×4 = 12 cups. The answer 7 incorrectly adds 3+4. Always multiply both parts by the same scale factor. |
| Evaluate: Is it reasonable to always use 20% as a tip? Discuss when a different percentage might be appropriate. | **20% is a generous standard, but tips should vary: exceptional service may warrant more, while takeout or counter service often uses 10-15%** — 20% is a common generous standard for sit-down dining, but context matters. Counter service or takeout typically warrants 10-15%. Exceptional service may justify 25%+. Group auto-gratuities are often 18%. Understanding percentages helps navigate these social norms appropriately. |
| A printer produces 120 pages in 4 minutes. Identify the two quantities and their units in this rate. | **120 pages (output) and 4 minutes (time)** — The two quantities are 120 pages (measuring printed output) and 4 minutes (measuring time). A rate always involves two measurements with different units, describing how one quantity relates to another. |
| | |

| | |
|---|---|
| Two trains leave at the same time. Train A goes 55 mph and Train B goes 70 mph. After 3 hours, how much farther has Train B traveled? | **45 miles** — Train A: 55×3=165 miles. Train B: 70×3=210 miles. Difference: 210-165=45 miles. Or: (70-55)×3 = 15×3 = 45 miles. The speed difference of 15 mph compounds over 3 hours. |
| A map uses two different scales: 1:25,000 for urban areas and 1:100,000 for rural areas. A road measures 4 cm in the urban section and 3 cm in the rural section. What is the total real distance? | **4 km (1 km + 3 km)** — Urban: 4 cm × 25,000 = 100,000 cm = 1 km. Rural: 3 cm × 100,000 = 300,000 cm = 3 km. Total: 1+3 = 4 km. Different map sections may use different scales — always check which scale applies to each measurement. |
| Two similar rectangles have areas of 18 sq cm and 50 sq cm. What is the ratio of their corresponding sides? | **3:5** — Area ratio: 18:50 = 9:25. Side ratio: √(9:25) = 3:5. Since area scales by the square of the linear scale factor, reverse by taking the square root. The ratio 9:25 is the area ratio, not the side ratio. |
| A bag has 4 red marbles and 6 blue marbles. What is the ratio of red to total marbles? | **4:10** — The total is 4 + 6 = 10 marbles. The ratio of red to total is 4:10. The ratio 4:6 is part-to-part, not part-to-whole. |

## 12. Advanced Integration & Synthesis

| ✂ Question (fold →) | Answer |
|---|---|
| A gear with 20 teeth meshes with a gear with 30 teeth. If the small gear turns 90 times, how many times does the large gear turn? | **60 times** — 20×90 = 30×x → 1,800 = 30x → x = 60 turns. Gears with more teeth turn fewer times (inverse proportion). Each time a tooth passes the mesh point, it must match — so turns × teeth is constant. |
| Solution A is 40% acid and Solution B is 70% acid. If you mix 6 liters of A with 4 liters of B, what is the acid concentration of the mixture? | **52%** — Acid from A: 0.40×6=2.4 L. Acid from B: 0.70×4=2.8 L. Total acid: 5.2 L in 10 L total. Concentration: 5.2/10=52%. The answer 55% incorrectly averages the two percentages. |
| A meal costs $65. You want to tip 18%. What is the total bill including tip? | **$76.70** — Tip = 0.18 × $65 = $11.70. Total = $65 + $11.70 = $76.70. Or: $65 × 1.18 = $76.70. The answer $11.70 is just the tip, not the total. |
| If 5 notebooks cost $12, which proportion correctly finds the cost of 8 notebooks? | **5/12 = 8/x** — 5/12 = 8/x sets up corresponding ratios: notebooks/dollars = notebooks/dollars. Solving: x = (8×12)/5 = 96/5 = $19.20. Each fraction must compare the same quantities in the same order. |
| Two routes on a map: Route A measures 5.2 cm and Route B measures 4.8 cm. Scale: 1 cm = 15 km. How much longer is Route A in actual distance? | **6 km** — Map difference: 5.2 - 4.8 = 0.4 cm. Actual difference: 0.4 × 15 = 6 km. Route A is 6 km longer. You can scale the difference directly or calculate each distance separately: 78 km - 72 km = 6 km. |
| In a class of 30 students, 12 wear glasses. What fraction of the class wears glasses? | **12/30 or 2/5** — The part-to-whole ratio is 12/30, which simplifies to 2/5 by dividing both by 6. The fraction 12/18 compares glasses-wearers to non-glasses-wearers, which is part-to-part. |
| Maria needs to mix paint in a 2:7 ratio (red to blue). She has 14 mL of red paint. How much blue paint does she need? | **49 mL** — The multiplier is 14÷2=7. So blue = 7×7 = 49 mL. The equivalent ratio is 14:49, which simplifies back to 2:7. Maria needs 49 mL of blue paint. |
| A room on a blueprint is 3 inches by 4 inches. Scale: 1 inch = 4 feet. What is the actual room area? | **192 square feet** — Actual dimensions: 3×4=12 ft by 4×4=16 ft. Area = 12×16 = 192 sq ft. The answer 48 only scales one dimension. The answer 12 gives the blueprint area. Always scale both dimensions before finding area. |
| A student measures a map distance as 7 cm, but the actual distance is known to be 280 km. Determine the map's scale and express it as 1 cm : ? km. | **1 cm : 40 km** — Scale = 280 ÷ 7 = 40 km per cm, so the scale is 1 cm : 40 km. This means each centimeter on the map represents 40 kilometers in reality. The answer 273 incorrectly subtracts 280-7. |

| | |
|---|---|
| Convert 1.25 to a percentage. | **125%** — 1.25 × 100 = 125%. A percentage greater than 100% is valid — it means more than the whole. For example, if sales grew by 125%, they more than doubled. |
| In a class of 12 boys and 18 girls, what is the ratio of boys to girls in simplest form? | **2:3** — Dividing both 12 and 18 by their GCF of 6 gives 2:3. The ratio 3:2 reverses the order, 12:18 is not simplified, and 6:9 is only partially simplified. |
| Which has a lower unit price: 8 oz for \$2.40 or 12 oz for \$3.00? | **12 oz for \$3.00** — \$2.40 ÷ 8 = \$0.30/oz and \$3.00 ÷ 12 = \$0.25/oz. Since \$0.25 < \$0.30, the 12 oz option is cheaper per ounce. Always compare unit rates to find the better deal. |
| Compare two tipping methods: (1) Calculate 15% of the pre-tax bill of \$80. (2) Calculate 15% of the post-tax bill of \$86.40 (8% tax). What is the difference in tip amount? | **\$0.96 difference (\$12.00 vs \$12.96)** — Pre-tax tip: 0.15 × \$80 = \$12.00. Post-tax tip: 0.15 × \$86.40 = \$12.96. Difference: \$0.96. Tipping on the post-tax amount gives a slightly larger tip because you are also tipping on the tax amount. |
| Analyze: A recipe mixes 2 solutions — one is 10% salt and another is 30% salt. Can you make a 50% salt solution by mixing only these two? Justify your answer. | **No, because the maximum concentration possible from mixing is 30% (using only the stronger solution). You cannot exceed the higher concentration by mixing.** — Impossible. When mixing two solutions, the result always falls between the two concentrations (10% to 30%). No combination of mixing can produce 50%. You would need a solution stronger than 50% to achieve this target. This is the weighted average principle applied to concentrations. |
| Purple paint is made by mixing red and blue in a 3:5 ratio. To make 24 liters of purple paint, how much blue paint is needed? | **15 liters** — Total parts = 3+5 = 8. Each part = 24÷8 = 3 liters. Blue = 5 parts × 3 liters = 15 liters. Red = 3×3 = 9 liters. Check: 9+15=24 liters total. |
| The ratio of boys to girls in School A is 3:4. In School B it is 5:7. School A has 210 students and School B has 360. Which school has more boys? | **School B with 150 boys vs School A with 90 boys** — School A: 210 × 3/7 = 90 boys. School B: 360 × 5/12 = 150 boys. School B has 60 more boys. Even though School A's ratio has fewer parts, the actual count depends on both the ratio and total students. |
| What is 2/5 as a percentage? | **40%** — 2 ÷ 5 = 0.40, and 0.40 × 100 = 40%. You can also see that 2/5 = 40/100 by multiplying both by 20. The answer 25% confuses 2/5 with 1/4. |
| A shopper sees: Small bag (5 lb) for \$8.75, Large bag (12 lb) for \$19.80. Determine which is the better value and explain your reasoning. | **Small bag at \$1.75/lb vs Large bag at \$1.65/lb** — Small: \$8.75÷5=\$1.75/lb. Large: \$19.80÷12=\$1.65/lb. The large bag is \$0.10/lb cheaper. When comparing value, always use unit rates rather than total prices or total quantities. |
| A store sells 6 cans of soup for \$9. What is the unit price per can? | **\$1.50** — Unit price = \$9 ÷ 6 = \$1.50 per can. A unit rate has 1 as the denominator. \$0.67 incorrectly divides 6÷9. \$3.00 divides 9÷3 (wrong divisor). \$15 incorrectly adds. |
| A photograph is 4 inches | |

| | |
|---|---|
| wide and 6 inches tall. If it is enlarged so the width is 10 inches, what will the height be? | **15 inches** — 4/6 = 10/x → 4x = 60 → x = 15 inches. The photo's aspect ratio (2:3) is maintained: 10:15 = 2:3. If the height were 12, the ratio would be 10:12 = 5:6, which distorts the image. |
| On a map, 2 cm represents 50 km. If two cities are 7 cm apart on the map, what is the actual distance? | **175 km** — 2/50 = 7/x → 2x = 350 → x = 175 km. The scale ratio is 1:25 (each cm = 25 km), so 7 cm × 25 km/cm = 175 km. The answer 350 forgot to divide by 2. |
| Gas Station A charges $3.60 for 1.2 gallons. Gas Station B charges $4.50 for 1.5 gallons. Which has the lower price per gallon? | **Both charge $3.00 per gallon** — Station A: $3.60÷1.2 = $3.00/gal. Station B: $4.50÷1.5 = $3.00/gal. Both charge exactly $3.00 per gallon. The different totals reflect different amounts purchased, not different rates. |
| Convert 3/4 to a percentage. | **75%** — 3 ÷ 4 = 0.75, and 0.75 × 100 = 75%. Alternatively, 3/4 = 75/100 by multiplying numerator and denominator by 25. The answer 34% misreads the fraction digits. |
| Your restaurant bill is $40. You want to leave a 20% tip. How much is the tip? | **$8.00** — Tip = 0.20 × $40 = $8.00. Total with tip = $40 + $8 = $48. A quick mental math trick: 10% = $4, so 20% = $4 × 2 = $8. The answer $48 is the total bill, not just the tip. |
| Arrange these from least to greatest: 0.35, 30%, 1/4, 0.28. | **1/4, 0.28, 30%, 0.35** — Converting all to decimals: 1/4 = 0.25, 0.28 = 0.28, 30% = 0.30, 0.35 = 0.35. Order: 0.25 < 0.28 < 0.30 < 0.35, so 1/4, 0.28, 30%, 0.35. |

## 13. Rates

| ✂ Question (fold →) | Answer |
|---|---|
| A phone's value depreciates from $1,000 to $600 in one year, then from $600 to $420 the next. Compare the percent decrease each year. | **Year 1: 40% decrease. Year 2: 30% decrease. Depreciation slowed.** — Year 1: (400/1000)×100 = 40%. Year 2: (180/600)×100 = 30%. Although dollar amounts differ ($400 vs $180), the percentage rate of decrease slowed from 40% to 30%, meaning the phone lost value more slowly in Year 2. |
| A store offers 30% off, then an additional 10% off the sale price. Is this the same as 40% off? Calculate for a $200 item. | **No. Sequential discounts give $126 (37% total), not $120 (40% off)** — First discount: $200 × 0.70 = $140. Second: $140 × 0.90 = $126. But 40% off: $200 × 0.60 = $120. Sequential discounts ($126) save less than a single 40% discount ($120) because the second discount applies to a smaller amount. |
| If 6:? = ?:15, and both blanks use the same ratio multiplier, what are the two missing numbers? | **6:10 = 9:15** — If the base ratio is 3:5, then 6:10 (×2) and 9:15 (×3) are both equivalent to 3:5. Check: 6/10=3/5 and 9/15=3/5. Both ratios express the same relationship. |
| A school has a student-to-teacher ratio of 20:1. If there are 5 teachers, how many students attend the school? | **100** — With a 20:1 ratio, 5 teachers means 20 × 5 = 100 students. The answer 25 incorrectly adds 20+5. The answer 105 incorrectly adds the teachers to the students. |
| A photocopier enlarges at 150%. If the original is 8 inches wide, what is the width after TWO consecutive 150% enlargements? | **18 inches** — First enlargement: 8 × 1.5 = 12 inches. Second: 12 × 1.5 = 18 inches. Or: 8 × $1.5^2$ = 8 × 2.25 = 18. Two 150% enlargements equal one 225% enlargement, not 200% (compound, not additive). |
| You are halving a bread recipe that calls for 3¾ cups of flour. How much flour do you need? Express as a mixed number. | **1⅞ cups** — 3¾ = 15/4. Half: 15/4 ÷ 2 = 15/8 = 1⅞ cups. This requires fraction division skill. The answer 1¾ incorrectly halves only the whole number part (3→1.5) and keeps ¾. |
| Critique this statement: 'A 1:100 scale is more detailed than a 1:1000 scale.' Is this correct? Explain using map examples. | **Correct — 1:100 means 1 cm represents 1 m (more detail), while 1:1000 means 1 cm represents 10 m (less detail, covers more area)** — Correct. At 1:100, 1 cm represents just 1 m — small features like furniture are visible. At 1:1000, 1 cm represents 10 m — you can see buildings but not individual rooms. Smaller scale denominators mean more detail but less area coverage. City plans use ~1:1000; country maps use ~1:1,000,000. |
| A student simplifies 14:21 as 7:10. Identify | **The student subtracted 7 from each instead of dividing; the correct answer is 2:3** — The student subtracted 7 from each (14-7=7, 21-7=10), which is wrong. Correct |

| | |
|---|---|
| the error and provide the correct simplification. | method: divide by GCF=7. 14÷7=2 and 21÷7=3, giving 2:3. Ratios are simplified by division, never subtraction. |
| On a map where 1 inch = 20 miles, two cities are 3.5 inches apart. What is the actual distance? | **70 miles** — 3.5 inches × 20 miles/inch = 70 miles. Map distance problems multiply the measured distance by the miles-per-unit scale. The answer 23.5 incorrectly adds 20+3.5. |
| A soup recipe for 6 people uses 4 cups broth, 2 cups vegetables, and 1 cup cream. Scale it for 9 people. | **6 cups broth, 3 cups vegetables, 1.5 cups cream** — Scale factor: 9/6 = 1.5. Broth: 4×1.5=6, Vegetables: 2×1.5=3, Cream: 1×1.5=1.5. Every ingredient must be scaled by the same factor to maintain the recipe's proportions. |
| Find the missing value: 5:8 = 15:? | **24** — The multiplier is 15 ÷ 5 = 3. So 8 × 3 = 24. The equivalent ratio is 15:24. Both parts must be multiplied by the same factor to maintain equivalence. |
| Which fraction converts to exactly 60%? | **3/5** — 60% = 60/100 = 3/5. Check: 3÷5 = 0.60 = 60%. The fraction 6/8 = 75%, 2/3 ≈ 66.7%, and 4/6 ≈ 66.7%. Only 3/5 equals exactly 60%. |
| Solve: 7/10 = x/30 | **x = 21** — Cross-multiplying: 7×30 = 10x → 210 = 10x → x = 21. Check: 7/10 = 0.7 and 21/30 = 0.7. The answer 210 forgot to divide by 10. |
| Convert 0.65 to a percentage. | **65%** — 0.65 × 100 = 65%. Moving the decimal point two places right converts any decimal to a percent. The answer 6.5% only moves one place, and 650% moves three places. |
| Convert 7/8 to a percentage. | **87.5%** — 7 ÷ 8 = 0.875, and 0.875 × 100 = 87.5%. Not all fractions convert to whole-number percentages. The answer 78% just concatenates the digits 7 and 8. |
| A store reduces prices by 35%. If the original price was $80, what is the new price? | **$52** — Discount = 0.35 × $80 = $28. New price = $80 - $28 = $52. Or: 0.65 × $80 = $52. The answer $28 is the amount saved, not the new price. |
| A pizza has 8 slices: 3 pepperoni, 2 mushroom, and 3 cheese. What is the part-to-whole ratio of mushroom slices? | **2:8** — There are 8 total slices. Mushroom to total = 2:8 (or 1:4 simplified). The ratio 2:6 and 2:3 incorrectly use only some slices as the whole. Always use the complete total for part-to-whole. |
| Explain why 5:10 and 1:2 represent the same ratio but 5:10 and 10:5 do not. | **5:10 simplifies to 1:2 by dividing by 5, but 10:5 reverses the comparison direction** — Dividing 5:10 by 5 gives 1:2, so they are equivalent. However, 10:5 = 2:1, which reverses the relationship — it means 2 of the first per 1 of the second, the opposite of 1:2. |
| Worker A assembles 15 toys in 3 hours. Worker B assembles 24 toys in 4 hours. Who works faster? | **Worker B at 6 toys/hour vs 5 toys/hour** — Worker A: 15÷3 = 5 toys/hr. Worker B: 24÷4 = 6 toys/hr. Worker B produces 1 more toy per hour. Comparing unit rates reveals who is faster, regardless of total output. |
| Which ratio is equivalent to 4:6? | **2:3** — Dividing both parts by 2: 4÷2=2 and 6÷2=3, giving 2:3. The ratio 3:2 reverses the order, 4:3 changes the relationship, and 8:18 = 4:9 which is a different ratio. |
| Construct a scenario where a 10% decrease followed by a 10% increase does NOT | **Start at $100: -10% = $90, then +10% of $90 = $99. You lose $1 because the 10% increase applies to a smaller base** — Starting at $100: 10% decrease gives $90. Then 10% increase of $90 = $9, giving $99. You do not return to $100 because the increase applies to the reduced amount ($90), not the original ($100). This asymmetry is a |

| | |
|---|---|
| return to the original value. Explain why. | fundamental property of percentage changes. |
| Why is the order important in a ratio? | **Changing the order changes which quantity is being compared to which** — The order in a ratio matters because it specifies which quantity is compared to which. A ratio of 3:5 means something different from 5:3 — the first represents 3 of one thing per 5 of another. |
| A team has 9 boys and 6 girls. Sarah says the ratio of boys to girls is 2:3. Is she correct? | **No, it should be 3:2** — Dividing both by 3: 9:6 = 3:2, not 2:3. Sarah reversed the order. Boys are listed first, so the correct simplified ratio is 3:2, meaning 3 boys for every 2 girls. |
| The ratio of fiction to nonfiction books on a shelf is 5:3. There are 40 books total. How many are fiction? | **25** — Total parts = 5+3 = 8. Each part = 40÷8 = 5 books. Fiction = 5 parts × 5 books = 25. Nonfiction = 3 × 5 = 15. Check: 25+15 = 40 total books. |
| A car uses 3 gallons of gas to travel 90 miles. Set up a proportion to find how far it can travel on 7 gallons. | **3/90 = 7/x** — The proportion 3/90 = 7/x keeps gallons on top and miles on the bottom in both ratios. Alternatively, 90/3 = x/7 also works. The key is consistency: same units in the same positions. |

## 14. Rates

| ✂ Question (fold →) | Answer |
|---|---|
| Which is a better deal: 20% off a $50 item, or $8 off the same item? | **20% off saves $10 vs $8 off, so 20% off is better** — 20% of $50 = $10 off, which saves $2 more than the flat $8 discount. Percentage discounts depend on the original price — 20% of a $20 item would only be $4, making $8 off better. Always convert to dollar amounts to compare. |
| A jacket costs $80 and is 25% off. What is the sale price? | **$60** — Discount = 0.25 × $80 = $20. Sale price = $80 - $20 = $60. Alternatively, paying 75% of original: 0.75 × $80 = $60. The answer $20 is the discount amount, not the sale price. |
| Two similar pentagons have a scale factor of 2:5. If the smaller one has a perimeter of 20 cm, what is the larger one's perimeter? | **50 cm** — Perimeter scale = side scale = 2:5. So 20/P = 2/5 → P = 50 cm. Perimeters scale linearly with the scale factor. The answer 100 incorrectly squares the ratio (that would be for area). |
| A fruit bowl has 6 bananas and 4 grapes. What is the part-to-part ratio of bananas to grapes in simplest form? | **3:2** — The ratio 6:4 simplifies to 3:2 by dividing both by the GCF of 2. This is part-to-part because it compares bananas directly to grapes, not to the total of 10 fruits. |
| A ratio table starts with 1:6. What is the fifth entry in the table? | **5:30** — Each entry multiplies both parts by the row number. The fifth entry: 1×5 = 5 and 6×5 = 30, giving 5:30. A ratio table generates equivalent ratios systematically. |
| Brand A: 16 oz for $3.20. Brand B: 24 oz for $4.32. Which is the better buy? | **Brand B at $0.18/oz** — Brand A: $3.20÷16 = $0.20/oz. Brand B: $4.32÷24 = $0.18/oz. Brand B is cheaper per ounce ($0.18 < $0.20). Even though Brand B costs more total, it gives more value per ounce. |
| Build a ratio table for 7:3 with four entries. What is the sum of all second-column values? | **30** — The four entries are 7:3, 14:6, 21:9, 28:12. The second-column values are 3, 6, 9, 12. Their sum is 3+6+9+12 = 30. Each value is a multiple of 3. |
| Two rectangles are similar. The first is 6 cm by 10 cm. If the second has a width of 9 cm, what is its length? | **15 cm** — Scale factor: 9/6=1.5. Length: 10×1.5=15 cm. Similar figures have all corresponding dimensions in the same ratio: 9/6 = 15/10 = 1.5. The answer 13 incorrectly adds 3. |
| What is 120% of 50? | **60** — 120% of 50 = 1.20 × 50 = 60. Since 120% is greater than 100%, the result (60) is greater than the original number (50). This represents the original plus 20% more. |
| A school surveys 50 students and finds 30 prefer basketball. If the school has 800 students, predict how many prefer basketball. | **480** — 30/50 = x/800 → 50x = 24,000 → x = 480. Or: 30/50 = 0.6 = 60%, and 0.6 × 800 = 480. This assumes the survey sample is representative of the whole school. |
| A stock price rises from $40 to $52. What is the percent increase? | **30%** — Change = $12. Percent increase = (12/40) × 100 = 30%. The answer 23.1% divides by 52 (the new value) instead of 40 (the original). 130% would be the new value as a percentage of the old. |
| A triangle with sides 5, 12, and 13 is similar to another triangle whose shortest side is 15. What are the | **36 and 39** — Scale factor: 15/5=3. Sides: 12×3=36 and 13×3=39. The triangle 15-36-39 is similar to 5-12-13. Check: $15^2+36^2=225+1296=1521=39^2$ |

| | |
|---|---|
| other two sides? | (Pythagorean theorem still holds). |
| A blueprint room measures 2.5 inches wide. At 1/8 inch = 1 foot scale, what is the actual width? | **20 feet** — 2.5 ÷ 0.125 = 20 feet. Each 1/8 inch represents 1 foot, so divide the measurement by 1/8 (or multiply by 8). The answer 10 incorrectly uses 1/4 inch scale. |
| Evaluate: A recipe is scaled up by 1/3 (making 4/3 of the original). Then the result is scaled up by 1/4 (making 5/4 of that). What is the total scale factor from the original? | **5/3 (the recipe makes 5/3 of the original)** — Total factor = (4/3) × (5/4) = 20/12 = 5/3 ≈ 1.667. The 4s cancel: (4/3)×(5/4) = 5/3. So the final recipe is 5/3 of the original — about 67% more. Sequential scaling multiplies; it does not add fractions. |
| Write the ratio of 15 to 25 as a fraction in simplest form. | **3/5** — Dividing numerator and denominator by 5 gives 15/25 = 3/5. The fraction 5/3 inverts the ratio. 15/25 is correct but not simplified. 5/25 equals 1/5, which is wrong. |
| How does a ratio table help you find equivalent ratios? | **It shows a pattern of multiplying both quantities by the same number** — A ratio table works by multiplying both quantities by the same whole number (1, 2, 3, ...), producing equivalent ratios. Adding the same number to both sides would NOT produce equivalent ratios. |
| A sale sign reads '33⅓% off.' What fraction of the original price do you save? | **1/3** — 33⅓% = 100/3 ÷ 100 = 100/300 = 1/3. This is the exact fraction, unlike 33/100 which only approximates it. Stores often use 33⅓% because it exactly equals the clean fraction 1/3. |
| A town's population grew from 5,000 to 6,000. What is the percent increase? | **20%** — Change = 1,000. Percent increase = (1,000 ÷ 5,000) × 100 = 20%. The answer 16.7% incorrectly divides by the new value (1000/6000). Always divide by the original amount. |
| A bike originally costs $300 but drops to $240. What is the percent decrease? | **20%** — Change = $60. Percent decrease = (60/300)×100 = 20%. The answer 25% divides by 240 (new value) instead of 300 (original). The answer 80% represents what fraction of the original the new price is. |
| A jar has 7 red and 3 green candies. What is the part-to-whole ratio of green candies to all candies? | **3:10** — The total is 7 + 3 = 10 candies. Green to total = 3:10. The ratio 3:7 is part-to-part (green to red). The ratio 7:10 is the part-to-whole ratio for red, not green. |
| Design a real-world scenario where the part-to-whole ratio is 1:4. What could the quantities represent? | **1 out of every 4 students chose pizza as their favorite lunch** — A 1:4 part-to-whole ratio means 1 out of every 4 items belongs to the subgroup, like 1 in 4 students choosing pizza. The option '4 cats and 1 dog' is part-to-part (1:4 dog-to-cat), not part-to-whole. |
| What is 45:60 in simplest form? | **3:4** — 45÷15=3 and 60÷15=4, so the simplest form is 3:4. The ratio 5:6 is wrong because 45/5=9 and 60/5=12, not 5 and 6. |
| An image is reduced to 60% of its original size, then that result is enlarged by 150%. What percentage of the original size is the final image? | **90%** — 0.60 × 1.50 = 0.90 = 90% of original. The reductions and enlargements multiply together. 210% incorrectly adds 60%+150%. Sequential percentage changes always compound multiplicatively. |
| An architect draws a 50-foot hallway using a 1/4 inch = 1 foot scale. How long is the hallway on the drawing? | **12.5 inches** — 50 × 0.25 = 12.5 inches on the drawing. At this scale, 4 inches on the blueprint represents 16 feet. The answer 200 incorrectly multiplies 50×4 instead of 50×0.25. |
| Explain why doubling the scale of a blueprint quadruples the area of | **Area depends on length × width, and both dimensions double, so the area multiplies by 2 × 2 = 4** — When scale doubles, both length and width |

| | |
|---|---|
| each room on the drawing, not just doubles it. | double: 2 cm → 4 cm and 3 cm → 6 cm. Original area = 6 sq cm. New area = 24 sq cm = 4× original. Area scales by the square of the linear scale factor because area = length × width. |

## 15. Rates

| ✂ Question (fold →) | Answer |
|---|---|
| Evaluate whether the ratio 6:9 or 8:12 better represents 'two-thirds'. Explain your reasoning. | **Both represent two-thirds because 6:9 = 2:3 and 8:12 = 2:3** — Both ratios simplify to 2:3, so both represent two-thirds equally well. 6÷3=2 and 9÷3=3; 8÷4=2 and 12÷4=3. Equivalent ratios represent the same relationship regardless of the actual numbers used. |
| The actual distance between two towns is 150 km. If the map scale is 1 cm : 25 km, how far apart are they on the map? | **6 cm** — Map distance = 150 ÷ 25 = 6 cm. When going from real distance to map distance, divide by the 'real' part of the scale. The answer 3,750 incorrectly multiplies 150 × 25. |
| Evaluate: 'If you double both terms of a ratio, the ratio stays the same. But if you double only one term, the ratio doubles.' Is the second claim correct? | **Not exactly — doubling the first term doubles the ratio value, but doubling the second term halves it** — Starting with 3:4 (=0.75): doubling the first gives 6:4 (=1.5, doubled). Doubling the second gives 3:8 (=0.375, halved). Doubling the numerator doubles the ratio; doubling the denominator halves it. They have opposite effects. |
| If 3 pounds of bananas cost $1.80, how much do 10 pounds cost? | **$6.00** — 3/1.80 = 10/x → 3x = 18 → x = $6.00. Or find the unit rate first: $1.80÷3 = $0.60/lb, then $0.60×10 = $6.00. Both methods give the same answer. |
| Juice boxes come in packs of 6 for $4.50 or packs of 10 for $7.00. Which pack has the lower unit cost? | **Pack of 10 at $0.70 each** — Pack of 6: $4.50÷6 = $0.75 each. Pack of 10: $7.00÷10 = $0.70 each. The 10-pack saves $0.05 per box. Larger quantities often (but not always) have lower unit costs. |
| If 2 workers can paint a room in 6 hours, how long would 3 workers take (assuming they work at the same rate)? | **4 hours** — Total work = 2 workers × 6 hours = 12 worker-hours. With 3 workers: 12 ÷ 3 = 4 hours. This is inverse proportionality — as workers increase, time decreases, but total work stays constant. |
| If the ratio of cats to dogs is 5:3, which fraction represents this ratio? | **5/3** — The ratio 5:3 means 5 cats for every 3 dogs, which as a fraction is 5/3. The fraction 3/5 reverses the comparison. 5/8 and 3/8 use the total as denominator. |
| On a blueprint at 1:100 scale, which is larger: a room drawn as a 3 cm × 4 cm rectangle or a room drawn as a 2 cm × 7 cm rectangle? By how much? | **The 2 cm × 7 cm room is larger: at 1:100 it is 2 m × 7 m = 14 sq m, versus the 3 cm × 4 cm room's 3 m × 4 m = 12 sq m — larger by 2 sq m** — At 1:100, multiply each drawn length by 100. Room 1: 3 cm × 4 cm → 3 m × 4 m = 12 sq m. Room 2: 2 cm × 7 cm → 2 m × 7 m = 14 sq m. Room 2 is larger by 2 sq m. |
| An item costs $25 before 8% sales tax. What is the total cost? | **$27.00** — Tax = 0.08 × $25 = $2.00. Total = $25 + $2 = $27.00. Or multiply by 1.08: $25 × 1.08 = $27.00. The answer $33 incorrectly calculates 8 × $25 ÷ 6. |
| A garden has 8 rose bushes and 12 tulip plants. Express the ratio of roses to tulips using a colon. | **8:12** — The ratio of roses to tulips is written as 8:12 using a colon (or 2:3 in simplest form). However, the question asks for the direct colon notation, which is 8:12. 12:8 reverses the order. |
| Three friends split profits in the ratio 2:3:5. If total profit is $200, what does the person with the largest share receive? | **$100** — Total parts = 2+3+5 = 10. Each part = $200÷10 = $20. Largest share (5 parts) = 5×$20 = $100. The other shares are $40 and $60. Check: $40+$60+$100=$200. |

| | |
|---|---|
| Compare two methods for checking if 8:14 and 20:35 are equivalent. | **Method 1: Simplify both (4:7 = 4:7). Method 2: Cross-multiply (8×35=280 = 14×20=280).** — Two valid methods: (1) Simplify both: 8:14÷2=4:7 and 20:35÷5=4:7, so they match. (2) Cross-multiply: 8×35=280 and 14×20=280, which are equal. Both methods confirm equivalence. |
| Faucet A fills a bucket at 2 gallons per 3 minutes. Faucet B fills at 3 gallons per 5 minutes. Which fills faster? | **Faucet A at 0.67 gal/min vs Faucet B at 0.6 gal/min** — Faucet A: 2÷3 ≈ 0.667 gal/min. Faucet B: 3÷5 = 0.6 gal/min. Faucet A fills faster because 0.667 > 0.6. Without converting to the same unit rate, the comparison would be misleading. |
| A phone battery is at 35%. If the full battery holds 4,000 mAh, how much charge remains? | **1,400 mAh** — 35% of 4,000 = 0.35 × 4,000 = 1,400 mAh remaining. The value 2,600 represents the charge already used (65%), not what remains. |
| A school's boy-to-girl ratio is 7:8. If 56 students are boys, how many are girls? | **64** — Each part = 56÷7 = 8 students. Girls = 8 parts × 8 students = 64. Total students = 56+64 = 120. The ratio 56:64 simplifies to 7:8, confirming the answer. |
| A recipe uses 2 teaspoons of salt for every 3 cups of flour. Is this a rate or a ratio? Justify your answer. | **It is a rate because teaspoons and cups are different units of measurement being compared** — This is a rate because it compares quantities in different units (teaspoons vs cups). Even though both measure volume, they are different units. A ratio would compare items in the same unit, like 2 cups of salt to 3 cups of flour. |
| Which of these ratios is already in simplest form? | **7:9** — 7 and 9 share no common factor other than 1 (7 is prime, $9=3^2$), so 7:9 is in simplest form. 4:8=1:2, 6:15=2:5, and 10:25=2:5 can all be simplified further. |
| A flag is 5 feet by 8 feet. A miniature version uses a scale factor of 1/4. What are the miniature's dimensions? | **1.25 feet by 2 feet** — 5 × 0.25 = 1.25 ft and 8 × 0.25 = 2 ft. The miniature is 1.25 by 2 feet. A scale factor less than 1 reduces the size. The ratio 1.25:2 = 5:8, maintaining the original proportions. |
| An exchange rate is $1 USD = €0.85 EUR. A jacket costs €119 in Paris. What is the price in USD? | **$140** — USD = €119 ÷ 0.85 = $140. Since €0.85 = $1, you need more dollars than euros to represent the same value. The answer $101.15 incorrectly multiplies 119×0.85 (that converts dollars to euros). |
| A machine fills 45 bottles in 9 minutes. How many bottles does it fill in 20 minutes? | **100 bottles** — 45/9 = x/20 → 9x = 900 → x = 100 bottles. Unit rate: 45÷9 = 5 bottles/min, then 5×20 = 100. The answer 225 incorrectly multiplies 45×5 then doesn't adjust. |
| A basket has 3 apples and 5 oranges. What is the ratio of apples to oranges? | **3 to 5** — A ratio compares two quantities in order. Since apples are mentioned first, the ratio is 3 to 5. The ratio 5 to 3 reverses the order, while 3 to 8 and 8 to 3 mix in the total. |
| A student says 1/3 = 33%. Is this exact or approximate? Explain the difference. | **Approximate — 1/3 = 33.333...% (repeating), not exactly 33%** — 1 ÷ 3 = 0.3333... (repeating forever), so 1/3 = 33.333...%, not exactly 33%. The difference is 0.333...%, which matters in precise calculations. Some fractions (like 1/3, 1/6, 1/7) produce repeating decimals. |
| How can you tell the difference between a part-to-part ratio and a part-to-whole ratio? | **Part-to-whole compares a subset to the entire group; part-to-part compares two subsets** — A part-to-part ratio compares two subgroups (like boys to girls), while a part-to-whole ratio compares one subgroup to the entire group (like boys to all students). They convey different information about the same data. |
| A map has a scale of 1 cm : 5 km. | **Every 1 centimeter on the map represents 5 kilometers in real life** — A scale of 1 cm : 5 km means 1 cm on the map equals 5 km in real life. The map is |

| | |
|---|---|
| What does this mean? | a reduced version of reality. This is a ratio comparing map distance to actual distance. |
| A cookie recipe for 24 cookies uses 2 eggs. You want to make 60 cookies. How many eggs do you need? | **5 eggs** — Scale factor: 60/24 = 2.5. Eggs: 2 × 2.5 = 5. The proportion 2/24 = x/60 gives 24x=120, x=5. Since you cannot use half an egg easily, 5 whole eggs works perfectly. |

## 16. Rates

| ✂ Question (fold →) | Answer |
|---|---|
| What is the scale factor if a 200-meter building is drawn as 10 cm on a blueprint? | **1:2,000** — Convert: 200 m = 20,000 cm. Scale = 10 cm : 20,000 cm = 1:2,000. The answer 1:20 forgets to convert meters to centimeters. Always use the same units when finding a scale factor. |
| A company's revenue went from $2 million to $2.5 million. Another went from $500,000 to $650,000. Which had a larger percent increase? | **The second company (30% vs 25%)** — Company 1: (500K/2M)×100 = 25%. Company 2: (150K/500K)×100 = 30%. Despite a smaller dollar increase ($150K vs $500K), Company 2 grew faster in percentage terms. Percent increase measures relative growth. |
| A car travels 150 miles in 3 hours. What is the rate? | **150 miles per 3 hours** — The rate is 150 miles per 3 hours (or equivalently, 50 miles per hour as a unit rate). The answer 450 incorrectly multiplies instead of dividing. Rates include both the quantities and their units. |
| A dollhouse is built at 1:12 scale. If a real dining table is 3 feet long and 1.5 feet wide, what are the dollhouse table's dimensions in inches? | **3 inches by 1.5 inches** — Real: 36 in × 18 in. Dollhouse: 36/12 = 3 in × 18/12 = 1.5 in. The 1:12 scale is standard for dollhouses — 1 inch represents 1 foot, making conversions intuitive. |
| A jar is 80% full with 240 mL of water. What is the jar's total capacity? | **300 mL** — 240 = 0.80 × capacity → capacity = 240 ÷ 0.80 = 300 mL. This is a 'reverse percentage' problem where you know the part and percent but need the whole. 192 incorrectly multiplies 240 × 0.80. |
| A factory makes 840 widgets in 7 hours. Another makes 1,080 in 9 hours. Which factory is more efficient? | **Both produce 120 widgets per hour, so they are equally efficient** — Factory 1: 840÷7 = 120 widgets/hr. Factory 2: 1080÷9 = 120 widgets/hr. Both produce at the same rate of 120 widgets per hour, making them equally efficient despite different totals. |
| A recipe serves 4 people and uses 6 eggs. Write a proportion to find eggs needed for 10 people. | **4/6 = 10/x** — 4/6 = 10/x maintains people/eggs in both ratios. Solving: 4x = 60, so x = 15 eggs. You can also write 6/4 = x/10 (eggs/people = eggs/people), which gives the same answer. |
| In a ratio table for 3:4, which pair does NOT belong? (6:8, 9:12, 12:15, 15:20) | **12:15** — 6:8 = 3:4, 9:12 = 3:4, 15:20 = 3:4, but 12:15 = 4:5 (dividing by 3). The pair 12:15 does not simplify to 3:4 and does not belong in the table. |
| If 8 workers can complete a job in 6 days, how many days would it take 12 workers? | **4 days** — Total work = 8 × 6 = 48 worker-days. With 12 workers: 48/12 = 4 days. Inverse proportion: when one quantity doubles, the other halves. The answer 9 incorrectly uses direct proportion. |
| You need to dilute a cleaning solution from 80% concentration to 20% concentration. For every 1 cup of concentrate, how many cups of water must you add? | **3 cups of water** — 0.80 × 1 = 0.20 × V → V = 4 cups total. Since 1 cup is concentrate, water = 4-1 = 3 cups. Check: 0.80/4 = 0.20 = 20%. The answer 4 gives total volume, not water added. |

| | |
|---|---|
| Simplify the ratio 18:24. | **3:4** — Dividing both by 6: 18÷6=3 and 24÷6=4, giving 3:4. While 9:12 and 6:8 are also equivalent, only 3:4 is fully simplified because 3 and 4 share no common factors other than 1. |
| A $120 pair of shoes is marked 40% off. What do you pay? | **$72** — 40% off means you pay 60%: 0.60 × $120 = $72. The discount is $48 (0.40 × 120), so $120 - $48 = $72. The answer $48 is the discount, not what you pay. |
| A paint mixture uses 3 parts blue to 2 parts white. How many liters of blue paint are needed to make 25 liters of the mixture? | **15 liters** — Total parts: 3+2=5. Each part: 25/5=5 L. Blue: 3×5=15 L. White: 2×5=10 L. Check: 15+10=25 L and 15:10=3:2. |
| A recipe for 4 servings uses 3 cups of flour. How much flour is needed for 10 servings? | **7.5 cups** — Per serving: 3/4 = 0.75 cups. For 10: 0.75 × 10 = 7.5 cups. Or: 3/4 = x/10 → x = 30/4 = 7.5. The answer 30 incorrectly multiplies 3×10 without adjusting for the original serving count. |
| Design a set of instructions to create a scale drawing of your classroom. What scale would you choose and why? | **Use 1 cm = 1 meter (1:100) for a room about 10m × 8m, producing a drawing about 10cm × 8cm that fits on standard paper** — Choosing 1 cm = 1 m (1:100) means a 10m × 8m room draws as 10cm × 8cm, fitting easily on paper. The scale should produce a drawing that fits the available paper while being large enough to show detail. Too large a scale and the drawing overflows; too small and details are lost. |
| Complete the ratio table: if 2:5, then 4:? | **10** — In a ratio table, both numbers are multiplied by the same factor. Since 2 × 2 = 4, then 5 × 2 = 10. The ratio 4:10 is equivalent to 2:5 because both simplify to the same relationship. |
| A car travels 240 miles in 4 hours. What is its average speed? | **60 mph** — Speed = 240 ÷ 4 = 60 miles per hour. Speed is a unit rate that compares distance to time. 960 incorrectly multiplies 240×4. Always divide distance by time to find speed. |
| What does a ratio compare? | **Two quantities** — A ratio is a comparison of two quantities that shows the relative size of one quantity to another. It does not compare operations, equations, or shapes. |
| Convert 45% to a fraction in simplest form. | **9/20** — 45% = 45/100. Dividing both by 5: 45÷5=9 and 100÷5=20, giving 9/20. The fraction 4/5 = 80% and 9/10 = 90%, both much larger than 45%. |
| A shadow problem: a 6-ft person casts a 4-ft shadow. A tree casts a 20-ft shadow. Set up a proportion for the tree's height. | **6/4 = x/20** — 6/4 = x/20 compares height/shadow for both objects. Solving: 4x = 120, x = 30 feet. Shadow problems rely on similar triangles where corresponding sides are proportional. |
| Generate three ratios equivalent to 5:2. | **10:4, 15:6, 20:8** — Multiplying both parts: ×2 gives 10:4, ×3 gives 15:6, ×4 gives 20:8. All simplify back to 5:2. The option 15:8 does not simplify to 5:2 (it would need 15/5=3 and 8/5=1.6, which is not whole). |
| In ratio notation, what does the colon (:) represent? | **The word 'to'** — The colon in ratio notation stands for the word 'to', so 3:5 is read as '3 to 5'. While ratios relate to division, the colon specifically represents the comparison relationship 'to'. |
| A lake's water level dropped from 50 feet to 42 feet. What | **16%** — Change = 8 ft. Percent decrease = (8/50)×100 = 16%. The answer 8% uses the raw change as a percentage. The answer 19% divides by 42 instead of 50. Always |

| | |
|---|---|
| is the percent decrease? | use the original as the denominator. |
| Water flows at 3/4 gallon per 1/2 minute. What is the unit rate in gallons per minute? | **1.5 gallons per minute** — (3/4) ÷ (1/2) = (3/4) × (2/1) = 6/4 = 1.5 gallons per minute. When dividing fractions, multiply by the reciprocal. This is a common middle school skill combining fractions with rates. |
| A recipe calls for 250 mL of oil and 750 mL of vinegar. What is the oil-to-vinegar ratio in simplest form? | **1:3** — 250÷250=1 and 750÷250=3, giving 1:3. This means for every 1 part oil, you need 3 parts vinegar. The ratio 1:4 is incorrect because 250×4=1000, not 750. |

## Keep going

You practiced **400 cards** from RatioRealm. Come back to the ones you missed — spaced-out practice makes them stick.

*Spark & Anvil is a 501(c)(3) public charity. Every app, story, and book is free, forever — no ads, no in-app purchases, no tracking.*


### Keep exploring online

| Play it now | Meet the cast | More free books |
|---|---|---|
| `https://spark-and-anvil.org/play/ratiorealm` | `https://spark-and-anvil.org/cast/ratiorealm` | `https://spark-and-anvil.org/books` |

*Scan a code with any phone camera, or type the address. Every app, story, and book is free, forever — no account, no tracking.*