

Numberverse — Flashcards

A Spark & Anvil printable flashcard deck. Quiz yourself on the Numberverse cast's curriculum — one card at a time.

How to use these cards

Fold each sheet down the middle line so the answers are hidden. Read the **question** on the left, say your answer out loud, then **unfold** to check the right side. Testing yourself — then checking — is one of the most powerful ways to remember. Get one wrong? That's the best kind of practice: try it again tomorrow.

Want real flashcards? Cut along the fold line and the rows into cards — question on one side, answer on the other.

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Numberverse — Flashcards

How to use these cards

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1. number-sense

⌂ Question (fold →)	Answer
Without calculating, determine which product is greater: $4 \times 50,000$ or 40×500 .	$4 \times 50,000$ is greater — $4 \times 50,000 = 200,000$ while $40 \times 500 = 20,000$. The first is 10 times greater. Students who choose 'equal' may think moving a factor of 10 between numbers doesn't change the product, but $4 \times 50,000$ has more total magnitude.
A library has 312,875 books. If the library gets 10,000 more books, what is the new total?	322,875 — $312,875 + 10,000 = 322,875$. The ten-thousands digit changes from 1 to 2. 313,875 adds 1,000 instead. 412,875 adds 100,000. 312,975 adds only 100.
Ms. Chen collected 8,462 bottle caps. What is the value of the 4 in this number?	400 — In 8,462: 8 is thousands, 4 is hundreds, 6 is tens, 2 is ones. The 4 in the hundreds place has a value of 400. Choosing 4 confuses digit with value; 40 is the tens place; 4,000 is the thousands place.
What number is represented by 4 ten-thousands + 3 hundreds + 7 ones?	40,307 — 4 ten-thousands = 40,000; 3 hundreds = 300; 7 ones = 7. Sum: 40,307. 43,007 puts the 3 in the thousands place. 40,370 puts the 7 in the tens place. 4,307 makes 4 just thousands.
Three students wrote the number eight hundred six thousand, forty. Who wrote it correctly: Alex (806,040), Jordan (860,040), or Priya (800,640)?	Alex: 806,040 — Eight hundred six thousand = 806,000. Forty = 40. Combined: 806,040 (Alex). Jordan wrote 860,040 (switched the 0 and 6 in thousands). Priya wrote 800,640 (misplaced the 6).
In the number 45,728, what is the value of the digit 5?	5,000 — The 5 is in the thousands place, so its value is 5,000. 500 confuses it with the hundreds place, 50 with the tens place, and 50,000 with the ten-thousands place.
Which statement about the number 880 is true?	The 8 in the hundreds place is 10 times the 8 in the tens place — The hundreds-place 8 equals 800 and the tens-place 8 equals 80. Since $800/80 = 10$, the hundreds 8 is 10 times the tens 8. They are not equal. Adding the digits ($8+8=16$) conflates digit with value.
A school has 2,375 students. If 1,000 more students enroll, how many students are there in total?	3,375 — $2,375 + 1,000 = 3,375$. Only the thousands digit changes. 2,475 adds 100 instead. 3,275 subtracts 100 while adding 1,000. 2,385 adds only 10.
The population of a town is 203,450. How many people is that rounded to the nearest ten-thousand?	200,000 — The ten-thousands digit is 0 (in 203,450). The thousands digit is 3, which is less than 5, so round down to 200,000. 210,000 rounds up incorrectly. 203,000 rounds to the nearest thousand. 204,000 also rounds to the wrong place.
A six-digit number has 7 in the hundred-thousands place, 0 in the ten-thousands place, 4 in the thousands place, and all other digits are 2. What is the number?	704,222 — Place by place: 7(hundred-thousands), 0(ten-thousands), 4(thousands), 2(hundreds), 2(tens), 2(ones) = 704,222. 740,222 swaps the 0 and 4. 700,422 puts the 4 in hundreds. 724,022 misplaces the 0.
Is it possible for a 5-digit	No, all 6-digit numbers are greater than all 5-digit numbers — The largest 5-digit number (99,999) is still less than the smallest 6-digit number (100,000). No

number to be greater than a 6-digit number?	matter the digits, a 6-digit number always exceeds a 5-digit number. Having larger digits cannot overcome a missing place value.
Which of these numbers has a 9 in the ten-thousands place?	194,320 — In 194,320: the ten-thousands digit is 9. In 901,234 the 9 is in the hundred-thousands place. In 219,430 the 9 is in the thousands place. In 349,210 the 9 is also in the thousands place.
Design a 7-digit number where the millions digit is half the ten-thousands digit, and the ten-thousands digit is 8. What could the millions digit be?	4 — Half of 8 is 4, so the millions digit is 4. The number could be 4,080,000 or many other possibilities. 16 is double rather than half. 2 is one-quarter of 8. 8 would make them equal, not half.
Which digit is in the hundred-thousands place of 5,831,046?	8 — Counting from the right: 6 is ones, 4 is tens, 0 is hundreds, 1 is thousands, 3 is ten-thousands, 8 is hundred-thousands. 3 is ten-thousands, 5 is millions, 1 is thousands.
In the number 555,555, how many times greater is the leftmost 5 than the rightmost 5?	100,000 times — The leftmost 5 = 500,000; the rightmost 5 = 5. Dividing: $500,000 / 5 = 100,000$ times greater. Each place is 10x the next, and there are 5 place shifts: $10^5 = 100,000$.
Lena's number has a 3 in the ten-thousands place. Kai's number has a 3 in the thousands place. How many times greater is Lena's 3 than Kai's 3?	10 times — $30,000 / 3,000 = 10$. Moving one place to the left always multiplies the value by 10. 3 times confuses the digit with the multiplier. 100 times would be two places apart.
Amir writes $50,000 + 7,000 + 200 + 60 + 8$ in standard form. What number did he write?	57,268 — $50,000 + 7,000 + 200 + 60 + 8 = 57,268$. 572,068 inserts an extra digit. 57,628 swaps the tens and hundreds. 50,7268 is not a valid standard form number.
Jamal says rounding 750,000 to the nearest hundred-thousand gives 800,000. Sara says it gives 700,000. Who is right and why?	Jamal is right because the ten-thousands digit (5) rounds up — To round to the nearest hundred-thousand, check the ten-thousands digit: 5. Since 5 or more rounds up, 750,000 rounds to 800,000. Sara incorrectly rounded down. The standard convention rounds 5 up.
Write $600,000 + 50,000 + 2,000 + 400 + 30 + 9$ in standard form.	652,439 — Adding each value: $600,000 + 50,000 = 650,000$; $+ 2,000 = 652,000$; $+ 400 = 652,400$; $+ 30 = 652,430$; $+ 9 = 652,439$. 650,439 omits the 2,000. 652,349 swaps the tens and hundreds digits.
Which number is 10,000 more than 489,235?	499,235 — $489,235 + 10,000 = 499,235$. The ten-thousands digit changes from 8 to 9. 490,235 adds only 1,000. 589,235 adds 100,000. 489,335 adds only 100.
How many times greater is the value of the 6 in 6,300 compared to the 6 in 630?	10 times — In 6,300 the 6 represents 6,000. In 630 it represents 600. Since $6,000 / 600 = 10$, the value is 10 times greater. 100 times would be two place shifts; 6 times confuses the digit with the multiplier.
A factory produced 146,500 toys last year. What is this number rounded to the nearest thousand?	147,000 — The hundreds digit is 5. When the rounding digit is 5 or more, round up: 146,500 rounds to 147,000. 146,000 rounds down incorrectly. 150,000 rounds to the nearest ten-thousand. 146,500 is the original, not rounded.
What is the value of the 7 in 372,150?	70,000 — The 7 is in the ten-thousands place, so its value is 70,000. 7,000 mistakes it for thousands, 700 for hundreds, and 700,000 for hundred-thousands.

Maya says the 3 in 43,609 and the 3 in 130,000 have the same value. Is she correct?	No, the second 3 is worth 30,000 while the first is worth 3,000 — In 43,609 the 3 is in the thousands place (value 3,000). In 130,000: 1 is in the hundred-thousands place, 3 is in the ten-thousands place (value 30,000). Since 3,000 does not equal 30,000, Maya is incorrect. 'Yes, both represent 3,000' wrongly assumes the same place. 'The first 3 is worth 300' misidentifies the place in 43,609.
Reina claims that switching the digits 3 and 8 in the number 38,000 would decrease its value by 50,000. Is she correct?	Yes, 38,000 becomes 83,000 which is actually 45,000 more, so she is incorrect — Switching gives 83,000. The difference is $83,000 - 38,000 = 45,000$, and it increases (not decreases). Reina was wrong on both the direction and the amount. 50,000 is a common error from $(8-3) \times 10,000$.

2. number-sense

⌂ Question (fold →)	Answer
Kai has \$5.60 and spends \$2.35 at the bookstore. How much money does he have left?	\$3.25 — $\$5.60 - \$2.35 = \$3.25$. Regroup: borrow from tenths, making it 5 tenths and 10 hundredths, then $10 - 5 = 5$ hundredths and $5 - 3 = 2$ tenths. \$3.35 skips the regrouping. \$3.15 subtracts tenths wrong.
Write the number that is 0.001 more than 4.999.	5.000 or 5 — $4.999 + 0.001 = 5.000$. The thousandths, hundredths, and tenths all carry over: $9+1=10$, carry 1 repeatedly up to the ones place. 4.1000 is incorrect addition. 4.9991 shows place value confusion.
Which decimal is equivalent to the fraction $\frac{7}{100}$?	0.07 — $\frac{7}{100} = 0.07$ (seven hundredths). $0.7 = \frac{7}{10}$ (tenths). $0.007 = \frac{7}{1000}$ (thousandths). $7.0 = \frac{7}{1}$ (ones). Matching the denominator to the place name is key.
What is the value of the 9 in 0.291?	9 hundredths or 0.09 — In 0.291: 2 is tenths, 9 is hundredths (0.09), 1 is thousandths. Choosing 0.9 confuses hundredths with tenths. Choosing 0.009 mistakes it for thousandths.
Between which two whole numbers does 7.824 lie?	Between 7 and 8 — The ones digit is 7, and since there's a nonzero decimal part, 7.824 is between 7 and 8. 'Between 78 and 79' ignores the decimal point. 'Between 7.8 and 7.9' gives tenths, not whole numbers. 'Between 8 and 9' rounds up incorrectly.
Express 2.350 in expanded form using fractions.	$2 + \frac{3}{10} + \frac{5}{100}$ — $2.350 = 2 + \frac{3}{10} + \frac{5}{100}$ (the trailing zero adds nothing). While $2 + \frac{35}{100}$ is numerically equivalent, it's not expanded form. $2 + \frac{3}{10} + \frac{5}{100} + \frac{0}{1000}$ is technically correct but unnecessarily includes $\frac{0}{1000}$.
Which is greater: 0.09 or 0.1?	0.1 is greater — $0.1 = 0.10$, which is greater than 0.09. The common misconception is that $9 > 1$, so $0.09 > 0.1$. But $0.10 > 0.09$ when compared in the same place value.
What is $0.4 + 0.08 + 0.003$ in standard decimal form?	0.483 — 0.4 (tenths) + 0.08 (hundredths) + 0.003 (thousandths) = 0.483. 0.4803 inserts an extra zero. 4.83 moves the decimal point. 0.0483 adds an extra zero before the 4.
Marcus says $0.1 + 0.2 = 0.3$. His calculator shows 0.30000000000000004. Which is correct for a math class?	Marcus is correct; 0.3 is the exact mathematical answer — Mathematically, $0.1 + 0.2 = 0.3$ exactly. The calculator artifact is due to floating-point representation in binary (a computer science concept). For a math class, Marcus's answer is correct. The calculator value is a representation error, not a math error.
Create a decimal number between 3.45 and 3.46 that uses the digit 7.	3.457 — 3.457 is between 3.45 (= 3.450) and 3.46 (= 3.460). It uses the digit 7 in the thousandths place. $3.467 > 3.46$, so it's outside the range. $3.475 > 3.46$. $3.447 < 3.45$.
What is 34.6 divided by 100?	0.346 — $34.6 \div 100 = 0.346$. The decimal shifts two places left. 3.46 divides by 10 (one place). 3,460 multiplies by 100. 0.0346 divides by 1,000 (three places).
Two students measure a pencil. Alex records 15.2 cm. Priya records 15.20 cm. Alex says his measurement is less precise. Analyze this claim.	Alex is right: 15.20 shows precision to hundredths, 15.2 only to tenths — While 15.2 and 15.20 are numerically equal, in measurement, 15.20 indicates precision to the hundredths place (the measurer confirmed zero hundredths). 15.2 is only precise to the tenths place. Equal values does not mean equal precision.
A store marks a shirt at	

\$19.99. About how much would 3 shirts cost? Which estimate is most reasonable?	About \$60 — \$19.99 rounds to \$20. $3 \times \$20 = \60 . The exact answer (\$59.97) is not 'about' — it's exact. \$50 underestimates. \$70 overestimates significantly. Rounding to a friendly number makes estimation easier.
A recipe calls for 0.750 kg of flour. Your scale shows only tenths of a kilogram. Can you measure exactly the right amount?	Yes, because 0.750 kg = 0.8 kg rounded to the nearest tenth — but exactly, it equals 0.75, which is not a single tenth — $0.750 = 0.75$ kg, which falls between 0.7 and 0.8. A scale showing only tenths cannot display 0.75 exactly — you would need to round. Rounding to 0.8 is the nearest tenth (since 5 rounds up). You cannot measure 0.75 exactly with a tenths-only scale.
A package weighs 1.065 kg. Round this to the nearest tenth.	1.1 kg — The tenths digit is 0. The hundredths digit is 6 (≥ 5), so round up: 1.1. Choosing 1.0 rounds down incorrectly. 1.07 rounds to the nearest hundredth. 1.065 is the original unrounded value.
Which digit is in the hundredths place of 12.738?	3 — In 12.738: 7 is tenths, 3 is hundredths, 8 is thousandths. Choosing 7 confuses tenths with hundredths. Choosing 8 confuses thousandths with hundredths. 2 is in the ones place.
A runner's time is 12.408 seconds. What does the 8 represent?	8 thousandths of a second — In 12.408: 4 is tenths, 0 is hundredths, 8 is thousandths. So the 8 represents 8 thousandths of a second (0.008 s). Hundredths would be the second decimal place. Tenths is the first.
How many times greater is 0.3 than 0.03?	10 times — $0.3 / 0.03 = 10$. Each place to the left is 10 times greater, just like with whole numbers. 3 times confuses the digit with the multiplier. 100 times would be two places apart.
A student writes that $0.6 > 0.59$ is wrong because $59 > 6$. Evaluate this reasoning.	The reasoning is flawed; $0.6 = 0.60$, which is greater than 0.59 — The student's error is comparing digits without aligning place values. Writing 0.6 as 0.60 makes it clear: 60 hundredths $>$ 59 hundredths. Ignoring place value and comparing 59 to 6 is the classic 'longer decimal is bigger' misconception.
If you multiply 0.008 by 10 three times in a row, what do you get?	8 — $0.008 \times 10 \times 10 \times 10 = 0.008 \times 1,000 = 8$. The decimal moves three places right. 0.8 is only one multiplication by 10. 80 is four multiplications. 0.08 is only two.
Dina says 0.30 is greater than 0.3 because it has more digits. Is she correct?	No, 0.30 and 0.3 are equal — Trailing zeros after the last nonzero decimal digit do not change the value: $0.30 = 0.3$. Dina's error is a common misconception that more digits always mean a larger number. This only applies to whole numbers, not decimals.
What is the value of the 6 in 3.614?	6 tenths or 0.6 — In 3.614: 3 is ones, 6 is tenths (0.6), 1 is hundredths (0.01), 4 is thousandths (0.004). Confusing tenths with hundredths or thousandths is a common decimal place value error.
How do you write 0.045 in words?	Forty-five thousandths — $0.045 = 45/1000 =$ forty-five thousandths. Forty-five hundredths would be 0.45. Four and five tenths would be 4.5. Four hundred fifty thousandths would be 0.450.
Arrange in order from least to greatest: 0.52, 0.502, 0.5	0.5, 0.502, 0.52 — Rewriting with equal places: $0.500 < 0.502 < 0.520$. A common error is thinking more digits means a larger number (placing $0.502 > 0.52$ because $502 > 52$), but $0.520 > 0.502$.
A scientist measures 0.725 grams of a chemical. She needs exactly 10 times that amount. How much does she	7.25 grams — $0.725 \times 10 = 7.25$. The decimal moves one place right. 72.5 multiplies by 100. 0.0725 divides by 10. Note that 7.250 is numerically equal to 7.25 (trailing zero), so 7.25 grams is the standard answer.

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3. operations

⌕ Question (fold →)	Answer
What is $8,000 - 3,000$?	5,000 — $8,000 - 3,000 = 5,000$. Simply subtract the thousands digits. 4,000 subtracts an extra thousand. 6,000 adds one too many. 11,000 adds instead of subtracting.
What is $4,562 + 3,215$?	7,777 — $2+5=7$ ones, $6+1=7$ tens, $5+2=7$ hundreds, $4+3=7$ thousands = 7,777. No regrouping. 7,677 has a subtraction error in one column. 7,787 and 7,767 swap digits.
Find two 4-digit numbers whose sum is exactly 10,000 and whose difference is 2,222.	6,111 and 3,889 — Using simultaneous conditions: $a + b = 10,000$ and $a - b = 2,222$. Adding: $2a = 12,222$, so $a = 6,111$. Then $b = 3,889$. Check: $6,111 + 3,889 = 10,000$ and $6,111 - 3,889 = 2,222$. $5,000+5,000$ sums to 10,000 but difference is 0. $7,111+2,889=10,000$ but difference is 4,222.
What is $10,000 - 4,567$?	5,433 — $10,000 - 4,567$: borrow across all zeros. $10-7=3$, $9-6=3$, $9-5=4$, $9-4=5$. Answer: 5,433. 5,443 has a tens error. 5,533 has a hundreds error. 6,433 has a thousands error.
Estimate $4,891 + 3,245$ by rounding each number to the nearest thousand.	8,000 — 4,891 rounds to 5,000 ($8 \geq 5$). 3,245 rounds to 3,000 ($2 < 5$). $5,000 + 3,000 = 8,000$. 7,000 rounds both down. 8,100 rounds to hundreds, not thousands. 9,000 rounds both up.
A school collected 12,458 cans in October and 9,763 cans in November. How many cans total?	22,221 — $8+3=11$ (1 carry), $5+6+1=12$ (2 carry), $4+7+1=12$ (2 carry), $2+9+1=12$ (2 carry), $1+0+1=2$. Total: 22,221. Each distractor has a single regrouping error in one column.
A student solves $7,012 - 3,856$ and gets 4,256. Without calculating, can you tell if this is reasonable?	No, it's too high; $7,000 - 4,000 = 3,000$ so the answer should be near 3,000 — Rounding: 7,012 rounds to 7,000 and 3,856 rounds to 4,000. Estimated difference: 3,000. The answer 4,256 is about 1,000 too high, indicating a regrouping error. The actual answer is 3,156.
What is $6,291 - 4,150$?	2,141 — $1-0=1$, $9-5=4$, $2-1=1$, $6-4=2$. Answer: 2,141. 2,041 subtracts the tens incorrectly. 2,151 adds 10 to the tens. 2,241 adds 100 to the hundreds.
What property does this show: $356 + 429 = 429 + 356$?	Commutative property of addition — The commutative property states $a + b = b + a$. The associative property changes grouping: $(a+b)+c = a+(b+c)$. The identity property uses 0: $a + 0 = a$. The distributive property involves multiplication over addition.
A student claims that when you subtract a 4-digit number from a 5-digit number, the answer is always a 4-digit number. Is this true?	Not always; $10,000 - 9,999 = 1$, which is a 1-digit number — Counterexample: $10,000 - 9,999 = 1$ (1 digit). Also $10,500 - 9,999 = 501$ (3 digits). The difference can have anywhere from 1 to 5 digits depending on how close the numbers are. The claim is false.
What is $5,003 - 2,687$?	2,316 — Regrouping across zeros: $5,003 = 4,000 + 900 + 90 + 13$. Then $13-7=6$, $9-8=1$ (after lending to ones), $9-6=3$, $4-2=2$. Answer: 2,316. 3,316 has a thousands-place error. 2,326 and 2,416 have regrouping errors.
A movie theater sold 3,845 tickets on Saturday and 2,976 on Sunday. Use compensation to add: round 2,976 to 3,000 and adjust.	6,821 — Compensation strategy: $3,845 + 3,000 = 6,845$, then subtract the 24 added: $6,845 - 24 = 6,821$. 6,845 forgets to compensate. 6,921 adds the 24 instead. 6,721 subtracts 100 instead of 24.

A delivery truck must visit three warehouses. The distances are: Warehouse A to B is 2,847 km, B to C is 1,956 km, and C back to A is 3,412 km. What is the total round-trip distance?	8,215 km — $2,847 + 1,956 = 4,803$. Then $4,803 + 3,412 = 8,215$ km. 8,115 loses 100 in regrouping. 8,315 adds an extra 100. 7,215 subtracts 1,000.
Two cities are bidding for a stadium. City A's bid is \$425,890 and City B's bid is \$398,450. How much less is City B's bid?	\$27,440 — $425,890 - 398,450$: $0-0=0$, $9-5=4$, $8-4=4$, 5-8 borrow: $15-8=7$, $12-9=2$ (after lending), $3-3=0$ (leading zero). Wait: recalculate. $425,890 - 398,450$: ones $0-0=0$, tens $9-5=4$, hundreds $8-4=4$, thousands 5-8 borrow $15-8=7$, ten-th 2-9 borrow $12-9=2$ (but 2 became 1), hundred-th $4-3-1=0$. Hmm, let me just verify: $398,450 + 27,440 = 425,890$. Yes, \$27,440.
Look at this work: $4,523 + 2,689 = 7,112$. Find and explain the error.	The tens column is wrong: $2 + 8 + 1(\text{carry}) = 11$, not 1; correct answer is 7,212 — Ones: $3+9=12$ (2, carry 1). Tens: $2+8+1=11$ (should be 1, carry 1). The student got 1 in tens but forgot the carry. Hundreds: $5+6+1=12$ (2, carry 1). Thousands: $4+2+1=7$. Correct: 7,212.
Fill in the blank: $2,748 + = 6,000$.	3,252 — $6,000 - 2,748 = 3,252$. Check: $2,748 + 3,252 = 6,000$. 3,352 adds an extra hundred. 3,152 subtracts an extra hundred. 3,262 has a tens-place error.
What is $3,486 + 2,957$?	6,443 — $6+7=13$ (write 3, carry 1). $8+5+1=14$ (write 4, carry 1). $4+9+1=14$ (write 4, carry 1). $3+2+1=6$. Answer: 6,443. 6,343 forgets one carry. 5,443 drops the thousands carry.
A farmer had 50,000 apples. After selling 18,375 apples, how many remain?	31,625 — $50,000 - 18,375 = 31,625$. Borrow: $50,000 = 49,999 + 1$, then $10-5=5$, $9-7=2$, $9-3=6$, $9-8=1$, $4-1=3$. Answer: 31,625. 32,625 has a thousands error. 31,725 and 31,635 have column errors.
Jordan says subtraction is commutative: $500 - 300 = 300 - 500$. Evaluate this claim.	Incorrect: $500 - 300 = 200$, but $300 - 500 = -200$; subtraction is NOT commutative — $500 - 300 = 200$, but $300 - 500 = -200$. The results differ, proving subtraction is not commutative. The commutative property ($a \text{ op } b = b \text{ op } a$) holds for addition and multiplication, but not subtraction or division.
The distance from Town A to Town B is 24,530 km. The distance from Town A to Town C is 16,875 km. How much farther is Town B?	7,655 km — $24,530 - 16,875$: regroup as needed. $10-5=5$, $12-7=5$, $14-8=6$, 3-6 needs borrow: $13-6=7$, $1-1=0$ (leading zero dropped). Wait: let me recompute. $24,530 - 16,875$: ones: $10-5=5$ (borrow), tens: 2-7 borrow: $12-7=5$ (adjusted 5 becomes 4, borrow from hundreds), hundreds: 4-8 borrow: $14-8=6$ (4 becomes 3), thousands: 3-6 borrow: $13-6=7$, ten-thousands: $1-1=0$. Answer: 7,655.
Which strategy is best for mentally calculating $5,998 + 3,467$?	Add $6,000 + 3,467 = 9,467$, then subtract 2 to get 9,465 — Compensation is ideal here: 5,998 is just 2 less than 6,000. Add $6,000 + 3,467 = 9,467$, then subtract 2 = 9,465. Paper algorithm works but isn't 'mental.' Rounding gives only an estimate. Left-to-right still requires regrouping.
Maya adds $37,564 + 8,249$ and gets 45,713. She checks by subtracting: $45,713 - 8,249$. What should she get?	37,564 — Inverse operations: if $37,564 + 8,249 = 45,713$, then $45,713 - 8,249$ must equal 37,564. This is how you check addition with subtraction. 37,464 has a digit error. 45,713 is the sum, not the check result.
Tom subtracted: $8,204 - 5,138 = 3,174$. Without computing fully, identify which column has an error.	The tens column: 0-3 requires borrowing, giving $10-3=7$ (not 7 tens — wait, 0-3 borrows to get $10-3=7$, but hundreds must adjust too) — Ones: 4-8 requires borrowing: $14-8=6$, not 4. Tom forgot to borrow in the ones column. Correct answer: $8,204 - 5,138 = 3,066$. The error is in the ones column — Tom wrote 4 instead of 6, likely subtracting $8-4$ instead of $14-8$.

Three classes raised money: Room A raised \$1,275, Room B raised \$2,890, and Room C raised \$1,435. What is the total?	\$5,600 — $1,275 + 2,890 = 4,165$. Then $4,165 + 1,435 = 5,600$. Total: \$5,600. \$5,500 loses a hundred in regrouping. \$5,700 adds an extra hundred. \$5,590 has a tens-place error.
What is $2,345 + 1,234$?	3,579 — $5+4=9$, $4+3=7$, $3+2=5$, $2+1=3$. No regrouping needed. Answer: 3,579. The distractors represent single-digit addition errors in various columns.

4. operations

⌂ Question (fold →)	Answer
Omar claims $50 \times 60 = 300$. Evaluate his answer.	Incorrect: $5 \times 6 = 30$, with two zeros = 3,000 — $5 \times 6 = 30$ (already has a zero). Add the two trailing zeros from 50 and 60: $30 \times 100 = 3,000$, not 300. Omar only used one trailing zero. 3,600 would be 60×60 . 30,000 adds too many zeros.
What is 85×36 ?	3,060 — $85 \times 6 = 510$. $85 \times 30 = 2,550$. Total: $510 + 2,550 = 3,060$. 2,960 subtracts 100 (carry error). 3,160 adds 100. 3,006 misplaces a zero.
A baker makes 175 cupcakes each day. How many cupcakes does she make in 8 days?	1,400 — 175×8 : $5 \times 8 = 40$ (0 carry 4), $7 \times 8 = 56 + 4 = 60$ (0 carry 6), $1 \times 8 = 8 + 6 = 14$. Answer: 1,400. 1,300 drops 100. 1,450 adds 50 (half-day). 1,200 multiplies only 150×8 .
Explain why 35×20 and 70×10 give the same product.	Both equal 700; doubling one factor and halving the other preserves the product — $35 \times 20 = 700$. $70 \times 10 = 700$. Doubling 35 gives 70; halving 20 gives 10. This is the doubling-halving strategy: $(a \times 2) \times (b / 2) = a \times b$. It's not a coincidence — it's a property of multiplication. Sums being equal is irrelevant to products.
A student multiplies 46×23 and writes partial products as 138 and 920. What error did the student make?	The partial products are correct ($46 \times 3 = 138$, $46 \times 20 = 920$); the answer should be 1,058 — $46 \times 3 = 138$ (correct). $46 \times 20 = 920$ (correct). Sum: 1,058. There is no error in the partial products — the student did it right. The question tests whether students can verify work. 920 includes the zero because we multiply by 20, not 2.
What is $1,205 \times 4$?	4,820 — $5 \times 4 = 20$ (0, carry 2). $0 \times 4 + 2 = 2$. $2 \times 4 = 8$. $1 \times 4 = 4$. Answer: 4,820. 4,800 drops the 20 from 5×4 . 4,920 adds an extra 100.
What is 99×56 ? Use a shortcut.	5,544 — Shortcut: $100 \times 56 = 5,600$. Then subtract $1 \times 56 = 56$. $5,600 - 56 = 5,544$. 5,644 adds 56 instead. 5,444 subtracts 156. 5,600 forgets the subtraction step.
Create a multiplication problem where both factors are 2-digit numbers and the product is between 1,000 and 1,100.	$33 \times 32 = 1,056$ — $33 \times 32 = 1,056$, which is between 1,000 and 1,100. The distractors: $25 \times 40 = 1,000$ (at the boundary, not between), $50 \times 22 = 1,100$ (at the boundary), $15 \times 72 = 1,080$ (also valid, actually). Multiple correct answers exist.
Estimate the product of 389×7 by rounding 389 to the nearest hundred.	2,800 — 389 rounds to 400 (hundreds digit 3, tens digit 8 ≥ 5 , round up). $400 \times 7 = 2,800$. 2,700 rounds 389 down to 390 and uses 390×7 . 3,500 rounds to 500. 2,723 is the exact answer, not an estimate.
Priya's method: to multiply 15×16 , she does $(15 \times 15) + 15 = 225 + 15 = 240$. Is this valid?	Yes, $15 \times 16 = 15 \times (15 + 1) = 15 \times 15 + 15 \times 1 = 225 + 15 = 240$ — This is the distributive property: $15 \times 16 = 15 \times (15 + 1) = 15 \times 15 + 15 \times 1 = 225 + 15 = 240$. Valid for any numbers, not just consecutive ones. $15 \times 15 = 225$ is correct.
What is 300×7 ?	2,100 — $300 \times 7 = 3 \times 7 \times 100 = 21 \times 100 = 2,100$. 210 drops a zero ($3 \times 7 \times 10$). 21,000 adds an extra zero. 2,010 misplaces the zero.
What is 47×12 ?	564 — $47 \times 2 = 94$. $47 \times 10 = 470$. Sum: $94 + 470 = 564$. 464 subtracts 100 (forgets a carry). 574 adds 10 extra. 554 has a tens-column error.
Which expression equals 23×15 ?	$(20 \times 15) + (3 \times 15)$ — $23 \times 15 = (20 + 3) \times 15 = 20 \times 15 + 3 \times 15 = 300 + 45 = 345$. $(23 \times 10) + (23 \times 50) = 230 + 1150 = 1380$ (wrong). $(20 \times 10) + (3 \times 5) = 200 + 15 = 215$

	(incomplete partial products). $(23+15) \times 2 = 76$ (wrong operation).
Without calculating, which is greater: 32×48 or 31×49 ?	$32 \times 48 = 1,536$ and $31 \times 49 = 1,519$; 32×48 is greater — Both pairs sum to 80, but 32 and 48 (difference 16) are closer together than 31 and 49 (difference 18). When the sum is fixed, closer numbers give a larger product. $32 \times 48 = 1,536 > 31 \times 49 = 1,519$. Equal sums does NOT mean equal products.
A farmer plants 9 rows with 2,450 seeds in each row. How many seeds total?	22,050 — $2,450 \times 9$: $0 \times 9 = 0$, $5 \times 9 = 45$ (5 carry 4), $4 \times 9 + 4 = 40$ (0 carry 4), $2 \times 9 + 4 = 22$. Answer: 22,050. 22,500 confuses $2,500 \times 9$. 21,050 subtracts 1,000. 22,150 has a tens error.
Yuki says $25 \times 40 = 25 \times 4 \times 10$. Is she using a valid property?	Yes, this uses the associative property: $25 \times (4 \times 10) = (25 \times 4) \times 10$ — $25 \times 40 = 25 \times (4 \times 10) = (25 \times 4) \times 10 = 100 \times 10 = 1,000$. This is the associative property (changing grouping). Commutative changes order ($a \times b = b \times a$). Distributive involves addition: $a(b+c)$.
What does the partial product 240 represent in the multiplication 68×34 ?	68×4 (ones digit of 34) minus adjustments — actually $8 \times 30 = 240$ — In partial products of 68×34 : $8 \times 4 = 32$, $60 \times 4 = 240$, $8 \times 30 = 240$, $60 \times 30 = 1800$. The value 240 could be either 60×4 or 8×30 . Both equal 240. $68 \times 30 = 2,040$ and $68 \times 4 = 272$, so those are the full row products.
A teacher says the answer to 238×5 must end in 0 or 5. Is this always true when multiplying by 5?	Yes, any whole number times 5 ends in 0 or 5 — Multiplying any whole number by 5 always gives a product ending in 0 or 5. This is because $5 \times \text{even} = \text{ends in } 0$, and $5 \times \text{odd} = \text{ends in } 5$. There are no exceptions. $238 \times 5 = 1,190$ (ends in 0).
Each classroom in a school has 28 desks. There are 32 classrooms. The principal wants to know how many desks there are in total. What is the answer?	896 — 28×32 : $28 \times 2 = 56$, $28 \times 30 = 840$. Total: $56 + 840 = 896$. 806 has an addition error (subtracts 90). 996 adds 100 too many. 886 has a tens error.
What is 256×3 ?	768 — $6 \times 3 = 18$ (8, carry 1). $5 \times 3 + 1 = 16$ (6, carry 1). $2 \times 3 + 1 = 7$. Answer: 768. 758 forgets a carry. 778 adds an extra carry. 668 under-multiplies the hundreds.
Use the distributive property to find 6×48 . Show it as $6 \times (50 - 2)$.	288 — $6 \times 50 = 300$. $6 \times 2 = 12$. $300 - 12 = 288$. 298 subtracts only 2. 278 subtracts 22. 308 adds instead of subtracting.
Tickets to a concert cost \$45 each. If 328 tickets were sold, what was the total revenue?	\$14,760 — $328 \times 5 = 1,640$. $328 \times 40 = 13,120$. Total: $1,640 + 13,120 = 14,760$. \$14,660 has a hundred-dollar error. \$14,860 adds an extra \$100. \$13,760 subtracts \$1,000.
A parking garage has 16 floors. Each floor holds 125 cars. How many cars fit in the garage?	2,000 — $125 \times 16 = 2,000$. Shortcut: $125 \times 8 = 1,000$, so $125 \times 16 = 2,000$. Or: $125 \times 6 = 750$, $125 \times 10 = 1,250$, total 2,000. $1,875 = 125 \times 15$. $2,125 = 125 \times 17$. $1,750 = 125 \times 14$.
What is 40×50 ?	2,000 — $40 \times 50 = 4 \times 5 \times 10 \times 10 = 20 \times 100 = 2,000$. 200 uses only one trailing zero. 20,000 uses too many zeros. 2,500 confuses 4×5 with a different computation.
A school orders 24 boxes of markers. Each box contains 36 markers. How many markers total?	864 — 24×36 : $24 \times 6 = 144$, $24 \times 30 = 720$. Total: $144 + 720 = 864$. 764 loses 100 in addition. 884 adds 20 extra. 824 has a partial product error.

5. operations

⌕ Question (fold →)	Answer
Marcus says that any number divided by 1 equals the number itself, and any number divided by itself equals 1. Is he right about both claims?	Yes, both are true (except dividing 0 by itself is undefined) — $n/1 = n$ for all numbers (identity property of division). $n/n = 1$ for all nonzero numbers. Both claims are correct. The edge case: $0/0$ is undefined (not 0 or 1). But for all positive numbers Marcus encounters at this level, both rules hold.
A gardener has 3,752 seeds to plant in rows of 7. How many complete rows and how many seeds are left?	536 rows with 0 seeds left — $3,752 / 7 = 536$ exactly. Check: $7 \times 536 = 3,752$. No remainder. 535 R7 is impossible (remainder equals divisor). $536 \text{ R}2$ gives $7 \times 536 + 2 = 3,754$. 537 gives $7 \times 537 = 3,759$.
What is the remainder when 29 is divided by 4?	1 — $29 / 4 = 7$ remainder 1. Check: $4 \times 7 + 1 = 29$. Remainder 3 would give $4 \times 7 + 3 = 31$. Remainder 2 gives 30. Remainder 0 gives 28.
Estimate $7,245 / 8$ by using a compatible number.	About 900 — 7,200 is close to 7,245 and divides evenly by 8: $7,200/8=900$. The exact answer is 905 R5, which is not an estimate. 800 uses $6,400/8$. 1,000 uses $8,000/8$.
What is $156 / 6$?	26 — $156 / 6 = 26$. Check: $6 \times 26 = 156$. 25 gives 150 (6 short). 27 gives 162 (6 too many). 24 gives 144.
If the remainder in a division by 6 is 7, what can you conclude?	An error was made; the remainder must be less than the divisor (less than 6) — A remainder of 7 when dividing by 6 is impossible — the remainder must be 0, 1, 2, 3, 4, or 5 (less than 6). This means the quotient should be 1 more and the remainder adjusted: $7 - 6 = 1$.
A bakery packs 6 muffins per box. They baked 1,458 muffins. How many full boxes and how many leftover muffins?	243 boxes with 0 left over — $1,458 / 6 = 243$ exactly. Check: $6 \times 243 = 1,458$. No remainder. $242 \text{ R}6$ is impossible (remainder equals divisor). $243 \text{ R}3$ gives $6 \times 243 + 3 = 1,461$. Divisibility test: sum of digits 18 is divisible by 3, and the number is even.
Create a division problem with a 3-digit dividend, a 1-digit divisor, and a remainder of exactly 3.	103 / 5 = 20 R3 — $103 / 5 = 20 \text{ R}3$. Check: $5 \times 20 + 3 = 103$. Many answers work: any (divisor $\times$ quotient + 3) with a 3-digit result. $100/5=20$ has R0. $103/4=25 \text{ R}3$ is also correct! $153/5=30 \text{ R}3$ is also correct!
Which equation can be used to check that $432 / 8 = 54$?	8 x 54 = 432 — To check division, multiply: divisor $\times$ quotient = dividend. So $8 \times 54 = 432$. The other equations show incorrect inverse operations. 432×8 gives 3,456. $54 + 8 = 62$.
What is $84 / 7$?	12 — $84 / 7 = 12$ because $7 \times 12 = 84$. 11 gives 77, 13 gives 91, 14 gives 98. Division is the inverse of multiplication.
What is $847 / 3$?	282 R1 — $847 / 3$: $8/3=2 \text{ R}2$, bring down 4: $24/3=8$, bring down 7: $7/3=2 \text{ R}1$. Answer: 282 R1. Plain 282 forgets the remainder. 283 rounds up instead of noting remainder. $281 \text{ R}2$: $3 \times 281 + 2 = 845$, not 847.
What is $5,607 / 9$?	623 — $5,607 / 9$: $56/9=6 \text{ R}2$, bring down 0: $20/9=2 \text{ R}2$, bring down 7: $27/9=3$. Answer: 623. Check: $9 \times 623 = 5,607$. $622 \text{ R}9$ is impossible (remainder $\geq$ divisor). $623 \text{ R}6$ gives $9 \times 623 + 6 = 5,613$.
A student divides 504 by 7 and gets 72. She checks with	Yes, $7 \times 72 = 504$ confirms the answer — $7 \times 72 = 504$ is correct, confirming $504/7=72$. Multiplication is the standard way to check division. $7 \times 71 = 497$ (not

multiplication: $7 \times 72 = 504$. Is her answer correct?	504). $7 \times 72 = 504$, not 514. You can always verify division by multiplying quotient by divisor.
A student claims: 'Division always makes numbers smaller.' Evaluate this claim.	Not always true: dividing by a number less than 1 (like 0.5) makes the result larger — Dividing by whole numbers greater than 1 does make numbers smaller ($12/3=4$). But dividing by fractions or decimals less than 1 makes numbers larger: $10/0.5=20$. And dividing by 1 keeps the number the same. The claim is only partially true.
A school bus holds 45 passengers. If 287 students need transport, how many buses are needed?	7 buses — $287 / 45 = 6 \text{ R}17$. But in context, you must round UP because the remaining 17 students need a bus too. So 7 buses. This is interpreting the remainder — you can't use 6.37 buses. 6 buses leaves students stranded.
In the division $1,000 / 3 = 333 \text{ R}1$, what does the remainder of 1 represent?	1 unit that cannot be equally split into 3 groups — $1,000 = 3 \times 333 + 1$. The remainder 1 is the amount left over that can't be evenly divided into 3 groups. It's not an error. While $1/3$ would be the fractional continuation, the remainder itself represents 1 whole unit left over.
A rope is 100 meters long. It is cut into pieces of 7 meters each. Is it possible to use ALL the rope with no waste?	No, because $100 / 7 = 14 \text{ R}2$; there will always be 2 meters left — $100/7 = 14 \text{ R}2$. No matter how carefully you cut, 14 pieces of 7m use 98m, leaving 2m that can't make another 7m piece. 7 does 'go into' 100 (14 times) — but not evenly. 15 pieces would need 105m.
Three friends divide \$100 equally. They each get \$33. What should happen to the remaining dollar?	The \$1 remainder must be decided by agreement — it cannot be split into equal whole dollars — $100/3 = 33 \text{ R}1$. Each gets \$33 (total \$99), leaving \$1. In real life they might split the dollar into cents (\$0.33 each with \$0.01 left), or make a group decision. \$34 each would need \$102. The remainder doesn't disappear.
What is $4,500 / 9$?	500 — $4,500 / 9$: think $45 / 9 = 5$, then attach the remaining zeros: 500. 50 drops a zero. 5,000 adds a zero. 450 divides by 10 instead of 9.
A warehouse has 2,184 boxes to load onto trucks. Each truck carries 8 boxes at a time. How many truck loads are needed to move all the boxes?	273 loads — $2,184 / 8 = 273$ exactly. Check: $8 \times 273 = 2,184$. No remainder, so exactly 273 loads. 272 loads would leave 8 boxes (an entire extra load needed). 274 loads is one too many.
A ribbon is 250 cm long. You cut pieces that are 8 cm each. How many complete pieces can you cut?	31 pieces — $250 / 8 = 31 \text{ R}2$. You can cut 31 complete pieces with 2 cm left over. Here, unlike the bus problem, you round DOWN because partial pieces don't count. 32 would need 256 cm. 31.25 isn't a whole piece count.
What is $365 / 5$?	73 — $365 / 5 = 73$. Long division: 5 into 36 = 7 (35), remainder 1, bring down 5 to make 15, $15/5 = 3$. Answer: 73. 72 would give 360. 75 would give 375. 74 would give 370.
A teacher has 135 stickers to share equally among 8 students. How many stickers does each student get?	16 stickers each with 7 left over — $135 / 8 = 16 \text{ R}7$. Each student gets 16 stickers with 7 remaining. Check: $8 \times 16 + 7 = 128 + 7 = 135$. 17 each would need 136. 16 with none left would be 128. 15 R15 is impossible (remainder > divisor).
What is $1,296 / 4$?	324 — $1,296 / 4 = 324$. Long division: 4 into 12=3, 4 into 9=2 R1, bring down 6: $16/4=4$. Check: $4 \times 324 = 1,296$. 314 loses 10 (tens error). 334 adds 10. 321 has ones and tens errors.
Look at this work: $462 / 3 = 154$.	154 is correct: $3 \times 154 = 462$ — $3 \times 154 = 462$ (correct). $3 \times 152 = 456$, not 462

A classmate says the answer is 152. Who is right?

(wrong by 6, which is a common error from miscounting during long division). $3 \times 156 = 468$. Only one answer can be right for a given division.

6. number-sense

⌕ Question (fold →)	Answer
A student rounds 445 to 400 when estimating 445×6 and gets 2,400. The exact answer is 2,670. Analyze the estimation error.	The estimate is 270 below the exact answer (about 10% low) because rounding down by 45 was too aggressive — Rounding 445 to 400 drops 45. Multiplied by 6: $45 \times 6 = 270$ undercount. The estimate 2,400 is about 10% below exact (2,670). Rounding to 450 (2,700) would be much closer. Rounding to 500 (3,000) overestimates by 330.
Create an estimation problem where rounding to the nearest hundred gives a worse estimate than rounding to the nearest ten.	450 + 350: rounding to hundreds gives $400 + 400 = 800$ (error 0); tens gives $450 + 350 = 800$ (same). Better: $449 + 351$: hundreds gives $400 + 400 = 800$, exact=800. Try $245 + 255$: hundreds $200 + 300 = 500$, tens $250 + 260 = 510$, exact=500. — Example: $148 + 152$. Hundreds: $100 + 200 = 300$ (exact: 300, error: 0). Better example: $847 + 153$. Hundreds: $800 + 200 = 1,000$ (exact: 1,000). Try: $549 + 251$. Hundreds: $500 + 300 = 800$ (exact: 800). The key idea: when digits in the tens place are large (close to 50), rounding to hundreds introduces more error.
A store charges \$12.95 per item. You buy 8 items. Estimate the total cost.	About \$104 — \$12.95 rounds to \$13. $\$13 \times 8 = \104 . Exact: \$103.60. $\$96 = \12×8 (rounds down too much). $\$120 = \15×8 (rounds up too far). \$103.60 is exact, not an estimate.
A project manager estimates a job will take between 80 and 120 hours. The team has 100 hours available. Is this enough?	Uncertain: 100 hours falls within the range, so the project might or might not finish on time — With a range of 80-120 hours and 100 available, the outcome is uncertain. Being at the midpoint of the estimate range means about 50% risk of going over. Planning for the maximum (120) is safer but may be unnecessary. The key insight is that range estimates express uncertainty.
A stadium holds 42,500 people. Tickets cost \$37 each. Estimate the total revenue if the stadium is full.	About \$1,600,000 — $40,000 \times \$40 = \$1,600,000$. Exact: $42,500 \times \$37 = \$1,572,500$. \$160,000 drops a zero. \$1,200,000 uses $40,000 \times 30$. \$2,000,000 uses $50,000 \times 40$.
A factory produces 4,872 widgets per day. Estimate production for a 5-day work week.	About 25,000 — 4,872 rounds to 5,000. $5,000 \times 5 = 25,000$. Exact: 24,360. $20,000 = 4,000 \times 5$ (rounds down too far). $30,000 = 6,000 \times 5$ (rounds up too far). 24,360 is exact.
Which is the best estimate for 298×5 ?	1,500 — 298 rounds to 300. $300 \times 5 = 1,500$. Exact answer: 1,490. 1,000 uses 200×5 (too low). 2,000 uses 400×5 (too high). 1,490 is exact, not an estimate.
Why does rounding	

both factors UP when estimating a product always give an overestimate?	Because increasing both factors increases the product beyond the exact value — If both factors increase (round up), the product increases too. For positive numbers: if $a \leq A$ and $b \leq B$, then $a \times b \leq A \times B$. This is always true — there are no exceptions for positive numbers. Commutativity is unrelated to overestimation.
Use compatible numbers to estimate $7,235 \div 8$.	About 900 — 7,200 is close to 7,235 and divides evenly by 8: $7,200/8=900$. Compatible numbers make mental math easier. 800 uses $6,400/8$. 1,000 uses $8,000/8$. 904 is near-exact.
Use clustering to estimate: $48 + 52 + 47 + 53 + 49$.	About 250 — All numbers are close to 50 (within 3). Clustering estimate: $5 \times 50 = 250$. Exact: 249. This is very accurate because the numbers cluster tightly. 200 and 300 are too far from the cluster.
A teacher says estimation is never useful once you have a calculator. Evaluate this claim.	Incorrect: estimation helps verify calculator answers, catch entry errors, and build number sense — Estimation remains valuable even with calculators: it catches data entry errors (typing 45 instead of 4.5), provides quick reasonableness checks, develops number sense, and works when technology isn't available. Calculators are more precise, but estimation is faster for checks.
Nadia drove 247 miles on Monday and 189 miles on Tuesday. About how many miles total?	About 440 miles — Rounding to nearest ten: $250 + 190 = 440$. Exact: 436. Rounding to hundred: $200+200=400$ (less precise). 500 rounds both up to $250+250$. 436 is exact.
Round 34.56 to the nearest whole number.	35 — The tenths digit is 5, so round up: 34.56 rounds to 35. 34 rounds down incorrectly. 34.6 rounds to the nearest tenth. 35.0 is the same as 35 (both correct, but 35 is standard).
Two students estimate 78×42 . Student A rounds to $80 \times 40 = 3,200$. Student B rounds to $80 \times 42 = 3,360$. Whose estimate is better?	Student A's is closer (error 76 vs 84) — Exact: $78 \times 42 = 3,276$. Student A: 3,200 (error 76). Student B: 3,360 (error 84). Student A's estimate is closer here. Rounding fewer factors often helps, but in this case rounding both to 80×40 landed nearer.
At a grocery store, items cost \$3.29, \$5.71, \$2.49, and \$4.51. Without a calculator, estimate the total using the strategy of your choice.	About \$16 (rounding: $\\$3+\\$6+\\$2+\\$5=\\$16$, or $\\$3.30+\\$5.70+\\$2.50+\\$4.50=\\$16$) — Compatible pairs: $\$3.29+\5.71 is almost exactly \$9, and $\$2.49+\4.51 is almost exactly \$7. Total: about \$16. Exact: \$16.00. By pairing numbers that sum to round values, we get a very accurate estimate. Rounding individually also gives \$16.
You have \$50 and want to buy items costing \$7.89, \$12.45, \$8.99, and \$15.50. Estimate whether you have enough.	About $\\$8+\\$12+\\$9+\\$16 = \\$45$, so yes, likely enough — Rounding up: $\$8+\$12+\$9+\$16=\$45$. Exact total: \$44.83. You have enough with about \$5 to spare. Rounding down (\$42) underestimates. The exact total doesn't exceed \$50.

Estimate 63×48 by rounding both to the nearest ten.	3,000 — 63 rounds to 60, 48 rounds to 50. $60 \times 50 = 3,000$. Exact answer: 3,024. 2,500 uses 50×50 . 3,500 uses 70×50 . 3,024 is exact, not an estimate.
Round 2,450 to the nearest thousand.	2,000 — The hundreds digit is 4, which is less than 5, so round down to 2,000. 3,000 rounds up incorrectly. 2,500 rounds to the nearest five hundred. 2,400 rounds to the nearest hundred.
When estimating $3,501 - 2,499$, is it better to round both numbers before or after subtracting?	After: exact difference is 1,002; rounding before gives $4,000 - 2,000 = 2,000$ (much worse) — Rounding before: $4,000 - 2,000 = 2,000$ (100% error!). Exact: 1,002. When numbers are close together, rounding before subtraction magnifies errors because both round in the same direction. Computing first, then rounding gives about 1,000.
Round 6,847 to the nearest hundred.	6,800 — The tens digit is 4, which is less than 5, so round down to 6,800. 6,900 incorrectly rounds up. 6,850 rounds to the nearest fifty, not hundred. 7,000 rounds to the nearest thousand.
Estimate $487 + 312$ by rounding each to the nearest hundred.	800 — 487 rounds to 500, 312 rounds to 300. $500 + 300 = 800$. The exact answer is 799. 700 rounds both down. 900 rounds both up. 799 is exact, not an estimate.
Estimate 9.87×4.12 by rounding each to the nearest whole number.	About 40 — 9.87 rounds to 10, 4.12 rounds to 4. $10 \times 4 = 40$. Exact: 40.6644. $36 = 9 \times 4$ (rounds 9.87 down too aggressively). $50 = 10 \times 5$ (rounds 4.12 up too much).
For estimating a restaurant bill of \$47.83, is it better to round to \$48 or \$50?	It depends on the goal: \$48 is more accurate; \$50 is faster and ensures enough money — The best rounding depends on context. \$48 is closer (better for accuracy). \$50 is easier to compute with and ensures you won't come up short — better for budgeting. Rounding down to \$47 could leave you short on cash. There's no single 'best' rounding.
Use front-end estimation for $5,234 + 3,871$.	About 8,000 — Front-end estimation uses the leading digit of each number: $5,000 + 3,000 = 8,000$. 9,000 would round each to the nearest thousand ($5,000 + 4,000$). 9,100 rounds and adds hundreds. 8,105 is close to exact.
Round 0.0849 to the nearest hundredth.	0.08 — Hundredths digit is 8, thousandths digit is 4 (< 5), so round down: 0.08. 0.09 rounds up incorrectly. 0.085 rounds to the nearest thousandth. 0.1 rounds to the nearest tenth.

7. number-sense

⌂ Question (fold →)	Answer
Evaluate: $48 \div 6 + 2 \times (9 - 5)$	16 — PEMDAS: $(9-5)=4$, then $48/6=8$ and $2 \times 4=8$ (left to right for same-level operations), then $8+8=16$. $12 = 48/4$. $20 = 48/(6+2) \times 4$. $40 = (48/6+2) \times (9-5)$.
A student says the distributive property works for multiplication over subtraction: $5 \times (8-3) = 5 \times 8 - 5 \times 3$. Is this correct?	Yes: $5(8-3) = 5 \times 5 = 25$, and $5 \times 8 - 5 \times 3 = 40 - 15 = 25$; it works — The distributive property works for multiplication over both addition AND subtraction: $a(b-c) = ab - ac$. Both sides equal 25 here. This is always true. It works because subtraction is adding a negative: $b-c = b+(-c)$.
Write an equivalent expression for $4 \times (6 + 9)$ using the distributive property.	$4 \times 6 + 4 \times 9 = 24 + 36 = 60$ — $4 \times (6+9) = 4 \times 6 + 4 \times 9 = 24 + 36 = 60$. Check: $4 \times 15 = 60$. $4 \times 6 \times 9$ multiplies all three (wrong). $(4+6) \times (4+9)$ adds 4 to each (wrong). $4 \times 6 + 9$ only distributes to the first term.
Is $(a - b) - c = a - (b - c)$ always true? Test with $a=10$, $b=5$, $c=2$.	No: $(10-5)-2 = 3$, but $10-(5-2) = 7$; subtraction is not associative — $(10-5)-2 = 3$ and $10-(5-2) = 7$. They are not equal, proving subtraction is NOT associative. In the second case, the minus distributes to make it $10-5+2=7$ (subtracting a subtraction becomes addition). Only when $c=0$ do they coincide.
Evaluate: $3 + 4 \times 5$	23 — Order of operations (PEMDAS): multiply first: $4 \times 5 = 20$, then add: $3 + 20 = 23$. $35 = (3+4) \times 5$ (adds before multiplying). $60 = 3 \times 4 \times 5$. $17 = 3 + 4 + 5 + 5$.
Which property does $7 + 9 = 9 + 7$ demonstrate?	Commutative property of addition — Switching the order of addends ($a + b = b + a$) is the commutative property. Associative changes grouping. Identity uses 0 ($a + 0 = a$). Distributive involves two operations.
Evaluate: $(8 + 2) \times (7 - 3)$	40 — Parentheses first: $(8+2)=10$ and $(7-3)=4$. Then $10 \times 4 = 40$. $20 = 10 \times 2$ (subtracts wrong). $52 = 8 + 2 \times 7 - 3$ (ignores parentheses). $74 = 8 \times 7 + 2 \times 3$.
Which is more efficient to calculate 99×87 : using the standard algorithm or using the distributive property as $(100-1) \times 87$?	Distributive is easier: $8,700 - 87 = 8,613$ — Using the distributive property: $(100-1) \times 87 = 8,700 - 87 = 8,613$. This requires only one easy multiplication and one subtraction. The standard algorithm requires two partial products, carries, and addition. Properties make computation more elegant.
Which expression uses the distributive property to rewrite 8×47 ?	$8 \times 40 + 8 \times 7$ — $8 \times 47 = 8 \times (40 + 7) = 8 \times 40 + 8 \times 7 = 320 + 56 = 376$. This is the distributive property. 47×8 is commutative. $8 + 40 \times 7 = 288$ (wrong). $(8 \times 40) \times 7 = 2,240$ (associative, wrong value).
Create a numerical expression that uses all three: parentheses, an exponent, and multiplication, and evaluates to exactly 100.	$(2 + 3)^2 \times 4 = 25 \times 4 = 100$ — $(2+3)^2 \times 4$: parentheses first: 5. Exponent: $5^2=25$. Multiply: $25 \times 4=100$. Uses parentheses, exponent, and multiplication. The distractors don't equal 100.
Evaluate: $100 - 6 \times (3 + 2^2)$	58 — Inside parentheses: $2^2=4$, then $3+4=7$. Multiply: $6 \times 7=42$. Subtract: $100-42=58$. $22 = 100 - 6 \times 13$. $376 = (100-6) \times 4$. $142 = 100 + 42$ (adds instead of subtracting).
Leo evaluates $2 + 3 \times 4^2$ as	He added $2+3=5$ first, then multiplied by 16; correct answer is $2+(3 \times 16) = 50$ — Leo likely did $(2+3) \times 4^2 = 5 \times 16 = 80$. Correct PEMDAS: $4^2=16$, then $3 \times 16=48$, then

80. Identify his error.	$2+48=50$. His error was adding before multiplying — violating order of operations.
What is the identity element for multiplication?	1 — The multiplicative identity is 1 because $a \times 1 = a$. The additive identity is 0 ($a + 0 = a$). 0 is wrong because $a \times 0 = 0$ (zero property, not identity). The number itself would give $a \times a = a^2$.
Insert parentheses to make this true: $6 + 3 \times 2 = 18$	$(6 + 3) \times 2 = 18$ — $(6+3) \times 2 = 9 \times 2 = 18$. Without parentheses, multiplication comes first: $6 + 6 = 12$. $6+(3 \times 2) = 6+6 = 12$ (same as no parentheses). Parentheses around just (2) don't change anything.
Simplify using properties: $25 \times 13 \times 4$	1,300 — Using commutative and associative properties: $25 \times 13 \times 4 = (25 \times 4) \times 13 = 100 \times 13 = 1,300$. Much easier than doing 25×13 first. $1,200 = 100 \times 12$. $1,000 = 25 \times 40$. $325 = 25 \times 13$.
Explain why $15 \times 0 + 15 = 15$ using properties of operations.	Zero property: $15 \times 0 = 0$. Identity property: $0 + 15 = 15$ — Step 1: $15 \times 0 = 0$ (zero property of multiplication). Step 2: $0 + 15 = 15$ (identity property of addition). Both properties are needed. Commutative property is not involved here.
What does the zero property of multiplication state?	Any number multiplied by 0 equals 0 — The zero property (or annihilation property): $a \times 0 = 0$ for any a . Adding 0 is the identity property of addition. 0 divided by a nonzero number IS defined ($= 0$); division BY 0 is undefined. 0 is not the multiplicative identity (that's 1).
Use the distributive property to factor: $36 + 24$.	$12 \times (3 + 2) = 12 \times 5 = 60$ — GCF of 36 and 24 is 12: $36=12 \times 3$, $24=12 \times 2$. So $36+24 = 12(3+2) = 12 \times 5 = 60$. Actually ALL options are valid: $6(6+4)=60$ and $4(9+6)=60$ too. The GCF version (12) is the most 'factored' form.
A store sells notebooks for \$3 each and pens for \$2 each. Write and evaluate an expression for 5 notebooks and 8 pens.	$5 \times 3 + 8 \times 2 = 31$ — 5 notebooks at \$3 = $5 \times 3 = \$15$. 8 pens at \$2 = $8 \times 2 = \$16$. Total: $\$15 + \$16 = \$31$. $(5+8) \times (3+2) = 13 \times 5 = 65$ (wrong: multiplies total items by total prices). $5 \times 8 + 3 \times 2 = 46$ (swaps quantities and prices).
Evaluate: $3 \times [2 + (5^2 - 10) / 3]$	21 — Step by step: $5^2=25$, $25-10=15$, $15/3=5$, $2+5=7$, $3 \times 7=21$. Each step follows PEMDAS within brackets. $15 = 3 \times 5$ (skips the 2). $27 = 3 \times 9$ (adds $15/3$ wrong). $45 = 3 \times 15$ (skips division).
Use the distributive property to mentally calculate 7×103 .	721 — $7 \times 103 = 7 \times (100 + 3) = 700 + 21 = 721$. $710 = 7 \times 100 + 10$ (adds 10 instead of 21). $731 = 7 \times 100 + 31$. $700 = 7 \times 100$ (forgets the 3).
Does the commutative property work for subtraction? Test with $12 - 5$ and $5 - 12$.	No: $12 - 5 = 7$ but $5 - 12 = -7$; subtraction is not commutative — $12-5=7$ and $5-12=-7$. Since 7 is not equal to -7, subtraction is NOT commutative. The issue isn't about negative numbers existing; it's that changing the order changes the result. Addition and multiplication are commutative; subtraction and division are not.
Evaluate: $2^3 + 4 \times 3$	20 — PEMDAS: $2^3=8$ (exponents first), $4 \times 3=12$ (multiplication next), $8+12=20$ (addition last). $72 = (2+4)^3$. $36 = 2^3 \times 4 + 3$ (incorrect order). $18 = 2 \times 3^2$.
Which property does $(3 \times 5) \times 4 = 3 \times (5 \times 4)$ demonstrate?	Associative property of multiplication — The parentheses (grouping) changed but the order stayed the same: $(3 \times 5) \times 4 = 3 \times (5 \times 4)$. This is the associative property. Commutative would change order: $3 \times 5 = 5 \times 3$. Distributive mixes operations.
Why is the expression $8 / 2(2+2)$ ambiguous?	It's unclear whether to read it as $(8/2)(2+2)=16$ or $8/(2(2+2))=1$ — The ambiguity comes from whether $2(2+2)$ is one term in the denominator or a separate multiplication. $(8/2)(2+2) = 4 \times 4 = 16$. $8/(2(2+2)) = 8/8 = 1$. Mathematicians use explicit fractions or parentheses to avoid this. Both interpretations follow valid (but

different) conventions.

8. number-sense

⌂ Question (fold →)	Answer
Is 1 a factor of every number?	Yes, because every number divided by 1 equals itself with no remainder — 1 is a universal factor: $n/1=n$ for every whole number, with zero remainder. This is true for odd, even, small, and large numbers alike. 1 is always included in any complete factor list.
What is the greatest common factor (GCF) of 18 and 24?	6 — Factors of 18: {1,2,3,6,9,18}. Factors of 24: {1,2,3,4,6,8,12,24}. Common: {1,2,3,6}. Greatest: 6. 3 is common but not the greatest. 12 is a factor of 24 but not 18. 72 is the LCM, not GCF.
Find all factor pairs of 24.	1x24, 2x12, 3x8, 4x6 — Factor pairs: (1,24), (2,12), (3,8), (4,6). Missing 4x6 is common (stopping at 3x8). Listing 6x4 separately from 4x6 is redundant — they're the same pair. Omitting 1x24 forgets the trivial pair.
What is the GCF of 36 and 60?	12 — Factors of 36: {1,2,3,4,6,9,12,18,36}. Factors of 60: {1,2,3,4,5,6,10,12,15,20,30,60}. Common: {1,2,3,4,6,12}. Greatest: 12. 6 is common but not greatest. 180 is the LCM. 18 is a factor of 36 but not 60.
If the GCF of two numbers is 1, what does that tell you about the numbers?	They are relatively prime (share no common factors other than 1) — GCF=1 means the numbers are 'relatively prime' or 'coprime' — they share no factors except 1. Neither needs to be actually prime: 8 and 15 are coprime but composite. Consecutive numbers are always coprime, but coprime numbers need not be consecutive.
Which pair of numbers has a GCF of 1 and an LCM equal to their product?	8 and 9 — GCF(8,9)=1, so LCM=8x9=72. For the others: GCF(4,6)=2, GCF(6,9)=3, GCF(10,15)=5. When numbers are coprime (GCF=1), their LCM equals their product. This is the relationship: GCF x LCM = product.
A student says the LCM of any two numbers is always greater than both numbers. Is this true?	Not always: if one number is a multiple of the other, the LCM equals the larger number — Counterexample: LCM(3,6)=6, which equals the larger number, not exceeding it. When one number divides the other, the LCM is just the larger number. LCM is not always the product either (LCM(4,6)=12, not 24).
What is the least common multiple (LCM) of 4 and 6?	12 — Multiples of 4: 4,8,12,16,20,24... Multiples of 6: 6,12,18,24... First common: 12. LCM = 12. 24 is a common multiple but not the least. 2 is the GCF, not LCM. 6 is just one of the numbers.
Two bells ring together at noon. One rings every 15 minutes, the other every 20 minutes. When do they next ring together?	At 1:00 PM (60 minutes later) — LCM(15,20) = 60 minutes. They next ring together 60 minutes after noon = 1:00 PM. 12:35 is 15+20 (adding instead of finding LCM). 12:30 is the LCM of 15 and 30 (wrong second value). 5:00 uses 15x20=300 minutes.
Two buses leave a station at the same time. Bus A returns every 12 minutes, Bus B every 18 minutes. In 2 hours (120 minutes), how many times do they return at the same time?	3 times (at 36, 72, and 108 minutes) — LCM(12,18) = 36. Multiples of 36 up to 120: 36, 72, 108. That's 3 times. The next would be 144 (>120). 2 times misses one. 4 times would need 144 minutes. 10 = 120/12 counts only Bus A's returns.
A number has exactly three factors: 1, 5, and 25. What must be true about	It is 25, which equals 5 squared (a perfect square of a prime) — The number is $25 = 5^2$. A number has exactly 3 factors when it's the square of a prime (p , p^2 have factors 1, p , p^2). 25 is not prime (it has 3 factors,

this number?	primes have 2). Not every multiple of 5 has exactly these factors.
A coach has 42 players to divide into equal teams with no one left out. What are the possible team sizes?	1, 2, 3, 6, 7, 14, 21, or 42 players per team — Factors of 42: 1,2,3,6,7,14,21,42. Each factor represents a valid team size. 'Only 6 or 7' misses many options. Omitting 1 and 42 forgets the extremes. Not any number works (e.g., 5 players leaves 2 out).
What are the first five multiples of 7?	7, 14, 21, 28, 35 — Multiples of 7: 7,14,21,28,35. While 0 is technically a multiple of every number, 'first five' typically starts at 7×1 . 36 is not a multiple of 7. 1 is not a multiple of 7 ($7 \times 1 = 7$, not 1).
Which divisibility rule helps you quickly determine if a number is divisible by 3?	Add the digits; if the sum is divisible by 3, the number is too — The divisibility rule for 3: sum all digits; if the sum is divisible by 3, so is the number. Last digit 0 or 5 is the rule for 5. Even number is the rule for 2. Last two digits for 4.
Using divisibility rules, is 738 divisible by 6?	Yes: it's even (divisible by 2) and $7+3+8=18$ is divisible by 3 — Divisible by 6 = divisible by both 2 and 3. 738 is even (8 is even), so divisible by 2. Digit sum: $7+3+8=18$, which is divisible by 3. Both conditions met: $738/6=123$. Ending in 8 doesn't guarantee divisibility by 6 alone.
Carlos says every even number is a multiple of 4. Is he correct?	No: 6 is even but not a multiple of 4 ($6/4 = 1.5$) — Counterexample: 6 is even but $6/4=1.5$ (not a whole number). Being even means divisible by 2, not necessarily by 4. However, every multiple of 4 IS even (since $4=2 \times 2$). The converse is not true.
Is 36 a multiple of 9?	Yes, because $9 \times 4 = 36$ — $36 = 9 \times 4$, so 36 is a multiple of 9 (and 9 is a factor of 36). $36/9 = 4$ with no remainder. Being greater than 9 alone doesn't make it a multiple ($37 > 9$ but isn't a multiple of 9).
A number is divisible by both 3 and 4. What is the smallest such number greater than 0?	12 — $\text{LCM}(3,4) = 12$. So 12 is the smallest positive number divisible by both. 7 is neither. 24 is divisible by both but not the smallest. 1 is not divisible by 3 or 4.
Is it possible for two different numbers to have the same set of factors?	No, different positive integers always have different factor sets — Every number n has n as a factor. If two different numbers had the same factor set, each would be a factor of the other, which is impossible for different positive integers (if a divides b and b divides a , then $a=b$). 6 and 12 have different factor sets.
Explain why 15 is the GCF of 15, 30, and 45.	15 divides all three ($15/15=1$, $30/15=2$, $45/15=3$) and no larger number divides all three — 15 is the GCF because: (1) it divides all three numbers evenly, and (2) no number larger than 15 divides 15 (since only 15 and 1 divide 15 besides 3 and 5). Being the smallest is not why it's the GCF. 45 is not odd (it's odd, but that's irrelevant).
List all the factors of 12.	1, 2, 3, 4, 6, 12 — Factor pairs of 12: 1×12 , 2×6 , 3×4 . All factors: 1,2,3,4,6,12. Missing 4 is a common error. Including 8 is wrong ($12/8$ has a remainder). Omitting 1 forgets that 1 is always a factor.
A florist has 28 roses and 42 tulips. She wants to make identical bouquets using all flowers. What is the maximum number of bouquets?	14 bouquets (2 roses and 3 tulips each) — $\text{GCF}(28,42) = 14$. So 14 bouquets: $28/14=2$ roses each, $42/14=3$ tulips each. 7 bouquets works (4 roses, 6 tulips) but isn't maximum. 28 bouquets can't divide 42 evenly. Same for 42.
Hot dogs come in packs of 8. Buns	24 of each (3 packs of hot dogs, 4 packs of buns) — $\text{LCM}(8,6) = 24$. Buy 3

come in packs of 6. What is the least number of each you must buy to have equal amounts?	packs of 8 (=24 hot dogs) and 4 packs of 6 (=24 buns). 48 is a common multiple but not the least. 12 works for buns (2 packs) but not hot dogs (12/8 isn't whole). 8 doesn't divide by 6.
Which of these numbers is a factor of 48: 5, 7, 8, or 9?	8 — $48/8 = 6$ exactly, so 8 is a factor. $48/5=9.6$ (remainder), $48/7=6.86$ (remainder), $48/9=5.33$ (remainder). Only 8 divides 48 evenly.
Design a number between 50 and 100 that has exactly 6 factors.	50 (its factors are 1, 2, 5, 10, 25, and 50 — exactly 6) — $50 = 2 \times 5^2$ has exactly 6 factors: 1,2,5,10,25,50. Other options: $75=3 \times 5^2$ (6 factors), $76=2^2 \times 19$ (6 factors), $98=2 \times 7^2$ (6 factors). 60 has 12 factors (too many). 97 is prime (only 2 factors).

9. number-sense

⌕ Question (fold →)	Answer
Create a composite number that has exactly three distinct prime factors, all less than 10.	$2 \times 3 \times 7 = 42$ — $42 = 2 \times 3 \times 7$ has three distinct prime factors (2, 3, and 7), all less than 10. Other valid answers include $30 = 2 \times 3 \times 5$ or $70 = 2 \times 5 \times 7$. 6 has only two prime factors. $8 = 2^3$ has only one distinct prime factor. $40 = 2 \times 2 \times 2 \times 5$ uses 4 which is not prime.
What does it mean for a number to be composite?	It has more than two factors — A composite number has more than two factors (at least 1, itself, and one other). 'Exactly two factors' defines prime numbers. Not all composites are even (e.g., 9, 15, 21).
Which number between 40 and 50 is prime?	43 — 43 is prime (not divisible by 2, 3, 5, or 7). $42 = 2 \times 21$ (even), $45 = 5 \times 9$ (ends in 5), $49 = 7 \times 7$ (perfect square). Between 40-50, the primes are 41, 43, and 47.
A school has 72 chairs to arrange into equal rows. Marcus wants each row to have a prime number of chairs. Which arrangement is NOT possible?	7 chairs per row — $72 \div 7 = 10 \text{ R}2$, so 7 does not divide evenly into 72. $72 \div 2 = 36$ and $72 \div 3 = 24$, both divide evenly. Therefore 7 chairs per row is not possible with 72 chairs.
What is the prime factorization of 60?	$2 \times 2 \times 3 \times 5$ — $60 = 2 \times 2 \times 3 \times 5$. All factors are prime. $2 \times 3 \times 10$ includes composite 10. 4×15 includes composites 4 and 15. 2×30 includes composite 30. Each must be broken down further.
Which number is composite?	51 — $51 = 3 \times 17$, so it is composite. 53, 59, and 61 are all prime numbers. Students often think 51 is prime because it ends in 1.
Find the prime factorization of 90.	$2 \times 3 \times 3 \times 5$ — $90 = 2 \times 3 \times 3 \times 5$. All factors are prime. $2 \times 5 \times 9$ includes composite 9. 3×30 includes composite 30. $2 \times 3 \times 15$ includes composite 15. Each distractor stops the factoring process too early.
Use a factor tree to find the prime factorization of 36.	$2 \times 2 \times 3 \times 3$ — $36 = 2 \times 2 \times 3 \times 3$. Using a factor tree: $36 \rightarrow 4 \times 9 \rightarrow (2 \times 2) \times (3 \times 3)$. The other options stop too early: 4×9 has composite factors; $2 \times 3 \times 6$ includes composite 6; $2 \times 2 \times 9$ includes composite 9.
Tomoko found that the prime factorization of a number is $2 \times 2 \times 2 \times 3$. What is the number?	24 — $2 \times 2 \times 2 \times 3 = 8 \times 3 = 24$. $12 = 2 \times 2 \times 3$ (missing one factor of 2). $18 = 2 \times 3 \times 3$ (wrong combination). $36 = 2 \times 2 \times 3 \times 3$ (has an extra 3 instead of an extra 2).
Which of the following is a prime number?	23 — 23 is prime because its only factors are 1 and 23. $21 = 3 \times 7$, $25 = 5 \times 5$, and $27 = 3 \times 9$, so all three are composite.
Lila says that all prime numbers greater than 2 must be odd. Evaluate her claim.	Correct, because any even number greater than 2 is divisible by 2 and therefore composite — Lila is correct. Any even number greater than 2 has at least three factors (1, 2, and itself), making it composite. Therefore all primes greater than 2 must be odd. This holds for ALL primes, not just those under 100. 4 is not prime ($4 = 2 \times 2$).
Two prime numbers add up to 20. One of them is 7. What is the other?	13 — $20 - 7 = 13$. Checking: 13 is prime (only factors are 1 and 13). So $7 + 13 = 20$. $12 = 2 \times 6$ (composite). 11 is prime but $7 + 11 = 18$, not 20. $14 = 2 \times 7$ (composite).
	No, $39 = 3 \times 13$, so it is composite — Kenji is wrong. While 39 is odd, odd

Kenji says 39 is prime because it is odd. Is he correct, and why?	numbers can still be composite. $39 = 3 \times 13$, so it has factors other than 1 and itself. 'All odd numbers are prime' is a common misconception. Note that $3 \times 12 = 36$, not 39.
A teacher places 24 students into equal groups with more than 1 student per group and more than 1 group. Which grouping uses a prime number of students per group?	3 students per group (8 groups) — The factor pairs of 24 include (3, 8), (4, 6), (6, 4), (8, 3). Among the group sizes listed, 3 is the prime number (factors: 1 and 3 only). 4, 6, and 8 are all composite.
How many prime numbers are there between 10 and 30?	6 — The primes between 10 and 30 are: 11, 13, 17, 19, 23, 29. That is 6 primes. Common errors: forgetting 29 (gives 5) or including $21 = 3 \times 7$ as prime (gives 7).
A garden has 48 flowers arranged in equal rows. If each row must have a prime number of flowers, which row size works?	2 flowers per row (24 rows) — 48's factor pairs include (2, 24), (3, 16), (4, 12), (6, 8). Among the choices, 2 is the only prime row size. $4 = 2 \times 2$, $6 = 2 \times 3$, and $8 = 2 \times 2 \times 2$ are all composite. (3 flowers per row would also work but is not listed.)
What is the only even prime number?	2 — 2 is the only even prime number. Its only factors are 1 and 2. Every other even number is divisible by 2 and at least one other number, making them composite. 0 is not prime (it has infinite factors). $4 = 2 \times 2$.
Diego says the number 91 must be prime because it is not divisible by 2, 3, or 5. Evaluate his reasoning.	Incorrect; $91 = 7 \times 13$, so he needed to check divisibility by 7 too — Diego's reasoning is flawed. To verify primality, you must check all primes up to the square root. Since the square root of 91 is approximately 9.5, he also needed to check 7. Indeed, $91 = 7 \times 13$. Just checking 2, 3, and 5 is insufficient for larger numbers.
Carlos says 57 is prime. Use divisibility rules to determine if he is correct.	No, $57 = 3 \times 19$ — The digit sum of 57 is $5 + 7 = 12$, which is divisible by 3, so 57 is divisible by 3. Indeed, $57 = 3 \times 19$. Carlos is incorrect. Being odd and not ending in 0 or 5 does not guarantee a number is prime.
A teacher says: 'There are no prime numbers between 23 and 29.' Is the teacher correct?	Yes, 24, 25, 26, 27, and 28 are all composite — The teacher is correct. 24 (even), 25 (5×5), 26 (even), 27 (3×9), and 28 (even) are all composite. 25 is not prime (5×5), and 27 is not prime (3×9). Gaps between consecutive primes can vary in size.
Why is 15 a composite number?	Because 15 has factors 1, 3, 5, and 15 — 15 is composite because it has four factors: 1, 3, 5, and 15. Being odd or ending in 5 does not automatically make a number composite (though ending in 5 does mean divisible by 5 for numbers > 5). 'Exactly 2 factors' would make it prime, not composite.
Priya claims that if you multiply two prime numbers, the result is always odd. Find a counterexample.	$2 \times 3 = 6$ (even) — 2 is the only even prime. Multiplying 2 by any other prime gives an even result: $2 \times 3 = 6$. This is a counterexample to Priya's claim. The other options all multiply two odd primes, which always give odd products.
Is the number 1 prime, composite, or neither?	Neither — The number 1 has only one factor (itself), so it does not meet the definition of prime (exactly 2 factors) or composite (more than 2 factors). It is classified as neither.
Aisha writes the prime factorization of 30. Which is correct?	$2 \times 3 \times 5$ — $30 = 2 \times 3 \times 5$, and all three factors (2, 3, 5) are prime. The other options contain composite factors: $6 = 2 \times 3$, $15 = 3 \times 5$, $10 = 2 \times 5$. Prime factorization must break the number down completely into primes.
Which pair of numbers are both	11 and 37 — 11 (factors: 1, 11) and 37 (factors: 1, 37) are both prime. In the other pairs: $9 = 3 \times 3$ is composite; $33 = 3 \times 11$ is composite; $15 = 3 \times 5$ is

prime?

composite.

10. number-sense

⌕ Question (fold →)	Answer
A number line shows points at -8, -3, 2, and 7. What is the pattern, and what comes next?	Add 5 each time; the next number is 12 — The pattern is +5: $-8 + 5 = -3$, $-3 + 5 = 2$, $2 + 5 = 7$, so the next term is $7 + 5 = 12$. Add 3 gives 10 (wrong constant). Multiply by -1 reverses signs. Add 4 gives 11 (close but wrong).
Which statement about negative numbers is true?	Every negative number is less than every positive number — Every negative number is to the left of every positive number on the number line, so every negative is less than every positive. Negative numbers get smaller (not larger) as you move left. $-10 < -1$ because -10 is farther from 0.
Four cities recorded these temperatures: City A: -12 degrees F, City B: 5 degrees F, City C: -3 degrees F, City D: -18 degrees F. Which city had the warmest temperature?	City B (5 degrees F) — $5 > -3 > -12 > -18$. City B at 5 degrees F is the warmest because it is the only positive temperature and thus the farthest right on the number line. City D is the coldest at -18 degrees F.
On a coordinate plane, point A is at (-3, 4). In which quadrant is point A located?	Quadrant II — (-3, 4) has a negative x and positive y. This puts it in Quadrant II (upper left). Quadrant I has both positive. Quadrant III has both negative. Quadrant IV has positive x and negative y.
Amara says -15 is greater than -8 because 15 is greater than 8. Evaluate her reasoning.	Incorrect; with negative numbers, the one closer to zero is greater, so -8 > -15 — Amara is incorrect. -8 is to the right of -15 on the number line, so $-8 > -15$. Her mistake is applying positive number ordering to negatives. With negative numbers, the number with the smaller absolute value is actually greater.
Which symbol correctly compares -3 and -7?	-3 > -7 — $-3 > -7$ because -3 is to the right of -7 on a number line. A common mistake is thinking $-7 > -3$ because $7 > 3$, but with negatives the number closer to zero is greater.
A parking garage has 3 underground levels (-1, -2, -3) and 5 above-ground levels (1, 2, 3, 4, 5), plus a ground floor (0). A car goes from level -3 to level 4. How many levels did it travel?	7 levels — $ 4 - (-3) = 4 + 3 = 7$ levels. $1 = 4 - 3 $ (ignores the negative sign). 4 only counts the above-ground part. 8 adds 3 + 5 (all levels in the garage rather than the distance traveled).
The Dead Sea is approximately -430 meters elevation. Denver, Colorado is approximately 1,609 meters elevation. What is the difference in elevation?	2,039 meters — $1,609 - (-430) = 1,609 + 430 = 2,039$ meters. $1,179 = 1,609 - 430$ (subtracts magnitudes instead of subtracting a negative). 1,609 ignores the Dead Sea's depth. 430 only considers the Dead Sea.
A student claims that the distance between -9 and 4 on a number line is 5 because $9 - 4 = 5$. Is this correct?	No, the distance is 13 because $4 - (-9) = 4 + 9 = 13$ — The correct distance is $ 4 - (-9) = 4 + 9 = 13$. The student subtracted the absolute values ($9 - 4 = 5$) instead of subtracting with the negative sign. Distance is always positive, so -13 is also wrong.
What is the opposite of -8?	8 — The opposite of -8 is 8. Both are 8 units from 0, but on opposite sides of the number line. -16 doubles the value instead of flipping the sign. The opposite of a negative is always positive.

A scuba diver descends from sea level at a rate of 8 meters per minute. After 6 minutes, what is the diver's depth?	-48 meters — Descending 8 meters/min for 6 minutes: $8 \times 6 = 48$ meters below sea level = -48 meters. 48 meters is the magnitude but missing the negative. $-14 = -(8 + 6)$ adds instead of multiplies. $-68 = -(8 \times 6 + 20)$ adds extra.
Points P(-5, 0) and Q(3, 0) are on the x-axis. What is the distance between them?	8 units — $ 3 - (-5) = 3 + 5 = 8$ units. Distance is always positive. $2 = 3 - 5 $ (forgot the negative). -2 is negative (distance cannot be negative). 3 just uses one coordinate.
A hiker starts at an elevation of -40 feet (below sea level) and climbs to 125 feet above sea level. How many feet did the hiker climb?	165 feet — Total climb = $125 - (-40) = 125 + 40 = 165$ feet. $85 = 125 - 40$ (subtracts instead of finding distance). 125 ignores the starting depth. 40 only counts the distance to sea level.
Which integer is located 5 units to the left of 0 on a number line?	-5 — Moving left from 0 on a number line goes into negative numbers. Five units left of 0 is -5. Moving right would give +5. Students sometimes confuse direction with sign.
Ravi says that 0 is a positive integer. Is he correct? Explain.	No, 0 is neither positive nor negative — 0 is neither positive nor negative. It is the origin on the number line that separates positive integers (right) from negative integers (left). 'Smallest positive integer' is actually 1. '0 is negative' is also false.
What integer represents 200 feet below sea level?	-200 — Below sea level is represented by negative integers. 200 feet below sea level = -200. Positive 200 would be above sea level. -20 uses the wrong magnitude.
Place the following integers in order from least to greatest: 4, -6, 0, -2, 3.	-6, -2, 0, 3, 4 — From least to greatest: -6 (farthest left), -2, 0, 3, 4 (farthest right). The distractor '-2, -6, 0, 3, 4' incorrectly orders negatives by ignoring that $-6 < -2$. '4, 3, 0, -2, -6' is greatest to least.
Which point on a coordinate plane is a reflection of (3, -5) across the x-axis?	(3, 5) — Reflecting (3, -5) across the x-axis gives (3, 5). The x-coordinate stays 3 and the y-coordinate flips from -5 to 5. (-3, -5) reflects across the y-axis. (-3, 5) reflects across the origin. (3, -5) is the original point unchanged.
A bank account has a balance of -\$35. A deposit of \$50 is made. What is the new balance?	\$15 — $-35 + 50 = 15$. The deposit covers the overdraft and leaves \$15 positive. $-$85$ subtracts instead of adding. $-$15$ gets the sign wrong. $$85$ adds the magnitudes ($35 + 50$) without the negative.
Plot the points (-2, -3), (4, -3), and (4, 2) on a coordinate plane. What is the length of the side from (-2, -3) to (4, -3)?	6 units — The distance between (-2, -3) and (4, -3) is $ 4 - (-2) = 4 + 2 = 6$ units. $2 = 4 - 2$ (forgets to subtract a negative). 5 units miscounts. $8 = 4 + -3 + 1$ (confuses coordinates).
Which integer is exactly halfway between -4 and 6 on a number line?	1 — The distance from -4 to 6 is 10 units. Half of 10 is 5. Starting from -4 and moving 5 units right: $-4 + 5 = 1$. Or average: $(-4 + 6) / 2 = 2 / 2 = 1$. 0 is 4 units from -4 but 6 from 6.
An elevator starts at floor -2 (basement level 2). It goes up 5 floors, then down 3 floors. What floor is it on now?	Floor 0 (ground level) — $-2 + 5 = 3$, then $3 - 3 = 0$. The elevator ends at the ground floor. Floor 4 forgets to subtract. Floor -6 subtracts both: $-2 - 5 + 3$. Floor 6 adds everything: $2 + 5 - 3$ (ignoring the negative start).
The temperature at noon was 5 degrees Celsius. By midnight it dropped 12 degrees. What was the temperature at midnight?	-7 degrees Celsius — $5 - 12 = -7$ degrees Celsius. The temperature went below zero. 7 degrees ignores the negative. -12 confuses the drop amount with the final temperature. 17 incorrectly adds instead of subtracts.

Design a real-world scenario where the integer -25 would be meaningful. Which scenario best fits?	A city's temperature drops to 25 degrees below zero Fahrenheit — A temperature of 25 degrees below zero is naturally represented as -25 degrees F. Selling 25 items, completing 25 laps, and scoring 25 points are all positive quantities with no meaningful negative interpretation in context.
A submarine is at -150 meters. It rises 60 meters. What is its new depth?	-90 meters — $-150 + 60 = -90$ meters. The submarine is still below sea level but closer to the surface. -210 incorrectly subtracts (goes deeper). -150 ignores the change. 60 confuses the change with the new position.

11. operations

⌂ Question (fold →)	Answer
A football team gains 8 yards, loses 13 yards, gains 5 yards, and loses 2 yards. What is the net yardage?	-2 yards — $8 + (-13) + 5 + (-2) = (8 + 5) + (-13 + -2) = 13 + (-15) = -2$ yards. They lost 2 net yards. 2 yards reverses the sign. -28 and 28 add all magnitudes.
What is the value of $-2 \times (8 - 13)$?	10 — $8 - 13 = -5$. Then $-2 \times (-5) = 10$ (negative times negative is positive). -10 forgets the sign rule. $-30 = -2 \times (8 + 13 - 5)$. $30 = 2 \times (8 + 13 - 6)$.
Mei calculates $-4 - (-4)$ and gets -8. What is her mistake?	She subtracted instead of adding; $-4 - (-4) = -4 + 4 = 0$ — $-4 - (-4) = -4 + 4 = 0$. Mei treated $-(-4)$ as -4 instead of +4. This is a common error where students distribute the subtraction sign but fail to flip the second negative. Any number minus itself equals 0.
A student claims that subtracting a negative number always gives a larger result than the original number. Is this true?	Yes, because subtracting a negative is equivalent to adding a positive — Yes. For any numbers a and b where b is negative: $a - b = a + b $. Adding a positive value always increases the result. This holds regardless of whether a is positive, negative, or zero. The result is always larger than a.
Which integer operation gives the greatest result: (A) $-5 + 12$, (B) $-5 - (-12)$, (C) $-5 \times (-2)$, or (D) $12 \div (-3)$?	C: $-5 \times (-2) = 10$ — $A = -5 + 12 = 7$. $B = -5 - (-12) = -5 + 12 = 7$. $C = -5 \times (-2) = 10$. $D = 12 \div (-3) = -4$. The greatest result is C = 10. Students may pick A or B (both = 7) without checking C.
Evaluate: $(-3)^3$.	-27 — $(-3)^3 = (-3) \times (-3) \times (-3) = 9 \times (-3) = -27$. An odd power of a negative number is negative. 27 incorrectly makes it positive (even power rule). $-9 = (-3) \times 3$ (only multiplies twice). $9 = (-3) \times (-3)$ (stops at the square).
Simplify: $4 \times (-6) + (-2) \times (-9)$.	-6 — $4 \times (-6) = -24$; $(-2) \times (-9) = 18$. Then $-24 + 18 = -6$. $42 = 24 + 18$ (ignores the negative). $-42 = -24 + (-18)$ (makes both products negative). 6 reverses the sign.
A deep-sea research vessel descends 45 meters every 3 minutes. Express the rate of descent as a unit rate.	-15 meters per minute — $-45 \div 3 = -15$ meters per minute. The negative sign indicates descent. 15 drops the negative. $-135 = -45 \times 3$ (multiplies instead of divides). $135 = 45 \times 3$.
Which expression is equivalent to $-3 - 5$?	$-3 + (-5)$ — $-3 - 5 = -3 + (-5) = -8$. Subtracting 5 is equivalent to adding -5. $-3 + 5 = 2$ (wrong sign on 5). $3 - 5 = -2$ (wrong sign on 3). $3 + 5 = 8$ (both signs wrong).
Calculate: $-12 + (-8) - (-5)$.	-15 — $-12 + (-8) - (-5) = -12 + (-8) + 5 = -20 + 5 = -15$. $-25 = -12 - 8 - 5$ (subtracts 5 instead of adding). $-5 = -12 + 8 - (-5)$ (wrong sign on 8). 25 adds all magnitudes.
What is $-42 \div 7$?	-6 — $-42 \div 7 = -6$. Negative divided by positive is negative. $42 \div 7 = 6$, so the answer is -6. 6 drops the negative. -7 divides wrong ($42 \div 6$). 7 reverses the operation.
Create an expression using three different integers (one positive, one negative, one zero) where the result is -10. Which expression works?	$0 - 4 \times 5 \div 2 = 0 - 10 = -10$ — $0 - 4 \times 5 \div 2$: by order of operations, $4 \times 5 = 20$, $20 \div 2 = 10$, then $0 - 10 = -10$. Other valid answers exist (e.g., $-10 + 0 \times 3$). The distractors all evaluate to values other than -10.
Calculate: $(-48) \div (-8) + (-3) \times 2$.	0 — $(-48) \div (-8) = 6$ (negative / negative = positive). $(-3) \times 2 = -6$. Then $6 + (-6) = 0$. $12 = 6 + 6$ (makes both positive). $-12 = -6 + (-6)$ (makes both negative). 6 only

	calculates the first part.
A stock loses \$4 per share each day for 5 consecutive days. What is the total change in value per share?	-\$20 — $(-4) \times 5 = -20$. The stock lost \$20 total over 5 days. \$20 drops the negative. $-\$9 = -4 + (-5)$ adds instead of multiplies. $\$9 = -4 + 5$ subtracts but gets wrong sign.
Jamal owes \$15 to one friend and \$22 to another. If he earns \$30, what is his net financial position?	-\$7 (still owes \$7) — $(-15) + (-22) + 30 = -37 + 30 = -7$. He owes \$37 total and earns \$30, leaving \$7 in debt. \$7 reverses the sign. \$67 adds all magnitudes. -\$37 ignores the earning.
What is $-6 \times (-4)$?	24 — $-6 \times (-4) = 24$. A negative times a negative gives a positive result. $6 \times 4 = 24$, and the two negatives cancel. -24 incorrectly keeps the negative. -10 and 10 add instead of multiply.
What is $-8 + 3$?	-5 — $-8 + 3 = -5$. Start at -8, move 3 to the right. Since $ -8 > 3 $, the result is negative: $-(8 - 3) = -5$. 5 drops the negative sign. -11 adds both as negatives. 11 adds both magnitudes.
The expression $(-1)^{50}$ equals what value?	1 — $(-1)^{50} = 1$. Since 50 is even, the negative signs cancel in pairs: $(-1 \times -1) = 1$, repeated 25 times. Any negative number raised to an even power is positive. -1 would be the answer for odd powers like $(-1)^{51}$.
What is $5 - (-7)$?	12 — $5 - (-7) = 5 + 7 = 12$. Subtracting a negative flips to addition. $-2 = 5 - 7$ (ignores the double negative). $2 = 7 - 5$ (reverses the subtraction). -12 incorrectly makes the result negative.
Suki says that -3×4 and $3 \times (-4)$ give different answers. Evaluate her claim.	Incorrect; both equal -12 because one negative factor always makes the product negative — $-3 \times 4 = -12$ and $3 \times (-4) = -12$. Both have exactly one negative factor, producing the same negative product. Multiplication is commutative, so the position of the negative sign does not change the result.
Is the following statement always true: 'A negative number divided by a negative number is always positive'?	Yes, negative divided by negative always gives a positive quotient — Yes. Negative divided by negative always gives a positive quotient. The sign rule for division matches multiplication: same signs yield positive, different signs yield negative. The magnitudes determine the size but not the sign.
Which property does this equation demonstrate: $-7 + 7 = 0$?	Additive inverse property — When a number and its opposite (additive inverse) are added, the sum is 0. This is the additive inverse property. Commutative is $a + b = b + a$. Identity is $a + 0 = a$. Distributive is $a(b + c) = ab + ac$.
What is the sign of the product $(-2) \times 3 \times (-5)$?	Positive, because there are two negative factors — There are 2 negative factors (-2 and -5), which is even, so the product is positive. $(-2) \times 3 \times (-5) = (-6) \times (-5) = 30$. An even count of negatives always yields a positive product.
Marcus says $(-2)^4 = -(2^4)$. Is he correct?	No; $(-2)^4 = 16$, but $-(2^4) = -16$ — $(-2)^4 = (-2)(-2)(-2)(-2) = 16$ (even power, positive). $-(2^4) = -(16) = -16$ (the negative is applied after the exponent). The parentheses make a critical difference. This is one of the most common exponent errors.
At dawn the temperature was -9 degrees F. By noon it had risen 22 degrees. What was the noon temperature?	13 degrees F — $-9 + 22 = 13$ degrees F. Since $22 > 9$, the result is positive: $22 - 9 = 13$. -31 subtracts instead of adding. -13 gets the sign wrong. 31 adds both magnitudes: $9 + 22$.

12. number-sense

Question (fold →)	Answer
Evaluate: $ -3 + 5 - -2 $.	6 — $ -3 = 3$, $ 5 = 5$, $ -2 = 2$. So $3 + 5 - 2 = 6$. $10 = 3 + 5 + 2$ (adds all). $0 = -3 + 5 - 2$ (forgets absolute value on -3). -6 makes the result negative incorrectly.
Nadia says $ a + b $ is always equal to $ a + b $. Find a counterexample.	a = 5, b = -3: $5 + (-3) = 2$, but $5 + -3 = 8$ — $a = 5$, $b = -3$: $ 5 + (-3) = 2 = 2$, but $ 5 + -3 = 5 + 3 = 8$. Since 2 is not equal to 8, this is a counterexample. Nadia's claim fails when a and b have different signs. The correct relationship is $ a + b \leq a + b $ (triangle inequality).
Which symbol makes this statement true: $ -12 $ ___ $ 8 $?	> — $ -12 = 12$ and $ 8 = 8$. Since $12 > 8$, the answer is >. Students may incorrectly think $-12 < 8$, but absolute value strips the sign first. We compare 12 and 8, not -12 and 8.
If $ x = 4$, what are the possible values of x?	x = 4 or x = -4 — $ x = 4$ means x is 4 units from 0. Both 4 and -4 are 4 units from 0. 'x = 4 only' forgets the negative solution. 'x = -4 only' forgets the positive solution. x = 0 has absolute value 0, not 4.
A student writes: 'Since $-3 < 5$, it follows that $ -3 < 5 $.' Evaluate this reasoning.	The conclusion happens to be true ($3 < 5$), but the reasoning is flawed — While $ -3 = 3 < 5 = 5 $ is true, the reasoning is flawed. Counterexample: $-10 < 2$ but $ -10 = 10 > 2 = 2$. The order of numbers does not transfer to the order of their absolute values because negation reverses order but absolute value strips the sign.
A weather station records high and low temperatures. Monday's high was 8 degrees C and low was -6 degrees C. What is the temperature range (difference between high and low)?	14 degrees — Range = $8 - (-6) = 8 + 6 = 14$ degrees. $2 = 8 - 6$ (subtracts magnitudes). -2 reverses the subtraction. 6 only uses the low temperature's magnitude. Subtracting a negative means adding.
A fish swims at a depth of -20 meters. A bird flies at an altitude of 35 meters. What is the total distance between them?	55 meters — $ 35 - (-20) = 35 + 20 = 55$ meters. From the fish to sea level is 20 m, then from sea level to the bird is 35 m: $20 + 35 = 55$ m. $15 = 35 - 20$ (subtracts magnitudes). Distance cannot be negative.
True or false: The absolute value of any number is always positive.	False; absolute value is always non-negative ($0 = 0$, which is not positive) — Absolute value is always non-negative (greater than or equal to 0), but not always positive. $ 0 = 0$, and 0 is neither positive nor negative. 'Always positive' excludes this case. Absolute value is never negative.
Three bank accounts have balances of -\$120, \$85, and -\$45. Which account is closest to a zero balance?	-\$45, because $-45 = 45$ is the smallest absolute value — $ -120 = 120$, $ 85 = 85$, $ -45 = 45$. The smallest absolute value is 45, so -\$45 is closest to zero. Being positive does not mean closest to zero. -\$120 is farthest from zero despite having the 'largest' magnitude.
A number and its absolute value differ by 16. What is the number?	-8 — If x is negative, $ x = -x$. The difference: $ x - x = -x - x = -2x$. Setting $-2x = 16$ gives $x = -8$. Check: $ -8 - (-8) = 8 + 8 = 16$. For positive x, $ x = x$ so the difference is always 0. -16 confuses the difference with the answer. 8 has no difference with its absolute value.
Which number is farther from 0 on a	-15, because $-15 = 15 > 11 = 11$ — $ -15 = 15$ and $ 11 = 11$. Since $15 > 11$, -15 is farther from 0. Being positive does not make a number farther

number line: -15 or 11?	from 0 — it depends on the magnitude. Their difference ($15 - 11 = 4$) is not relevant to which is farther from 0.
Evaluate: $ 7 - 15 $.	8 — $7 - 15 = -8$, then $ -8 = 8$. -8 forgets to apply absolute value. $22 = 7 + 15$ (adds instead of subtracts). -22 adds and makes negative. Always compute inside first, then take absolute value.
What is the absolute value of -9?	9 — $ -9 = 9$. Absolute value measures distance from 0, which is always non-negative. -9 is 9 units from 0. Choosing -9 confuses the number with its absolute value. 0 and 1 are unrelated.
Points A(-8, 0) and B(6, 0) are on the x-axis. Use absolute value to find the distance between them.	$-8 - 6 = -14 = 14$ units — Distance = $ -8 - 6 = -14 = 14$ units. This can also be computed as $ 6 - (-8) = 14 = 14$. The distractor $ (-8) + 6 = 2$ adds instead of subtracts. Interestingly, $ -8 + 6 = 14$ gives the right answer here because the points are on opposite sides of 0, but this method does not generalize.
A thermometer reads -18 degrees F. Another reads -7 degrees F. Which location is colder, and by how many degrees?	-18 degrees F is colder; the difference is 11 degrees — $-18 < -7$, so -18 degrees F is colder. The difference is $ -18 - (-7) = -11 = 11$ degrees. $25 = 18 + 7$ (adds magnitudes). Saying -7 is colder confuses 'larger magnitude' with 'colder.'
Arrange these numbers from greatest to least: $ -3 $, -5, $ 2 $, $ -4 $, 1.	-3, 2, 1, -4, -5 which is 3, 2, 1, -4, -5 — Simplify: $ -3 = 3$, $ 2 = 2$, $ -4 = -4$. Values are 3, -5, 2, -4, 1. Greatest to least: 3, 2, 1, -4, -5. The first distractor is least to greatest. The third distractor misorders the positive values.
A diver is at -35 feet and another diver is at -12 feet. What is the distance between them?	23 feet — $ -35 - (-12) = -35 + 12 = -23 = 23$ feet. $47 = 35 + 12$ (adds magnitudes instead of finding distance). -23 forgets to take absolute value. 35 only uses one diver's depth.
Without calculating, determine which is greater: $ -25 + 10 $ or $ 25 - 10 $.	They are equal; both equal 15 — $-25 + 10 = -15$, so $ -15 = 15$. And $25 - 10 = 15$, so $ 15 = 15$. They are equal. This illustrates that $ a = -a $ for any number a. The sign inside does not affect the absolute value.
Order the following from least to greatest: -4, $ 3 $, $ -6 $, $ 0 $.	-6, -4, 0, 3 which is -6, -4, 0, 3 — Simplify: -4 stays as -4. $ 3 = 3$. $ -6 = -6$. $ 0 = 0$. Ordering: $-6 < -4 < 0 < 3$. The key trap is $ -6 $: the absolute value of -6 is 6, then the outer negative makes it -6.
Explain what $ -5 = 5$ means in terms of a number line.	-5 is 5 units away from 0 on the number line — $ -5 = 5$ means -5 is located 5 units from 0 on the number line. It does not mean -5 is greater than or equal to 5. -5 is to the left of 0 (and much farther left than 5), but its distance from 0 is 5 units.
What is $ 0 $?	0 — $ 0 = 0$. Zero is 0 units away from itself on the number line. It is the only number whose absolute value is 0. 1 is a common wrong answer from thinking absolute value must be at least 1.
Miguel claims that if $ a > b $, then $a > b$. Is he always correct?	No; for example $(-5) > 3$ but $-5 < 3$ — Miguel is wrong. $ -5 = 5 > 3 = 3$, but $-5 < 3$. A negative number can have a larger absolute value than a positive number yet still be less than it. Absolute value measures magnitude (distance from 0), not actual position on the number line.
Design a real-world problem where absolute value is needed to find the answer. Which scenario requires	Finding the distance between two temperatures: -14 degrees F and 22 degrees F — Finding the distance between -14 degrees F and 22 degrees F requires absolute value: $ 22 - (-14) = 36 = 36$ degrees. The other scenarios involve only positive numbers where absolute value is

absolute value?	unnecessary (adding scores, averaging heights, totaling costs).
For which values of x is $ x > x$ true?	All negative values of x — When x is negative, $ x = -x$ which is positive, so $ x > x$. For example, if $x = -3$, $ x = 3 > -3$. When $x \geq 0$, $ x = x$, so $ x > x$ is false (they are equal). The statement is true only for negative x .
Two numbers have the same absolute value: $ a = b = 7$. The numbers are different. What could a and b be?	$a = 7$ and $b = -7$ — 7 and -7 are both 7 units from 0 but are different numbers. 7 and 7 are the same number (violates 'different'). 0 and 7 have different absolute values. 14 and -14 have absolute value 14, not 7.

13. Fractions Fundamentals

Question (fold →)	Answer
What is $\frac{4}{8}$ in simplest form?	$\frac{1}{2}$ — Divide numerator and denominator by their GCF (4). 4 divided by 4 = 1, 8 divided by 4 = 2. So $\frac{4}{8} = \frac{1}{2}$.
Order from least to greatest: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$.	$\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ — Convert $\frac{1}{2}$ to $\frac{2}{4}$. Now compare: $\frac{1}{4} < \frac{2}{4} < \frac{3}{4}$, so the order is $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$.
Explain whether the fraction $\frac{5}{5}$ is greater than, less than, or equal to 1.	Equal to 1 — When numerator equals denominator ($\frac{5}{5}$), you have all the pieces of the whole, which equals exactly 1.
What is the GCF of 6 and 9, and how does it help simplify $\frac{6}{9}$?	GCF is 3; $\frac{6}{9}$ simplifies to $\frac{2}{3}$ — Factors of 6: 1, 2, 3, 6. Factors of 9: 1, 3, 9. GCF = 3. Divide both by 3: $\frac{6}{3} = 2$, $\frac{9}{3} = 3$. So $\frac{6}{9} = \frac{2}{3}$.
A number line is marked from 0 to 1. You need to place $\frac{5}{6}$ and $\frac{2}{3}$ on it. Describe where each fraction goes and which is closer to 1.	$\frac{5}{6}$ is closer to 1; $\frac{2}{3} = \frac{4}{6}$ is farther — $\frac{2}{3} = \frac{4}{6}$. On the number line, $\frac{4}{6}$ is at two-thirds and $\frac{5}{6}$ is at five-sixths. $\frac{5}{6}$ is closer to 1 because it is only $\frac{1}{6}$ away, while $\frac{2}{3}$ ($\frac{4}{6}$) is $\frac{2}{6}$ away.
Invent a real-world scenario where you would need to compare the fractions $\frac{2}{5}$ and $\frac{3}{8}$, and explain which is larger.	Answers vary; $\frac{2}{5} > \frac{3}{8}$ ($\frac{16}{40} > \frac{15}{40}$) — Example: Two runners finish $\frac{2}{5}$ and $\frac{3}{8}$ of a track. LCD = 40: $\frac{2}{5} = \frac{16}{40}$, $\frac{3}{8} = \frac{15}{40}$. Since $16 > 15$, $\frac{2}{5}$ is the larger fraction.
Is $\frac{7}{3}$ greater than or less than 2? Justify your answer.	Greater than 2 — $2 = \frac{6}{3}$. Since $\frac{7}{3} > \frac{6}{3}$, the fraction $\frac{7}{3}$ is greater than 2. As a mixed number, $\frac{7}{3} = 2 \frac{1}{3}$.
Which symbol makes this true: $\frac{2}{5}$ ___ $\frac{3}{5}$?	$<$ — When denominators are equal, compare numerators. Since $2 < 3$, then $\frac{2}{5} < \frac{3}{5}$.
A recipe calls for $\frac{3}{4}$ cup of flour. You only have a $\frac{1}{4}$ cup measuring cup. How many scoops do you need?	3 scoops — To find how many $\frac{1}{4}$ cups make $\frac{3}{4}$ cup: $\frac{3}{4}$ divided by $\frac{1}{4} = 3$. You need 3 scoops.
Maria ate $\frac{1}{3}$ of a pie and her brother ate $\frac{1}{4}$. Maria says she ate more. Her brother says they ate the same because the numerators are equal. Who is right and why?	Maria is right; $\frac{1}{3} > \frac{1}{4}$ because thirds are larger pieces than fourths — Maria is correct. With equal numerators, the fraction with the smaller denominator is larger because the whole is divided into fewer (bigger) pieces. $\frac{1}{3} > \frac{1}{4}$.
Why are $\frac{2}{4}$ and $\frac{1}{2}$ called equivalent fractions?	They represent the same amount — Equivalent fractions represent the same value. $\frac{2}{4}$ simplified (divide top and bottom by 2) equals $\frac{1}{2}$.
Convert the mixed number $2 \frac{1}{3}$ to an improper fraction.	$\frac{7}{3}$ — Multiply whole number by denominator: 2 times 3 = 6. Add numerator: $6 + 1 = 7$. Keep denominator: $\frac{7}{3}$.
Which fraction is NOT equivalent to $\frac{2}{3}$?	$\frac{4}{9}$ — $\frac{2}{3} = \frac{4}{6}$, $\frac{6}{9}$, and $\frac{8}{12}$ (multiply top and bottom by 2, 3, 4). But $\frac{4}{9}$ does not simplify to $\frac{2}{3}$ since 4 times 3 = 12, not 9 times 2 = 18.
Place the fraction $\frac{3}{4}$ on a number line. Between which two whole numbers does it fall?	0 and 1 — Since $\frac{3}{4} < 1$ (numerator < denominator), it falls between 0 and 1 on the number line.
A jar has 12 marbles: 5 red, 3 blue, 4 green. What fraction of the marbles are blue?	$\frac{3}{12}$ — The fraction of blue marbles is blue count over total count: $\frac{3}{12}$ (which simplifies to $\frac{1}{4}$).

What is the numerator in the fraction $\frac{5}{12}$?	5 — The numerator is the number above the fraction bar. In $\frac{5}{12}$, the numerator is 5.
What fraction of a pizza is shaded if 3 out of 8 equal slices are shaded?	$\frac{3}{8}$ — A fraction represents parts over whole. 3 shaded out of 8 total = $\frac{3}{8}$.
Compare $\frac{3}{5}$ and $\frac{4}{7}$. Which is greater? Explain your strategy.	$\frac{3}{5}$ is greater — Find LCD: 5 and 7 have LCD 35. Convert: $\frac{3}{5} = \frac{21}{35}$, $\frac{4}{7} = \frac{20}{35}$. Since $21 > 20$, $\frac{3}{5} > \frac{4}{7}$.
Convert the improper fraction $\frac{7}{4}$ to a mixed number.	$1 \frac{3}{4}$ — 7 divided by 4 = 1 with remainder 3. So $\frac{7}{4} = 1 \frac{3}{4}$.
What fraction of 1 hour is 15 minutes?	$\frac{1}{4}$ — 15 minutes out of 60 minutes = $\frac{15}{60}$. Simplify by dividing both by 15: $\frac{15}{15} = 1$, $\frac{60}{15} = 4$. So $\frac{15}{60} = \frac{1}{4}$.
Sam says any fraction with a larger numerator is always greater. Find a counterexample to prove Sam wrong.	Example: $\frac{5}{100} < \frac{1}{2}$ even though $5 > 1$ — Sam is wrong. Counterexample: $\frac{5}{100} = 0.05$, but $\frac{1}{2} = 0.5$. Even though $5 > 1$, the fraction $\frac{5}{100} < \frac{1}{2}$ because the denominator matters.
A student says $\frac{1}{3} > \frac{1}{2}$ because $3 > 2$. Is this correct? Why or why not?	Incorrect; larger denominator means smaller pieces — The student is wrong. A larger denominator means the whole is divided into more (and therefore smaller) pieces. $\frac{1}{3} < \frac{1}{2}$ because thirds are smaller than halves.
If you eat 2 slices of a pizza cut into 6 equal slices, what fraction is left?	$\frac{4}{6}$ — You ate 2 out of 6 slices, so $6 - 2 = 4$ slices remain. The fraction left is $\frac{4}{6}$ (which simplifies to $\frac{2}{3}$).
Which fraction is equivalent to $\frac{1}{2}$?	$\frac{3}{6}$ — Multiplying numerator and denominator of $\frac{1}{2}$ by 3 gives $\frac{3}{6}$. Both equal the same amount.
What is the denominator in the fraction $\frac{7}{10}$?	10 — The denominator is the number below the fraction bar. In $\frac{7}{10}$, the denominator is 10.

14. Fraction Operations

⌕ Question (fold →)	Answer
What is $\frac{1}{2}$ divided by 3?	$\frac{1}{6}$ — Dividing by a whole number: $\frac{1}{2}$ divided by 3 = $\frac{1}{2}$ times $\frac{1}{3}$ = $\frac{1}{6}$.
What is $\frac{2}{3} + \frac{5}{6}$?	$1\frac{1}{2}$ or $\frac{3}{2}$ — LCD = 6. Convert $\frac{2}{3} = \frac{4}{6}$. Add: $\frac{4}{6} + \frac{5}{6} = \frac{9}{6}$. Simplify: $\frac{9}{6} = \frac{3}{2} = 1\frac{1}{2}$.
A baker needs $2\frac{1}{2}$ cups of flour but only has a $\frac{1}{3}$ cup measuring cup. Explain step-by-step how many scoops are needed.	$7\frac{1}{2}$ scoops — Convert: $2\frac{1}{2} = \frac{5}{2}$. Divide: $\frac{5}{2}$ divided by $\frac{1}{3} = \frac{5}{2}$ times 3 = $\frac{15}{2} = 7\frac{1}{2}$ scoops.
Is $\frac{2}{3} + \frac{3}{4}$ greater than 1? How can you tell without computing the exact sum?	Yes; both fractions are greater than $\frac{1}{2}$, so their sum exceeds 1 — $\frac{2}{3} > \frac{1}{2}$ and $\frac{3}{4} > \frac{1}{2}$. Since both exceed $\frac{1}{2}$, their sum $> \frac{1}{2} + \frac{1}{2} = 1$. You can estimate without finding an exact answer.
What is $3\frac{1}{2} + 2\frac{1}{4}$?	$5\frac{3}{4}$ — Whole numbers: $3 + 2 = 5$. Fractions: $\frac{1}{2} = \frac{2}{4}$, so $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$. Total: $5\frac{3}{4}$.
What is the first step when adding $\frac{1}{3} + \frac{1}{6}$?	Find a common denominator — Before adding fractions with unlike denominators, find a common denominator so the pieces are the same size. LCD of 3 and 6 is 6.
A board is $\frac{5}{8}$ foot long. You cut off $\frac{1}{4}$ foot. How long is the remaining piece?	$\frac{3}{8}$ foot — LCD = 8. Convert $\frac{1}{4} = \frac{2}{8}$. Subtract: $\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$ foot.
What is $4\frac{1}{3} - 1\frac{2}{3}$?	$2\frac{2}{3}$ — Since $\frac{1}{3} < \frac{2}{3}$, borrow 1 from 4: $4\frac{1}{3} = 3 + \frac{3}{3} + \frac{1}{3} = 3\frac{4}{3}$. Subtract: $3\frac{4}{3} - 1\frac{2}{3} = 2\frac{2}{3}$.
You jogged $\frac{3}{4}$ mile in the morning and $\frac{1}{2}$ mile in the afternoon. How far did you jog in total?	$1\frac{1}{4}$ miles — LCD = 4. Convert $\frac{1}{2} = \frac{2}{4}$. Add: $\frac{3}{4} + \frac{2}{4} = \frac{5}{4} = 1\frac{1}{4}$ miles.
What is $\frac{3}{5} + \frac{1}{5}$?	$\frac{4}{5}$ — Like denominators: add numerators. $3 + 1 = 4$. Answer: $\frac{4}{5}$.
Evaluate whether this statement is always true: When you divide a fraction by another fraction, the answer is always greater than 1.	Not always true; $\frac{1}{4}$ divided by $\frac{1}{2} = \frac{1}{2}$, which is less than 1 — Counterexample: $\frac{1}{4}$ divided by $\frac{1}{2} = \frac{1}{4}$ times $\frac{2}{1} = \frac{2}{4} = \frac{1}{2} < 1$. The result depends on the relative sizes of the fractions.
What is $\frac{5}{6} - \frac{1}{3}$?	$\frac{1}{2}$ — LCD = 6. Convert $\frac{1}{3} = \frac{2}{6}$. Subtract: $\frac{5}{6} - \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$.
Why does multiplying fractions NOT require a common denominator?	Multiplication counts parts of parts, not combining same-sized pieces — Multiplication finds a fraction of a fraction (part of a part). You multiply numerators and denominators directly because you are scaling, not combining like-sized pieces.
A recipe needs $\frac{2}{3}$ cup of sugar. You want to make half the recipe. How much sugar	$\frac{1}{3}$ cup — Half the recipe: $\frac{1}{2}$ times $\frac{2}{3} = \frac{2}{6} = \frac{1}{3}$ cup of sugar.

do you need?	
What is $\frac{2}{3}$ times $\frac{4}{5}$?	$\frac{8}{15}$ — Multiply numerators: 2 times 4 = 8. Multiply denominators: 3 times 5 = 15. Answer: $\frac{8}{15}$.
A student says $\frac{1}{2}$ times $\frac{1}{3} = \frac{1}{6}$. Another says the answer should be bigger than $\frac{1}{2}$ since you are multiplying. Who is correct?	The first student; multiplying by a fraction less than 1 makes the result smaller — The first student is correct. Multiplying by a fraction less than 1 yields a smaller result: $\frac{1}{2}$ times $\frac{1}{3} = \frac{1}{6}$. The second student confuses fraction multiplication with whole-number multiplication.
Why does "keep, change, flip" work for dividing fractions?	Dividing by a fraction is the same as multiplying by its reciprocal — Dividing by a number is the same as multiplying by its reciprocal. Since the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$, dividing by $\frac{a}{b} =$ multiplying by $\frac{b}{a}$.
Compare two methods for computing $\frac{2}{5} + \frac{3}{10}$: converting to decimals vs. finding a common denominator. Which is more precise for fractions?	Common denominator is more precise — Common denominator gives exact results: $\frac{2}{5} = \frac{4}{10}$, so $\frac{4}{10} + \frac{3}{10} = \frac{7}{10}$. Decimals work here but can produce repeating decimals for fractions like $\frac{1}{3}$.
What is $\frac{3}{4}$ divided by $\frac{1}{2}$?	$\frac{3}{2}$ or $1\frac{1}{2}$ — Divide: keep-change-flip. $\frac{3}{4}$ divided by $\frac{1}{2} = \frac{3}{4}$ times $\frac{2}{1} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$.
Design a word problem that requires both addition and multiplication of fractions to solve. Include the solution.	Answers vary. Example: You have $\frac{3}{4}$ yard of fabric and buy $\frac{1}{2}$ yard more. You use $\frac{2}{3}$ of the total for a project. How much did you use? Solution: $\frac{3}{4} + \frac{1}{2} = \frac{5}{4}$; $\frac{2}{3}$ times $\frac{5}{4} = \frac{10}{12} = \frac{5}{6}$ yard. — Example: You have $\frac{3}{4}$ yard of ribbon and buy $\frac{1}{2}$ yard more. You use $\frac{2}{3}$ of all your ribbon. Total: $\frac{3}{4} + \frac{1}{2} = \frac{5}{4}$. Used: $\frac{2}{3}$ times $\frac{5}{4} = \frac{10}{12} = \frac{5}{6}$ yard.
Explain why you need a common denominator to add fractions.	You can only add pieces that are the same size — A common denominator ensures both fractions describe pieces of the same size. Just as you cannot add 1 foot + 1 inch directly, you cannot add $\frac{1}{3} + \frac{1}{4}$ without matching denominators.
What is $\frac{1}{3} + \frac{1}{4}$?	$\frac{7}{12}$ — LCD of 3 and 4 is 12. Convert: $\frac{1}{3} = \frac{4}{12}$ and $\frac{1}{4} = \frac{3}{12}$. Add: $\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$.
What is $\frac{1}{4} + \frac{1}{4}$?	$\frac{2}{4}$ or $\frac{1}{2}$ — With like denominators, add numerators: $1 + 1 = 2$. Result: $\frac{2}{4} = \frac{1}{2}$.
A student computed $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$ by adding numerators and denominators. What is wrong with this approach?	You cannot add numerators and denominators separately; you need a common denominator — Adding numerators and denominators separately does not work. The correct approach: LCD = 6, $\frac{1}{2} = \frac{3}{6}$, $\frac{1}{3} = \frac{2}{6}$, sum = $\frac{5}{6}$.
What is $\frac{7}{8} - \frac{3}{8}$?	$\frac{4}{8}$ or $\frac{1}{2}$ — Like denominators: subtract numerators. $7 - 3 = 4$. Answer: $\frac{4}{8} = \frac{1}{2}$.

15. Ratios and Proportions

✂ Question (fold →)	Answer
Simplify the ratio 12:8.	3:2 — GCF of 12 and 8 is 4. Divide both terms: $12/4 = 3$, $8/4 = 2$. Simplified ratio: 3:2.
In a bag with 4 red marbles and 6 blue marbles, what is the ratio of red to total?	4:10 or 2:5 — Total = $4 + 6 = 10$. Ratio of red to total = 4:10, which simplifies to 2:5.
Which is the better deal: 3 apples for \$2.25 or 5 apples for \$3.50?	5 apples for \$3.50 (\$0.70 each vs \$0.75 each) — Unit prices: $\$2.25 \div 3 = \0.75 per apple vs $\$3.50 \div 5 = \0.70 per apple. The 5-apple deal is cheaper per apple.
A graph shows a straight line through the origin. Does this represent a proportional relationship? Why?	Yes; a straight line through the origin means y/x is constant — A proportional relationship graphs as a straight line through the origin, because $y = kx$ for some constant k , meaning the ratio y/x is always the same.
If 5 pencils cost \$2.50, what is the cost per pencil?	\$0.50 — Unit rate = total cost $\div$ number of items = $\$2.50 \div 5 = \0.50 per pencil.
Two stores sell the same juice. Store A: 64 oz for \$3.20. Store B: 48 oz for \$2.16. Is more quantity always the better deal?	No; Store B is cheaper per ounce (\$0.045/oz vs \$0.05/oz) — Store A: $\$3.20 \div 64 = \$0.05/\text{oz}$. Store B: $\$2.16 \div 48 = \$0.045/\text{oz}$. Store B is cheaper per ounce, so more quantity does not always mean a better deal.
A photograph is 4 inches wide and 6 inches tall. You want to enlarge it so it is 10 inches wide. What will the height be if the proportions are kept the same?	15 inches — Proportion: $4/6 = 10/x$. Cross multiply: $4x = 60$. Divide: $x = 15$ inches. The enlarged photo is 10 by 15 inches.
A car travels 240 miles on 8 gallons of gas. What is the fuel efficiency in miles per gallon?	30 miles per gallon — Fuel efficiency = miles / gallons = $240 / 8 = 30$ miles per gallon.
Are these ratios proportional? 6:9 and 10:15	Yes, both simplify to 2:3 — Simplify: $6/3 = 2$, $9/3 = 3 \rightarrow 2:3$. And $10/5 = 2$, $15/5 = 3 \rightarrow 2:3$. Both equal 2:3, so they are proportional.
A map scale is 1 inch = 50 miles. Two cities are 3.5 inches apart on the map. How far apart are they in real life?	175 miles — Scale: 1 inch = 50 miles. Distance: 3.5 times 50 = 175 miles.
A scale model is 1:24. If the model car is 7 inches long, how long is the real car?	168 inches or 14 feet — Scale 1:24 means real = model times 24. Real length = 7 times 24 = 168 inches = 14 feet.
A paint mixture uses red and blue in a 2:3 ratio. If you have 10 cups of blue paint, how much red paint do you need? What if you want to make the color slightly more purple — how would you adjust the ratio?	6 2/3 cups red; increase blue proportion (e.g., 2:4 ratio) — Proportion: $2/3 = x/10$. Cross multiply: $3x = 20$, $x = 6 \frac{2}{3}$ cups red. For more purple, increase the blue proportion (e.g., 2:4 ratio).
What is a unit rate?	A rate with a denominator of 1 — A unit rate is a comparison where the second quantity is 1, like per 1 pound or 60 miles per 1 hour.
	They are equivalent ratios that represent the same

What does it mean for two ratios to be proportional?	relationship — Proportional ratios are equivalent — they simplify to the same ratio. For example, 2:4 and 3:6 are proportional because both equal 1:2.
Explain why the ratio 3:4 is different from the ratio 4:3.	Order matters in ratios; 3:4 means 3 of the first for every 4 of the second — Ratios are ordered. 3:4 means 3 of the first quantity for every 4 of the second. Reversing gives a different relationship (4 of the first for every 3 of the second).
A recipe for 4 servings uses 2 cups of rice. How much rice is needed for 10 servings?	5 cups — Unit rate: $2/4 = 0.5$ cups per serving. For 10 servings: $0.5 \times 10 = 5$ cups.
You are designing a scale model of your school building. The real building is 60 feet tall and 200 feet long. Your display space is 2 feet wide. Determine an appropriate scale and calculate all model dimensions.	Scale 1:100; model is 0.6 feet tall and 2 feet long — Space constraint: $2 \text{ ft} / 200 \text{ ft} = 1/100$ scale. At 1 ¹⁰⁰ height = $60/100 = 0.6 \text{ ft}$ (7.2 in), length = $200/100 = 2 \text{ ft}$. Model fits the display.
Solve the proportion: $4/x = 12/15$.	x = 5 — Cross multiply: 4 times 15 = 60, 12 times x = 12x. So $60 = 12x$, $x = 5$.
The ratio of boys to girls in a class is 3:5. If 6 more girls join, the ratio becomes 3:7. How many boys are in the class?	9 boys — Let boys = 3x, girls = 5x. New ratio: $3x/(5x+6) = 3/7$. Cross multiply: $21x = 15x + 18$, $6x = 18$, $x = 3$. Boys = $3(3) = 9$.
Solve: $5/8 = 15/x$. Find x.	x = 24 — Cross multiply: $5x = 8 \times 15 = 120$. Divide: $x = 120/5 = 24$.
A town is growing proportionally. In 2020 it had 5,000 people. In 2023 it had 6,500. A planner says at this rate it will have 8,000 by 2026. Evaluate this claim.	The claim assumes a constant increase of 500/year, giving 8,000 in 2026 — this is reasonable if growth stays linear — Growth: 1,500 over 3 years = 500 people/year. Projecting: $6,500 + 3(500) = 8,000$. The claim is consistent with linear growth, but real populations may not grow linearly.
What is the ratio of 3 cats to 5 dogs?	3:5 — A ratio compares two quantities in order. 3 cats to 5 dogs is written as 3:5.
A runner claims she maintains a constant speed because she ran 2 miles in 16 minutes and 5 miles in 40 minutes. Is her speed constant?	Yes; both give a rate of 8 minutes per mile — Rate 1: $16 \text{ min} / 2 \text{ mi} = 8 \text{ min/mi}$. Rate 2: $40 \text{ min} / 5 \text{ mi} = 8 \text{ min/mi}$. Both unit rates are equal, confirming constant speed.
At a school, the ratio of students to teachers is 22:1. If there are 5 teachers, how many students are there?	110 students — Ratio 22:1 means 22 students per teacher. With 5 teachers: $22 \times 5 = 110$ students.
If 3 notebooks cost \$4.50, how much do 7 notebooks cost?	\$10.50 — Unit rate: $.50 / 3 = .50$ per notebook. For 7: $.50 \times 7 = .50$.

16. Algebraic Thinking and Expressions

⌂ Question (fold →)	Answer
Explain the difference between an expression and an equation.	An expression has no equals sign; an equation states two expressions are equal — An expression (like $3x + 2$) is a mathematical phrase with no equals sign. An equation (like $3x + 2 = 8$) states that two expressions are equal.
Evaluate whether this statement is always, sometimes, or never true: $2(a + b) = 2a + b$.	Sometimes true (only when $b = 0$) — Distribute: $2(a + b) = 2a + 2b$. For this to equal $2a + b$, we need $2b = b$, so $b = 0$. The statement is sometimes true — only when $b = 0$.
What is a term in the expression $6y - 2 + y$?	A single number, variable, or product of numbers and variables separated by + or - — Terms are the parts of an expression separated by addition or subtraction. In $6y - 2 + y$, there are three terms: $6y$, -2 , and y .
Create a real-world situation that can be modeled by the equation $4x + 10 = 50$. Solve it and explain what x represents.	Example: a streaming service charges a \$10 signup fee plus \$4/month; after \$50 total, $4x + 10 = 50$, so $x = 10$ months — A \$10 signup fee plus \$4/month gives $4x + 10 = 50$. Subtract 10: $4x = 40$. Divide by 4: $x = 10$ months. So x represents the number of months subscribed.
Simplify the expression: $2(3x + 4) - 5x + 1$.	$x + 9$ — Distribute: $2(3x + 4) = 6x + 8$. Combine like terms: $6x - 5x = x$ and $8 + 1 = 9$. Result: $x + 9$.
In the expression $3x + 5$, what is the coefficient of x ?	3 — The coefficient is the number multiplied by a variable. In $3x + 5$, the coefficient of x is 3.
Write an expression and equation for this situation: A school bus has some students. At the first stop, 8 students get on. At the second stop, 3 get off. There are now 27 students. How many were on the bus originally?	$s + 8 - 3 = 27$; $s = 22$ — Let s = original students. After stops: $s + 8 - 3 = s + 5 = 27$. Solve: $s = 27 - 5 = 22$ students.
What does it mean to "solve" an equation?	Find the value of the variable that makes the equation true — Solving an equation means finding the value of the variable that makes both sides of the equals sign equal.
What is a constant in the expression $4n + 7$?	7 — A constant is a term with no variable. In $4n + 7$, the constant is 7 because it does not change regardless of n .
A student simplifies $5x + 3 + 2x$ to $10x$. What error did they make?	They incorrectly combined 3 (a constant) with the x terms; correct answer is $7x + 3$ — Like terms: $5x + 2x = 7x$. The constant 3 is not a like term and cannot be combined with x terms. Correct: $7x + 3$.
Is it possible for $x + 5 = x + 3$ to have a solution? Explain.	No; subtracting x from both sides gives $5 = 3$, which is false — Subtract x from both sides: $5 = 3$. This is a contradiction — no value of x makes it true. The equation has no solution.
Write an expression for the perimeter of a rectangle with length L and width W .	$2L + 2W$ or $2(L + W)$ — Perimeter = $L + W + L + W = 2L + 2W$, or equivalently $2(L + W)$.
	$7x$ — Combine like terms: $4x$ and $3x$ both have the variable x .

Simplify: $4x + 3x$.	Add coefficients: $4 + 3 = 7$. Result: $7x$.
You have and spend d dollars. Then you earn . Write an expression for how much money you have.	$50 - d + 20$ or $70 - d$ — Start: 50. Spend d : $50 - d$. Earn 20: $50 - d + 20 = 70 - d$.
What is a variable in mathematics?	A letter or symbol that represents an unknown number — A variable is a letter or symbol (like x or n) that represents an unknown or changing value in a mathematical expression or equation.
Two students write expressions for "8 less than twice a number n ." Student A writes $8 - 2n$. Student B writes $2n - 8$. Who is correct?	Student B; "8 less than" means subtract 8 from $2n$ — "Twice a number n " = $2n$. "8 less than" means subtract 8 from that: $2n - 8$. Student B is correct. The phrase order differs from the operation order.
Solve: $x / 4 = 7$.	$x = 28$ — Multiply both sides by 4: $x = 7$ times 4 = 28.
Write an algebraic expression for: five more than a number n .	$n + 5$ — "Five more than n " translates to $n + 5$. "More than" indicates addition.
Evaluate the expression $2x + 3$ when $x = 4$.	11 — Substitute $x = 4$: $2(4) + 3 = 8 + 3 = 11$.
Solve: $x + 7 = 12$.	$x = 5$ — Subtract 7 from both sides: $x + 7 - 7 = 12 - 7$, so $x = 5$.
Solve: $y - 9 = 15$.	$y = 24$ — Add 9 to both sides: $y = 15 + 9 = 24$.
Solve: $3n = 18$.	$n = 6$ — Divide both sides by 3: $3n/3 = 18/3$, so $n = 6$.
A gym charges a membership fee plus per visit. Write an expression for the total cost of v visits.	$25 + 10v$ — Fixed fee = 25 (constant). Variable cost = $10v$ (10 dollars times v visits). Total: $25 + 10v$.
Are the expressions $3(x + 2)$ and $3x + 6$ equivalent? How do you know?	Yes; distributing 3 across $(x + 2)$ gives $3x + 6$ — The distributive property: $3(x + 2) = 3$ times $x + 3$ times 2 = $3x + 6$. The expressions are equivalent for all values of x .
Look at the pattern: 2, 5, 8, 11, 14, ... What is the rule, and what is the 10th term?	Rule: start at 2, add 3 each time; 10th term = 29 — Common difference = 3. Formula: $2 + 3(n - 1)$. 10th term: $2 + 3(9) = 2 + 27 = 29$.

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