LocusForge — Flashcards

A Spark & Anvil printable flashcard deck. Quiz yourself on the LocusForge cast's curriculum — one card at a time.

How to use these cards

Fold each sheet down the middle line so the answers are hidden. Read the **question** on the left, say your answer out loud, then **unfold** to check the right side. Testing yourself — then checking — is one of the most powerful ways to remember. Get one wrong? That's the best kind of practice: try it again tomorrow.

Want real flashcards? Cut along the fold line and the rows into cards — question on one side, answer on the other.

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1. Constructions — compass and straightedge

✂ Question (fold →)	Answer
A compass and straightedge CANNOT (classically) trisect a general angle — this shows constructions:	**have precise, provable limits** — Some tasks (trisecting a general angle) are provably impossible with compass and straightedge — constructions have exact bounds.
A compass is used to:	**draw circles and copy lengths** — A compass draws arcs/circles of a set radius and transfers lengths exactly.
A straightedge differs from a ruler in that it:	**has no measurement markings** — A straightedge draws lines but carries no scale, unlike a marked ruler.
Constructing a line PARALLEL to a given line through a point uses the fact that:	**copying an angle creates equal corresponding angles** — Copying the angle a transversal makes creates equal corresponding angles, guaranteeing parallel lines.
The intersection of the arcs in a construction gives a point that:	**satisfies the exact distance conditions of both arcs** — An arc intersection is exactly the point at the specified radius from both centers — precise by construction.
Constructing a line perpendicular to a given line through a point uses:	**equal arcs to locate two equidistant points, then their bisector** — Equal arcs locate points equidistant from the given point; their perpendicular bisector is the perpendicular line.
To bisect an ANGLE with construction, you:	**draw an arc across both sides, then equal arcs from those points, and connect** — The construction produces a ray with equal arcs on both sides — cutting the angle exactly in half.
The two arcs used to construct a perpendicular bisector must have:	**the same radius (greater than half the segment)** — Equal radii from each endpoint guarantee the intersection points are equidistant from both.
A construction that produces a 60° angle uses the fact that:	**an equilateral triangle has 60° angles** — Constructing an equilateral triangle yields exact 60° angles.
To construct a PERPENDICULAR BISECTOR of a segment, you:	**draw equal arcs from both endpoints and connect the intersections** — Equal-radius arcs from each endpoint intersect at two points equidistant from both — their line is the perpendicular bisector.
Two intersecting arcs of equal radius from the ends of a segment locate points on the:	**perpendicular bisector** — Equal-radius arc intersections are equidistant from both endpoints — on the perpendicular bisector.
To copy an angle, you use a compass to:	**transfer arc measurements that reproduce the angle exactly** — Arcs transferred with a compass reproduce the angle's opening exactly.
Why can't a construction use a ruler's measurement marks?	**classical constructions must be exact and independent of a measuring scale** — The tradition (and rigor) of construction rests on exact geometric relationships, not scale readings that introduce approximation.
Every point on the perpendicular bisector of a segment is:	**equidistant from the two endpoints** — The perpendicular bisector is exactly the set of points equidistant from the two endpoints.
The reason a construction is exact is that it relies on:	**the defining properties of circles and lines** — Constructions use exact geometric properties (equal radii, straight lines), not measurement.

A straightedge is used to:	**draw straight line segments (without measuring)** — A straightedge draws straight lines but has no scale for measuring.
Why are constructions a foundation of formal geometry?	**they build exact figures whose properties can be proven from first principles** — Constructions ground geometry in exact, provable figures — the starting point for reasoning and proof.
Mastering constructions builds the foundation for proof because it develops:	**reasoning about exact geometric relationships** — Constructions train the exact, relationship-based reasoning that proofs require.
A geometric CONSTRUCTION uses only:	**a compass and straightedge (no measuring)** — Classical constructions use an unmarked straightedge and a compass — exact, not measured.
Copying a segment with a compass works because the compass:	**preserves the exact length between two points** — Setting the compass to a segment's length lets you transfer that exact length elsewhere.
The point where a segment's perpendicular bisector meets the segment is the:	**midpoint** — The perpendicular bisector crosses the segment at its midpoint.
An equilateral triangle can be constructed by:	**drawing two arcs of the same radius as the base from each endpoint** — Two arcs equal to the base length from each endpoint meet at the apex, giving three equal sides.
A valid construction step must be:	**justifiable by geometric reasoning** — Each construction step should be justifiable — that's what makes the result provably correct.
Constructing the midpoint of a segment is a special case of constructing the:	**perpendicular bisector** — The perpendicular bisector crosses the segment exactly at its midpoint.
A construction is valued in geometry because it is:	**exact and justifiable, not just an approximation** — Constructions produce provably exact figures whose correctness can be justified — the basis of rigorous geometry.

2. Angles and lines — parallel lines and angle relationships

✂ Question (fold →)	Answer
If two angles are both congruent AND supplementary, each must be:	**90°** — Equal angles summing to 180° are each 90°.
With parallel lines, co-interior (same-side interior) angles are:	**supplementary** — Same-side interior angles are supplementary (sum to 180°) with parallel lines.
If two parallel lines are cut by a transversal, same-side EXTERIOR angles are:	**supplementary** — Same-side exterior angles are supplementary with parallel lines.
If a triangle has angles 50° and 60°, the third angle is:	**70°** — 180 − 50 − 60 = 70°.
Angle relationships turn a diagram into equations because:	**known relationships give equations to solve for unknowns** — Vertical, supplementary, and parallel-line relationships convert geometry into solvable algebra.
The angles in a triangle sum to:	**180°** — The interior angles of any triangle add to 180°.
An EXTERIOR angle of a triangle equals:	**the sum of the two remote interior angles** — A triangle's exterior angle equals the sum of the two non-adjacent (remote) interior angles.
Vertical angles being congruent follows from:	**both being supplementary to the same angle** — Each vertical angle is supplementary to the same adjacent angle, so they're equal.
With parallel lines and a transversal, ALTERNATE INTERIOR angles are:	**congruent** — Alternate interior angles (opposite sides of the transversal, between the lines) are congruent when the lines are parallel.
The exterior angle of a triangle is always __ than either remote interior angle:	**larger** — Since the exterior angle equals the SUM of the two remote interior angles, it exceeds either one alone.
VERTICAL angles (formed by two intersecting lines) are:	**congruent (equal)** — Vertical angles — opposite angles at an intersection — are always congruent.
Why are angle and line relationships foundational to geometry?	**they let you deduce unknown angles and prove figures' properties** — These relationships are the basic deductions geometry builds proofs from.
Two angles forming a straight line (a linear pair) are:	**supplementary (sum to 180°)** — A linear pair lies along a straight line and sums to 180°.
A right angle measures:	**90°** — A right angle is exactly 90°.
If one angle of a linear pair is 110°, the other is:	**70°** — 180 − 110 = 70°.
Two angles are COMPLEMENTARY if they sum to:	**90°** — Complementary angles add to 90° (a right angle).
A common misconception is judging an angle relationship from how a diagram LOOKS. Instead you should:	**use the given information and theorems** — Diagrams may not be to scale; reason from the givens and theorems, not the picture.

Alternate EXTERIOR angles with parallel lines are:	**congruent** — Alternate exterior angles are congruent when the lines are parallel.
Two angles that share a vertex and a side, with no overlap, are:	**adjacent angles** — Adjacent angles share a vertex and a side without overlapping.
When a transversal crosses two PARALLEL lines, corresponding angles are:	**congruent** — Parallel lines cut by a transversal make congruent corresponding angles.
Perpendicular lines meet at:	**a 90° angle** — Perpendicular lines intersect at right angles (90°).
Two angles are SUPPLEMENTARY if they sum to:	**180°** — Supplementary angles add to 180° (a straight angle).
An angle bisector divides an angle into:	**two equal angles** — An angle bisector splits an angle into two congruent halves.
If two lines are cut by a transversal and corresponding angles are EQUAL, the lines are:	**parallel** — Equal corresponding angles guarantee the lines are parallel (the converse).
Angle relationships let you find unknown angles by:	**setting up equations from the relationships (equal or supplementary)** — Known relationships (vertical = equal, linear pair = 180°, etc.) let you solve for unknowns algebraically.

3. Triangle congruence — SSS, SAS, ASA, and CPCTC

✂ Question (fold →)	Answer
A congruence proof should rely on GIVEN information and theorems, not on:	**how congruent the triangles appear in the diagram** — Valid proofs use givens and theorems; appearances (which may be misleading) are never a justification.
A common congruence error is pairing NON-corresponding parts. The correct order comes from:	**the correspondence in the congruence statement (matching vertices)** — The order of letters in △ABC ≅ △DEF tells you A↔D, B↔E, C↔F — pair by correspondence, not appearance.
If △ABC ≅ △DEF, then side AB corresponds to side:	**DE** — The correspondence A↔D, B↔E means AB ↔ DE.
If you prove two triangles congruent, you may then conclude any pair of corresponding parts is equal by:	**CPCTC** — CPCTC lets you conclude corresponding parts are equal once congruence is established.
To use SAS, the equal angle must be:	**between the two equal sides (the included angle)** — SAS requires the angle to be included (between) the two equal sides.
Two triangles are CONGRUENT if:	**all corresponding sides and angles are equal (same size and shape)** — Congruent triangles match exactly — all corresponding sides and angles are equal.
CPCTC stands for:	**Corresponding Parts of Congruent Triangles are Congruent** — Once triangles are proven congruent, CPCTC lets you conclude any pair of corresponding parts is equal.
SSS congruence means two triangles are congruent if:	**all three pairs of corresponding sides are equal** — Side-Side-Side: three equal side pairs guarantee congruence.
'AAA' (all three angles equal) proves triangles are:	**similar, but NOT necessarily congruent** — Equal angles give the same shape (similar) but say nothing about size, so AAA is not a congruence criterion.
You use CPCTC AFTER:	**you have already proven the triangles congruent** — CPCTC is a follow-up step: first prove congruence, then conclude corresponding parts are equal.
ASA congruence means two triangles are congruent if:	**two angles and the INCLUDED side are equal** — Angle-Side-Angle: two angles and the side between them guarantee congruence.
Why is triangle congruence central to geometry?	**it lets you prove that figures are exactly equal and deduce further properties** — Congruence is a core proof tool: establishing it lets you deduce equal parts (CPCTC) and build larger arguments.
AAS congruence means two triangles are congruent if:	**two angles and a NON-included side are equal** — Angle-Angle-Side: two angles and a side not between them guarantee congruence.
SAS congruence means two triangles are congruent if:	**two sides and the INCLUDED angle are equal** — Side-Angle-Side: two sides and the angle BETWEEN them guarantee congruence.
Congruent triangles have corresponding sides that are:	**equal in length** — Corresponding sides of congruent triangles are equal (not merely proportional).

Congruence proofs matter because they let you establish that:	**two figures are exactly equal, unlocking further deductions** — Establishing congruence rigorously enables a chain of further deductions (via CPCTC).
'SSA' is NOT a valid congruence criterion because:	**two sides and a non-included angle can form two different triangles** — SSA (the 'ambiguous case') can produce two non-congruent triangles, so it doesn't guarantee congruence.
Reflexive property in a proof lets you state a shared side is:	**congruent to itself** — A side shared by two triangles is congruent to itself (reflexive) — often a key given.
A rigid motion mapping one triangle exactly onto another proves they are:	**congruent** — If a rigid motion maps one triangle onto another, they're congruent by definition.
Marking a diagram with the given equal parts helps a congruence proof by:	**showing which criterion (SSS/SAS/ASA) applies** — Marking equal parts reveals which congruence criterion the givens satisfy.
To prove two triangles congruent, you must show enough parts equal to match:	**a valid criterion (SSS, SAS, ASA, AAS, or HL)** — A congruence proof matches the givens to one of the valid criteria.
If $\triangle ABC \cong \triangle DEF$, angle B corresponds to angle:	**E** — The order A↔D, B↔E, C↔F gives angle B ↔ angle E.
HL congruence applies specifically to:	**right triangles (Hypotenuse-Leg)** — Hypotenuse-Leg proves congruence for right triangles when the hypotenuse and one leg match.
Two triangles that share a side and have it as a congruence part use the __ property:	**reflexive** — A shared side is congruent to itself by the reflexive property.
Congruent triangles have equal:	**areas and perimeters (all corresponding measures)** — Because all corresponding parts are equal, congruent triangles share every measure, including area and perimeter.

4. Similarity — same shape, proportional size

✂ Question (fold →)	Answer
Two rectangles are similar only if:	**their side ratios are equal** — All rectangles have equal angles, so similarity requires equal side (length:width) ratios.
Why is similarity a powerful tool in geometry?	**it lets you relate figures of different sizes and measure the inaccessible indirectly** — Similarity underlies scale drawings, maps, indirect measurement, and trigonometry — relating figures across sizes.
Shadows and similar triangles let you find a tall object's height by:	**comparing its shadow to a known object's shadow (proportion)** — Similar triangles from shadows relate heights and shadow lengths in proportion — indirect measurement.
If two triangles are similar with scale factor 2, a side of 3 in the small one corresponds to:	**6 in the large one** — Multiply by the scale factor: $3 \times 2 = 6$.
All squares are similar to each other because:	**they have the same shape (equal angles, proportional sides)** — Every square has four right angles and equal sides in the same ratio, so all squares are similar.
A scale drawing (like a map) uses __ to represent real distances:	**similarity (a constant scale factor)** — Scale drawings shrink real distances by a constant factor — a use of similarity.
If two triangles are similar, their corresponding sides are in a constant:	**ratio (the scale factor)** — Similar triangles' corresponding sides share one constant ratio — the scale factor.
In similar figures, corresponding ANGLES are:	**equal** — Similar figures have equal corresponding angles (same shape).
Similar triangles are used to find unknown lengths by:	**setting up a proportion between corresponding sides** — A proportion of corresponding sides lets you solve for an unknown length.
A common misconception is treating similar figures as CONGRUENT. The difference is:	**congruent figures are the same size; similar figures may differ in size** — Congruence requires equal size; similarity only requires the same shape (proportional sizes).
Similarity underlies trigonometry because right triangles with the same acute angle are:	**similar, giving constant side ratios (sin, cos, tan)** — Same-angle right triangles are similar, so their side ratios are constant — the basis of trig functions.
Two figures are SIMILAR if they have:	**the same shape but possibly different sizes (proportional)** — Similar figures share the same shape; their corresponding sides are proportional and angles equal.
The SCALE FACTOR between similar figures is:	**the ratio of corresponding side lengths** — The scale factor is the constant ratio of a side in one figure to the corresponding side in the other.
If two triangles are similar with ratio 1:3, and the small has area 4, the large has area:	**36** — Area ratio = $3^2 = 9$, so $4 \times 9 = 36$.
	the equal included angle between them — SAS similarity: two

SAS similarity requires two proportional sides and:	proportional sides with the equal included angle guarantee similarity.
Two similar figures with scale factor 3 have areas in the ratio:	**9 : 1** — Area ratio = 3^2 = 9, so 9 : 1.
A claim that two figures are similar 'because they both have a right angle' is:	**insufficient (one equal angle doesn't guarantee similarity)** — One equal angle isn't enough; AA requires TWO equal angle pairs.
In similar figures, corresponding SIDES are:	**proportional (in a constant ratio)** — Corresponding sides of similar figures share a constant ratio — the scale factor.
The RATIO of areas of two similar figures equals:	**the square of the scale factor** — If sides scale by k, areas scale by k^2 (a common surprise).
AA similarity says two triangles are similar if:	**two pairs of angles are equal** — Angle-Angle: two equal angle pairs guarantee similarity (the third angle follows).
SSS similarity says triangles are similar if all three side pairs are:	**proportional** — SSS similarity: three pairs of proportional sides guarantee similarity.
Congruent figures are a special case of similar figures where the scale factor is:	**1** — Congruence is similarity with scale factor 1 (same size).
The key difference between similarity and congruence is:	**similarity allows a scale change; congruence does not** — Congruence is similarity with scale factor 1; general similarity permits any scale factor.
A midsegment of a triangle (connecting two side midpoints) is __ the third side:	**half the length of and parallel to** — The midsegment is parallel to the third side and half its length (a similarity result).
Similar figures need BOTH equal angles AND proportional sides — having only equal angles guarantees similarity for triangles because:	**for triangles, equal angles force proportional sides (AA)** — In triangles, two equal angles (AA) automatically make the sides proportional — so AA is enough.

5. Right-triangle trigonometry and the Pythagorean theorem

✂ Question (fold →)	Answer
To choose the right trig ratio, identify which sides are __ relative to the angle:	**opposite, adjacent, or the hypotenuse** — Labeling sides as opposite, adjacent, and hypotenuse (relative to the angle) selects the ratio.
Trig ratios stay the same for a given angle regardless of triangle size because such triangles are:	**similar** — Same-angle right triangles are similar, so their side ratios (the trig values) are constant.
SINE of an angle in a right triangle equals:	**opposite / hypotenuse** — sin = opposite/hypotenuse (the 'SOH' of SOH-CAH-TOA).
The converse of the Pythagorean theorem says if $a^2 + b^2 = c^2$, the triangle is:	**a right triangle** — If the side lengths satisfy $a^2 + b^2 = c^2$, the triangle must be right.
A key trig misconception is treating the sides' roles as fixed. Actually opposite and adjacent depend on:	**which angle you're using** — Which side is 'opposite' or 'adjacent' switches depending on the reference angle.
Right-triangle trig lets surveyors and navigators find distances they cannot:	**measure directly** — Trig computes inaccessible distances from measurable angles and lengths — indirect measurement.
The mnemonic SOH-CAH-TOA helps you remember:	**the sine, cosine, and tangent ratios** — SOH-CAH-TOA encodes sin=O/H, cos=A/H, tan=O/A.
The HYPOTENUSE of a right triangle is:	**the side opposite the right angle (the longest side)** — The hypotenuse is opposite the right angle and is the longest side.
TANGENT of an angle equals:	**opposite / adjacent** — tan = opposite/adjacent ('TOA').
A right triangle with legs 3 and 4 has a hypotenuse of:	**5** — $\sqrt{(3^2 + 4^2)} = \sqrt{(9 + 16)} = \sqrt{25} = 5$.
The Pythagorean theorem applies ONLY to:	**right triangles** — $a^2 + b^2 = c^2$ holds specifically for right triangles.
For a 45-45-90 triangle, the hypotenuse is __ times a leg:	**$\sqrt{2}$** — In a 45-45-90 triangle, the hypotenuse is $\sqrt{2}$ times each equal leg.
A common trig error is using the wrong side as the hypotenuse. The hypotenuse is always:	**opposite the right angle** — Regardless of orientation, the hypotenuse is opposite the right angle.
For an angle θ, if the opposite side is 6 and the hypotenuse is 10, sin θ =:	**0.6** — sin θ = opposite/hypotenuse = 6/10 = 0.6.
Trigonometric ratios work because right triangles with the same angle are:	**similar (so their side ratios are constant)** — Same-angle right triangles are similar, so their side ratios (sin, cos, tan) depend only on the angle.
In a 30-60-90 triangle, the side opposite 30° is __ the hypotenuse:	**half** — The side opposite the 30° angle is half the hypotenuse.
To find a missing ANGLE from two sides,	**an inverse trig function (arcsin, arccos, or arctan)** — Inverse trig

you use:	functions recover the angle from a known ratio of sides.
The angle of elevation to the top of a building can be used with trig to find the:	**building's height (given a distance)** — Given the horizontal distance and the elevation angle, tangent gives the height.
A triangle with sides 5, 12, 13 is:	**a right triangle ($5^2 + 12^2 = 13^2$)** — 25 + 144 = 169 = 13^2, so it's a right triangle by the converse.
The tangent of 45° equals:	**1** — In a 45-45-90 triangle the legs are equal, so tan 45° = opposite/adjacent = 1.
The Pythagorean theorem states that in a right triangle:	**$a^2 + b^2 = c^2$ (legs squared sum to the hypotenuse squared)** — For legs a, b and hypotenuse c: $a^2 + b^2 = c^2$.
If sin θ = opposite/hypotenuse, then for a 30° angle in a right triangle with hypotenuse 10, the opposite side is:	**5** — sin 30° = 0.5, so opposite = 0.5 × 10 = 5.
To find a missing SIDE using an angle and one side, you use:	**a trig ratio (sine, cosine, or tangent)** — Choose the ratio relating the known and unknown sides to the given angle, then solve.
Why is right-triangle trigonometry so useful?	**it relates angles and lengths, enabling indirect measurement and modeling** — Trigonometry connects angles and distances, powering navigation, surveying, physics, and engineering.
COSINE of an angle equals:	**adjacent / hypotenuse** — cos = adjacent/hypotenuse ('CAH').

6. The unit circle — angles and coordinates

✂ Question (fold →)	Answer
Values of sine range between:	**-1 and 1** — Since sine is a y-coordinate on a radius-1 circle, it ranges from -1 to 1.
On the unit circle, the COSINE of an angle is the:	**x-coordinate of the point** — Cosine gives the x-coordinate of the terminal point.
The UNIT CIRCLE is a circle with:	**radius 1 centered at the origin** — The unit circle has radius 1 and is centered at (0, 0).
The unit circle connects to right-triangle trig because for an acute angle, the coordinates match:	**the cosine (adjacent) and sine (opposite) side ratios** — For an acute angle, the unit-circle x and y coordinates are exactly the cosine and sine ratios of a right triangle.
At 0° (or 0 radians), the point on the unit circle is:	**(1, 0)** — At 0°, the point is (cos 0, sin 0) = (1, 0).
A full revolution around the circle is:	**360° (or 2π radians)** — One full turn is 360° = 2π radians.
At 90° (π/2), the point on the unit circle is:	**(0, 1)** — At 90°, the point is (cos 90°, sin 90°) = (0, 1).
The unit circle shows sine and cosine are related by a 90° SHIFT because:	**sin θ = cos(90° − θ) (co-functions)** — Sine and cosine are co-functions, offset by 90° — visible on the unit circle.
On the unit circle, the SINE of an angle is the:	**y-coordinate of the point** — Sine gives the y-coordinate of the terminal point.
The maximum value of cosine is:	**1** — Cosine (an x-coordinate on a radius-1 circle) reaches a maximum of 1.
On the unit circle, tangent of an angle equals:	**sin θ / cos θ (y/x)** — tan θ = sin θ/cos θ = y/x on the unit circle.
Radians measure an angle by:	**the arc length on a unit circle** — One radian is the angle subtending an arc equal to the radius; on the unit circle, radians equal arc length.
The unit circle EXTENDS trig to angles beyond a right triangle by:	**allowing angles of any size (including past 90° and negative)** — The unit circle defines sine and cosine for all angles, not just the acute angles of a right triangle.
At 270° the point on the unit circle is:	**(0, -1)** — At 270°, the point is (cos 270°, sin 270°) = (0, -1).
The unit circle is foundational because it extends trig to:	**all angles and connects to periodic (wave) functions** — It defines trig for every angle and links to periodic functions modeling waves and cycles.
Sine and cosine are called PERIODIC because their values:	**repeat every 360° (2π)** — Sine and cosine repeat every full revolution — they're periodic with period 360°.
180° in radians is:	**π** — 180° corresponds to π radians (half a revolution).
In the second quadrant (90°–180°), cosine (the x-coordinate) is:	**negative** — In the second quadrant, x-coordinates are negative, so cosine is negative there.

For an angle θ on the unit circle, the point on the circle is:	**(cos θ, sin θ)** — The terminal point for angle θ is (cos θ, sin θ).
Why is the unit circle a powerful tool?	**it defines trig for all angles and links geometry, coordinates, and periodic functions** — The unit circle unifies trigonometry, coordinates, and periodic behavior — foundational for advanced math and physics.
Because the radius is 1, for any angle on the unit circle:	**$\sin^2\theta + \cos^2\theta = 1$** — The point (cos θ, sin θ) lies on $x^2 + y^2 = 1$, giving $\sin^2\theta + \cos^2\theta = 1$.
At 360° the point on the unit circle is the same as at:	**0° (they coincide)** — A full revolution returns to the start, so 360° coincides with 0°.
In the third quadrant (180°–270°), both sine and cosine are:	**negative** — In the third quadrant both x and y are negative, so cosine and sine are negative.
At 30°, the coordinates on the unit circle are approximately:	**(√3/2, 1/2)** — At 30°, (cos 30°, sin 30°) = (√3/2, 1/2).
An angle of 45° corresponds to how many radians?	**π/4** — 45° = 180°/4 = π/4 radians.

7. Circles — inscribed and central angles, chords

✂ Question (fold →)	Answer
A chord that passes through the center is a:	**diameter (the longest chord)** — A chord through the center is a diameter — the circle's longest chord.
A common circle error is confusing inscribed and central angles. If a central angle is 80°, the inscribed angle on the same arc is:	**40°** — The inscribed angle is half the central angle: 80°/2 = 40°.
A tangent line is __ to the radius at the point of tangency:	**perpendicular** — The radius drawn to the point of tangency is perpendicular to the tangent.
A radius perpendicular to a chord __ that chord:	**bisects** — A radius perpendicular to a chord bisects it (splits it into two equal halves).
A TANGENT to a circle:	**touches the circle at exactly one point** — A tangent touches the circle at exactly one point.
An INSCRIBED angle has its vertex:	**on the circle** — An inscribed angle's vertex lies on the circle itself.
Two tangent segments drawn to a circle from the same external point are:	**equal in length** — Tangent segments from a common external point to a circle are congruent.
A central angle of 90° intercepts an arc of:	**90°** — A central angle equals its intercepted arc, so 90° gives a 90° arc.
An ARC of a circle is:	**a portion of the circle's circumference** — An arc is a connected part of the circle's edge.
The DIAMETER of a circle is:	**twice the radius (across through the center)** — The diameter is a full chord through the center — twice the radius. (Confusing the two is a common error.)
A CHORD of a circle is:	**a segment with both endpoints on the circle** — A chord connects two points on the circle (the diameter is the longest chord).
If a circle has radius 6, its diameter is:	**12** — Diameter = 2 × radius = 12.
Circle theorems let engineers and designers work with exact:	**angle, arc, and chord relationships** — Precise circle relationships support gear design, architecture, and more.
An inscribed angle equals __ its intercepted arc (or central angle):	**half** — The inscribed angle theorem: an inscribed angle is half the central angle (arc) it subtends.
A CENTRAL angle has its vertex at:	**the center of the circle** — A central angle's vertex is the circle's center; it equals its intercepted arc.
The set of all points on a circle is a fixed distance (the radius) from:	**the center** — Every point on a circle is the radius distance from the center.
An inscribed angle intercepting a 100° arc measures:	**50°** — An inscribed angle is half its intercepted arc: 100°/2 = 50°.
The AREA of a circle is:	**πr^2** — Area = πr^2.
The RADIUS of a circle is:	**the distance from the center to any point on the circle** — The

	radius runs from the center to the edge.
An angle inscribed in a SEMICIRCLE (subtending a diameter) is:	**a right angle (90°)** — Thales' theorem: an inscribed angle subtending a diameter is 90°.
The CIRCUMFERENCE of a circle is:	**2πr (the distance around it)** — Circumference = 2πr (or πd) — the distance around the circle.
Why are circle theorems important in geometry?	**they reveal precise relationships among angles, arcs, and chords for proof and design** — Circle theorems give exact angle/arc/chord relationships used throughout geometry, engineering, and design.
The area of a circle with radius 4 is:	**16π** — Area = $\pi r^2 = \pi(4^2) = 16\pi$.
A common error is using the diameter where the radius is needed in πr^2. For a circle of diameter 10, the area is:	**25π (radius 5)** — The radius is 5 (half the diameter), so area = $\pi(5^2) = 25\pi$.
Inscribed angles that subtend the SAME arc are:	**equal to each other** — All inscribed angles subtending the same arc are equal (each is half that arc).

8. Coordinate geometry — distance, midpoint, and slope proofs

✂ Question (fold →)	Answer
The midpoint formula finds a point that:	**is exactly halfway between two points** — The midpoint is the exact center of a segment, found by averaging coordinates.
The MIDPOINT of a segment from (x_1,y_1) to (x_2,y_2) is:	**$((x_1+x_2)/2, (y_1+y_2)/2)$** — The midpoint averages the coordinates: $((x_1+x_2)/2, (y_1+y_2)/2)$.
Two lines both with slope 3 are:	**parallel (or the same line)** — Equal slopes mean the lines are parallel (or identical).
Why is coordinate geometry a powerful proof method?	**it links algebra and geometry, letting you prove geometric facts by computation** — Coordinate geometry bridges algebra and geometry, enabling exact, computational proofs (a Descartes legacy).
A vertical line has a slope that is:	**undefined** — A vertical line has zero run, making the slope undefined (division by zero).
To prove a quadrilateral is a PARALLELOGRAM using coordinates, you can show:	**both pairs of opposite sides have equal slopes (parallel)** — Equal slopes for both pairs of opposite sides prove they're parallel — a parallelogram.
Coordinate geometry unified algebra and geometry, an idea credited to:	**René Descartes** — Descartes' coordinate system (Cartesian plane) linked algebra and geometry.
The SLOPE of a line through (x_1,y_1) and (x_2,y_2) is:	**$(y_2-y_1)/(x_2-x_1)$** — Slope = rise/run = $(y_2-y_1)/(x_2-x_1)$.
To show a triangle is a RIGHT triangle with coordinates, you can check that two sides have:	**perpendicular slopes (product −1)** — If two sides' slopes multiply to −1, they're perpendicular, making a right angle.
Placing a general triangle with one vertex at the origin and one side on the x-axis:	**simplifies the algebra while keeping generality** — Smart coordinate placement simplifies computation without losing the proof's generality.
The midpoint of (0,0) and (8,6) is:	**(4, 3)** — ((0+8)/2, (0+6)/2) = (4, 3).
The DISTANCE between two points (x_1,y_1) and (x_2,y_2) is:	**$\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$** — The distance formula comes from the Pythagorean theorem: $\sqrt{\Delta x^2 + \Delta y^2}$.
A line with slope 2 and a line with slope −1/2 are:	**perpendicular (negative reciprocals)** — 2 × (−1/2) = −1, so the lines are perpendicular.
To prove a quadrilateral is a RHOMBUS with coordinates, show:	**all four sides are equal (distance formula)** — Equal side lengths (distance formula) for all four sides prove a rhombus.
The distance between (0,0) and (3,4) is:	**5** — $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$.
A coordinate proof avoids relying on the diagram because it uses:	**exact algebraic calculation** — Coordinate proofs compute exactly, avoiding the appearance-based errors of diagram reading.
Choosing convenient coordinates	**simpler without loss of generality** — Placing the figure conveniently

(like placing a vertex at the origin) makes a coordinate proof:	simplifies the algebra while keeping the proof fully general.
A horizontal line has a slope of:	**0** — A horizontal line has zero rise, so its slope is 0.
Two lines are PERPENDICULAR if their slopes are:	**negative reciprocals (product = −1)** — Perpendicular lines have slopes that are negative reciprocals ($m_1 \cdot m_2 = -1$).
The midpoint of (2,4) and (6,8) is:	**(4, 6)** — ((2+6)/2, (4+8)/2) = (4, 6).
The slope of the line through (1,2) and (3,6) is:	**2** — (6 − 2)/(3 − 1) = 4/2 = 2.
To show a triangle is ISOSCELES with coordinates, you check that two sides have:	**equal lengths (using the distance formula)** — Equal side lengths (distance formula) prove an isosceles triangle.
A coordinate proof is powerful because it:	**turns geometric claims into algebra you can verify exactly** — Placing figures on axes lets you prove properties with exact algebra rather than trusting a picture.
Two lines are PARALLEL if their slopes are:	**equal** — Parallel lines have equal slopes.
The distance formula is derived from the:	**Pythagorean theorem** — The distance formula applies the Pythagorean theorem to coordinate differences.

9. Transformations — rigid motions and dilation

✂ Question (fold →)	Answer
Two reflections over PARALLEL lines produce a:	**translation** — Reflecting over two parallel lines is equivalent to a translation.
A 180° rotation about the origin sends (x, y) to:	**(−x, −y)** — A 180° rotation about the origin negates both coordinates.
A square has ROTATIONAL symmetry of order:	**4 (it maps onto itself every 90°)** — A square looks the same after 90°, 180°, 270°, and 360° turns — order 4.
Transformations connect to congruence and similarity by providing:	**the motions that define them (rigid motions and dilations)** — Rigid motions define congruence; dilations extend to similarity — transformations underpin both.
A rigid motion produces an image that is __ to the original:	**congruent** — Because rigid motions preserve size and shape, the image is congruent.
Reflecting a point over the x-axis changes (x, y) to:	**(x, −y)** — Reflection over the x-axis negates the y-coordinate: (x, y) → (x, −y).
A figure has LINE symmetry if:	**it can be reflected over a line onto itself** — Line (reflectional) symmetry means a reflection maps the figure onto itself.
A dilation with scale factor 2 about the origin sends (x, y) to:	**(2x, 2y)** — A dilation multiplies coordinates by the scale factor: (2x, 2y).
A rotation is determined by a center point AND:	**an angle of rotation** — A rotation needs a center and an angle.
A DILATION changes a figure by:	**resizing it by a scale factor from a center point** — A dilation scales the figure larger or smaller by a factor from a center, producing a similar figure.
Composing two rigid motions produces:	**another rigid motion (still congruent)** — A sequence of rigid motions is itself a rigid motion, so the result is congruent.
A reflection over the y-axis sends (x, y) to:	**(−x, y)** — Reflecting over the y-axis negates the x-coordinate: (−x, y).
Two figures are congruent if and only if:	**one can be mapped to the other by a sequence of rigid motions** — The modern definition: congruence means a rigid-motion mapping exists between the figures.
Composing a reflection with itself (over the same line) returns the figure to:	**its original position** — Reflecting twice over the same line undoes the reflection, restoring the original.
Why are transformations a powerful modern foundation for geometry?	**they DEFINE congruence and similarity and unify how figures relate** — Transformations give precise, motion-based definitions of congruence (rigid motions) and similarity (dilations) — a unifying framework.
A translation is determined by a:	**vector (distance and direction)** — A translation is specified by a vector giving how far and which way to slide.

A TRANSLATION moves a figure by:	**sliding it a fixed distance and direction (no turn or flip)** — A translation slides every point the same distance in the same direction.
A translation that moves a figure right 3 and up 2 sends (x, y) to:	**(x + 3, y + 2)** — Right 3, up 2 adds to the coordinates: (x + 3, y + 2).
A ROTATION moves a figure by:	**turning it around a fixed point** — A rotation turns the figure about a center point by an angle.
An equilateral triangle has rotational symmetry of order:	**3** — It maps onto itself every 120° — order 3.
A dilation (scale factor ≠ 1) produces an image that is __ to the original:	**similar (same shape, different size)** — A dilation preserves shape but changes size, giving a similar figure.
A figure with 4 lines of symmetry and rotational symmetry of order 4 is a:	**square** — A square has four lines of symmetry and order-4 rotational symmetry.
A REFLECTION moves a figure by:	**flipping it over a line (the line of reflection)** — A reflection flips the figure across a line, producing a mirror image.
A figure has ROTATIONAL symmetry if:	**it can be turned less than a full circle onto itself** — Rotational symmetry means some rotation (< 360°) maps the figure onto itself.
Translations, rotations, and reflections are called RIGID motions because they:	**preserve size and shape (distances and angles)** — Rigid motions preserve lengths and angles, so the image is congruent to the original.

10. Quadrilaterals — properties and classification

✂ Question (fold →)	Answer
A common misconception is that a shape with four equal sides must be a SQUARE. Actually it could be a:	**rhombus (equal sides but not necessarily right angles)** — Four equal sides make a rhombus; it's a square only if the angles are also 90°.
A TRAPEZOID has:	**at least one pair of parallel sides** — A trapezoid has at least one pair of parallel sides.
The diagonals of a RECTANGLE are:	**equal in length** — A rectangle's diagonals are equal (and bisect each other).
Why is classifying quadrilaterals by properties important?	**knowing a shape's category tells you which properties and theorems apply** — Correct classification lets you apply the right properties (equal diagonals, right angles, etc.) in proofs and problems.
If a quadrilateral has diagonals that bisect each other, it is a:	**parallelogram** — Diagonals bisecting each other is a defining property of a parallelogram.
A KITE has:	**two pairs of consecutive equal sides** — A kite has two distinct pairs of adjacent congruent sides.
The most SPECIFIC name for a quadrilateral with four equal sides and four right angles is:	**square** — While it's also a rhombus, rectangle, and parallelogram, the most specific name is square.
A SQUARE is:	**both a rectangle and a rhombus (four equal sides AND four right angles)** — A square has four equal sides and four right angles, so it's both a rectangle and a rhombus.
The interior angles of a quadrilateral sum to:	**360°** — A quadrilateral's interior angles sum to 360°.
The diagonals of a SQUARE are:	**equal AND perpendicular (and bisect each other)** — A square (both rectangle and rhombus) has diagonals that are equal, perpendicular, and bisecting.
Knowing a quadrilateral's exact type lets you apply its specific:	**diagonal, side, and angle properties in proofs** — The precise classification determines which properties (equal diagonals, perpendicular diagonals, etc.) you can use.
In a parallelogram, consecutive (adjacent) angles are:	**supplementary** — Adjacent angles of a parallelogram sum to 180° (supplementary).
Classifying quadrilaterals from most general to most special goes:	**quadrilateral → parallelogram → rectangle/rhombus → square** — Each step adds properties: parallelograms, then rectangles/rhombi, then the square (both).
The diagonals of a parallelogram:	**bisect each other** — In a parallelogram, the diagonals bisect each other (cross at their midpoints).
A trapezoid is NOT a parallelogram because it has:	**at most one pair of parallel sides** — A (exclusive) trapezoid has only one pair of parallel sides, unlike a parallelogram's two.
	four right angles — A rectangle is a parallelogram whose angles are all

A RECTANGLE is a parallelogram with:	90°.
A rectangle that is also a rhombus must be a:	**square** — Having right angles (rectangle) and equal sides (rhombus) makes a square.
In a parallelogram, opposite angles are:	**equal** — Opposite angles of a parallelogram are congruent.
A PARALLELOGRAM has:	**both pairs of opposite sides parallel** — A parallelogram has two pairs of parallel opposite sides (and equal opposite sides/angles).
Every rectangle is a parallelogram, but not every parallelogram is a rectangle, because:	**a rectangle needs right angles that a general parallelogram may lack** — Rectangles are a special case of parallelograms (with right angles); the general parallelogram need not have them.
The diagonals of a RHOMBUS are:	**perpendicular to each other** — A rhombus's diagonals are perpendicular (and bisect each other).
A RHOMBUS is a parallelogram with:	**all four sides equal** — A rhombus is a parallelogram with four equal sides.
An ISOSCELES trapezoid has:	**one pair of parallel sides and equal legs (and equal base angles)** — An isosceles trapezoid has parallel bases, congruent legs, and congruent base angles.
In a parallelogram, opposite sides are:	**equal in length** — Opposite sides of a parallelogram are equal.
A QUADRILATERAL is a polygon with:	**four sides** — A quadrilateral has four sides (and four angles).

11. Polygons and area — perimeter, area, and regular polygons

✂ Question (fold →)	Answer
A common error is confusing area and perimeter. Perimeter is measured in:	**linear units (e.g. cm); area in square units (e.g. cm²)** — Perimeter is a length (linear units); area is a region (square units) — a frequently confused distinction.
A hexagon has interior angles summing to:	**720°** — (6 − 2) × 180° = 4 × 180° = 720°.
Comparing two shapes, a larger perimeter does NOT guarantee a larger:	**area** — A long, thin shape can have a large perimeter but small area — the two aren't directly linked.
Area and perimeter formulas let us solve real problems like:	**flooring, fencing, and material estimation** — Area (covering) and perimeter (bordering) drive practical estimation in construction and design.
Why are area and perimeter fundamental measures?	**they quantify a figure's boundary and interior for real-world problems** — Perimeter (boundary) and area (region) are basic measurements used in construction, design, and everyday problem-solving.
Each interior angle of a REGULAR pentagon is:	**108°** — 540° ÷ 5 = 108° per angle.
The area of a circle uses π because π relates a circle's:	**circumference (and area) to its radius/diameter** — π is the constant linking a circle's circumference and area to its radius/diameter.
The perimeter of a regular polygon is:	**the number of sides × the side length** — For a regular polygon, perimeter = n × side length.
Doubling every side of a square multiplies its AREA by:	**4** — Area scales as the square of the linear factor: 2^2 = 4.
The area of a parallelogram is:	**base × height** — Parallelogram area = base × perpendicular height.
The area of a triangle with base 10 and height 6 is:	**30 square units** — ½ × 10 × 6 = 30.
A rectangle 5 by 3 has an area of:	**15 square units** — 5 × 3 = 15 square units.
A REGULAR polygon has:	**all sides equal AND all angles equal** — A regular polygon is both equilateral (equal sides) and equiangular (equal angles).
Two rectangles with the same perimeter can have:	**different areas** — Rectangles of equal perimeter vary in area depending on their proportions (a square maximizes it).
A pentagon's interior angles sum to:	**540°** — (5 − 2) × 180° = 3 × 180° = 540°.
The area of a triangle is:	**½ × base × height** — Triangle area = ½ base × height.
The AREA of a polygon is:	**the amount of space it covers** — Area measures the two-dimensional space a figure covers.

A rectangle 5 by 3 has a perimeter of:	**16 units** — Perimeter = 2(5 + 3) = 16.
The sum of interior angles of an n-sided polygon is:	**(n − 2) × 180°** — The interior angle sum of an n-gon is (n − 2)·180°.
To find the area of an irregular polygon, you can:	**decompose it into triangles/rectangles and sum their areas** — Splitting an irregular shape into simple pieces and summing their areas gives the total.
Each interior angle of a regular hexagon is:	**120°** — 720° ÷ 6 = 120° per angle.
The sum of the EXTERIOR angles of any convex polygon is:	**360°** — The exterior angles of any convex polygon always sum to 360°, regardless of the number of sides.
The PERIMETER of a polygon is:	**the total distance around it (sum of the side lengths)** — Perimeter is the sum of all side lengths — the distance around.
The area of a rectangle is:	**length × width** — Area of a rectangle = length × width.
The area of a trapezoid is:	**½(b_1 + b_2) × height** — Trapezoid area = ½ (sum of the parallel bases) × height.

12. Three-dimensional solids — volume, surface area, and cross-sections

✂ Question (fold →)	Answer
A horizontal slice of a cylinder (parallel to its base) is a:	**circle** — Slicing a cylinder parallel to its circular base gives a circle.
The volume of a sphere is:	**$(4/3)\pi r^3$** — A sphere's volume is $(4/3)\pi r^3$.
Why are 3D solids and cross-sections important?	**they model real objects and develop spatial reasoning for science and design** — Understanding solids and their sections underlies architecture, engineering, medical imaging, and manufacturing.
Understanding cross-sections is crucial in fields like medical imaging because:	**scans show 2D slices of 3D anatomy** — CT/MRI scans present 2D cross-sections of 3D structures — reading them is spatial reasoning.
The volume of a solid doubles the base while keeping height, so the volume:	**doubles** — Volume is proportional to base area (at fixed height), so doubling the base doubles the volume.
A vertical slice through the apex of a cone gives a cross-section that is a:	**triangle** — Slicing a cone vertically through its apex produces a triangular cross-section.
A CROSS-SECTION of a solid is:	**the 2D shape formed when a plane slices through it** — Slicing a solid with a plane reveals a 2D cross-section.
Slicing a sphere with any plane through it gives a cross-section that is a:	**circle** — Every planar slice of a sphere is a circle.
The VOLUME of a solid measures:	**the amount of space it occupies (in cubic units)** — Volume is the 3D space a solid fills, measured in cubic units.
A slanted (oblique) slice of a cylinder produces a cross-section that is:	**an ellipse** — Cutting a cylinder at an angle (not parallel to the base) gives an ellipse.
The volume of a pyramid is __ that of a prism with the same base and height:	**one-third** — A pyramid's volume is 1/3 of the corresponding prism ($V = \frac{1}{3} \times$ base area $\times$ height).
The volume of a CYLINDER is:	**$\pi r^2 h$ (base area × height)** — A cylinder's volume is its circular base area (πr^2) times height.
Nets are flattened patterns that fold into a solid and help compute:	**surface area** — A net lays out all a solid's faces flat, making surface area easy to sum.
The number of faces on a cube is:	**6** — A cube has six square faces.
Doubling every dimension of a cube multiplies its VOLUME by:	**8** — Volume scales as the cube of the linear factor: $2^3 = 8$.

Predicting a cross-section before slicing is a valuable spatial-reasoning skill because it:	**builds the ability to visualize 3D-to-2D relationships** — Anticipating cross-sections develops spatial reasoning used in engineering, medicine (imaging), and design.
The surface area of a cube with edge 3 is:	**54 square units (6 faces of 3×3)** — Each face is 3 × 3 = 9; six faces give 6 × 9 = 54.
A vertical slice of a cylinder (through its axis) is a:	**rectangle** — Cutting a cylinder along its axis produces a rectangular cross-section.
The volume of a rectangular PRISM (box) is:	**length × width × height** — Volume of a box = l × w × h.
A box 2 × 3 × 4 has a volume of:	**24 cubic units** — 2 × 3 × 4 = 24 cubic units.
The SURFACE AREA of a solid measures:	**the total area of all its faces (in square units)** — Surface area sums the areas of all the solid's surfaces.
A cube with edge length 2 has a volume of:	**8 cubic units** — $2^3 = 8$ cubic units.
The volume of a cone is __ the volume of a cylinder with the same base and height:	**one-third** — A cone's volume is 1/3 of the cylinder with the same base and height ($V = \frac{1}{3}\pi r^2 h$).
The surface area of a sphere is:	**$4\pi r^2$** — A sphere's surface area is $4\pi r^2$.
The volume of a solid measured in units cubed reflects that it is:	**three-dimensional** — Cubic units (length^3) reflect volume's three dimensions.

13. Conic sections — slicing a cone

✂ Question (fold →)	Answer
Slicing through BOTH nappes (halves) of a double cone gives a:	**hyperbola** — A steep slice cutting both nappes of a double cone gives a hyperbola (two branches).
The four conic sections all come from:	**one source — slicing a cone at different angles** — Circle, ellipse, parabola, and hyperbola are unified as slices of a single cone.
A CIRCLE is the set of points:	**equidistant from a center** — A circle is all points at a fixed distance (radius) from a center.
Slicing a cone PARALLEL to its base gives a:	**circle** — A slice parallel to the base of a cone is a circle.
The two fixed points that define an ellipse are called its:	**foci** — An ellipse is defined by the constant sum of distances to its two foci.
Slicing a cone perpendicular to its axis (parallel to the base) gives a:	**circle** — A slice perpendicular to the axis (parallel to the base) is a circle.
The path of a thrown ball (ignoring air resistance) is a:	**parabola** — Projectile motion traces a parabola — a real-world conic section.
A parabola has a single fixed point (the focus) and a fixed line called the:	**directrix** — A parabola is defined by equal distance to a focus and a directrix line.
An ELLIPSE is the set of points where:	**the sum of distances to two foci is constant** — An ellipse is defined by a constant SUM of distances to two foci.
The standard equation $(x^2/a^2) + (y^2/b^2) = 1$ represents an:	**ellipse** — This standard form is the equation of an ellipse.
A circle is a special case of an ellipse where the two foci:	**coincide (at the center)** — When an ellipse's foci merge into one point, it becomes a circle.
Why are conic sections important across math and science?	**they describe orbits, projectile paths, lenses, antennas, and more** — Conics model planetary orbits, projectile motion, reflector shapes, and countless physical phenomena.
Slicing a cone at a SLIGHT angle (not parallel to the base or side) gives an:	**ellipse** — A tilted slice (not through the base plane) produces an ellipse.
The equation $x^2 + y^2 = r^2$ represents a:	**circle of radius r centered at the origin** — $x^2 + y^2 = r^2$ is the standard equation of a circle centered at the origin.
CONIC SECTIONS are the curves formed by:	**slicing a cone with a plane at different angles** — The circle, ellipse, parabola, and hyperbola all arise from slicing a cone at different angles.
Conic sections were studied by ancient Greeks and later became essential to:	**understanding planetary motion (Kepler, Newton)** — Ancient conic studies later underpinned Kepler's and Newton's laws of planetary motion.
	two separate branches — A hyperbola consists of two open branches (from

A hyperbola has:	cutting both nappes of a cone).
Satellite dishes and headlights use __ shapes to focus signals or light:	**parabolic** — Parabolic reflectors focus incoming rays to (or from) the focus — used in dishes and headlights.
Recognizing conics unifies seemingly different curves as:	**slices of the same cone** — The power of conic sections is seeing circle, ellipse, parabola, and hyperbola as one family from a cone.
Slicing a cone PARALLEL to its slant side gives a:	**parabola** — A slice parallel to the cone's slant edge produces a parabola.
Planetary orbits are (approximately):	**ellipses** — Kepler showed planets orbit in ellipses with the Sun at one focus.
A PARABOLA is the set of points:	**equidistant from a focus and a directrix line** — A parabola is equidistant from a focus point and a directrix line.
Conic sections connect geometry to algebra because each conic:	**has a characteristic equation** — Each conic corresponds to a specific type of quadratic equation — linking the slice to algebra.
The eccentricity of a conic measures:	**how much it deviates from being circular** — Eccentricity quantifies how 'stretched' a conic is (0 for a circle, larger for ellipses/hyperbolas).
A HYPERBOLA is the set of points where:	**the difference of distances to two foci is constant** — A hyperbola is defined by a constant DIFFERENCE of distances to two foci.

14. Vectors — magnitude, direction, and operations

✂ Question (fold →)	Answer
Two vectors are equal if they have:	**the same magnitude AND direction** — Vectors are equal when both magnitude and direction match, regardless of position.
A UNIT vector has a magnitude of:	**1** — A unit vector has length 1 and gives a direction.
The COMPONENTS of a vector are:	**its horizontal and vertical parts** — A vector splits into horizontal (x) and vertical (y) components.
Subtracting vector b from a (a − b) is the same as:	**adding a and the negative of b** — a − b = a + (−b): add a to the reverse of b.
A vector can be represented as:	**an arrow (with length and direction)** — Vectors are drawn as arrows whose length is the magnitude and whose orientation is the direction.
A VECTOR is a quantity with:	**both magnitude and direction** — A vector has a size (magnitude) and a direction — unlike a scalar (magnitude only).
In physics, adding force vectors gives the:	**net (resultant) force** — The vector sum of forces is the net force determining motion.
Subtracting (5, 3) − (2, 1) gives:	**(3, 2)** — (5 − 2, 3 − 1) = (3, 2).
Vectors matter in computer graphics because they represent:	**positions, directions, and movements precisely** — Graphics use vectors for positions, directions, and motion of objects.
To ADD two vectors, you add their:	**corresponding components** — Add the x-components and the y-components separately.
The magnitude of the vector (6, 8) is:	**10** — $\sqrt{(6^2 + 8^2)} = \sqrt{100} = 10$.
Adding vectors (2, 3) and (1, 4) gives:	**(3, 7)** — (2 + 1, 3 + 4) = (3, 7).
The MAGNITUDE of a vector with components (3, 4) is:	**5** — Magnitude = $\sqrt{(3^2 + 4^2)} = \sqrt{25} = 5$ (the Pythagorean theorem).
Geometrically, vector addition can be done by:	**placing them head-to-tail** — Head-to-tail (tip-to-tail) placement gives the resultant vector from start to finish.
Adding a vector and its negative gives the:	**zero vector** — A vector plus its opposite cancels to the zero vector.
Resolving a force into components is useful because it lets you:	**analyze its horizontal and vertical effects separately** — Splitting a vector into components lets you handle each direction independently (as in physics).
The direction of a vector (3, 3) makes an angle with	**45°** — Equal components give $\tan \theta = 3/3 = 1$, so $\theta = 45°$.

the x-axis of:	
Resolving a vector into components uses:	**trigonometry (the magnitude and direction angle)** — Components come from magnitude × cos θ (x) and magnitude × sin θ (y).
Multiplying a vector by a SCALAR (like 2) changes its:	**magnitude (and reverses direction if negative)** — Scalar multiplication scales the length; a negative scalar also reverses direction.
Velocity is a vector because it has:	**both speed (magnitude) and direction** — Velocity combines speed and direction — a vector quantity.
Multiplying the vector (2, 3) by the scalar 3 gives:	**(6, 9)** — 3 × (2, 3) = (6, 9).
A vector and its NEGATIVE point:	**in opposite directions with the same magnitude** — The negative of a vector has equal length but opposite direction.
Two vectors are PARALLEL if one is a __ of the other:	**scalar multiple** — Parallel vectors point the same (or opposite) way — one is a scalar multiple of the other.
Why are vectors important across math, physics, and engineering?	**they model quantities with direction (force, velocity, displacement) precisely** — Vectors precisely represent directional quantities essential to physics, engineering, navigation, and computer graphics.
The zero vector has:	**magnitude 0 and no defined direction** — The zero vector has no length and no particular direction.

15. Proof and justification — from conjecture to certainty

✂ Question (fold →)	Answer
A definition in geometry is:	**an agreed, precise meaning used as a valid reason in proofs** — Definitions give precise meanings and count as valid justifications in proofs.
Deductive reasoning moves from:	**accepted premises to a guaranteed conclusion** — Deduction derives conclusions that MUST follow from accepted premises.
The 'given' information in a proof is:	**the facts you may assume true to start** — Givens are the starting facts you're allowed to use; the goal is the conclusion.
A proof is INVALID if it:	**uses a step that isn't justified by the givens or prior results** — An unjustified leap makes a proof invalid, regardless of its length.
The CONVERSE of a statement 'if P then Q' is:	**'if Q then P'** — The converse swaps hypothesis and conclusion: 'if Q then P'.
A single example that a statement holds is:	**not a proof (it doesn't cover all cases)** — One example can't prove a general statement — you must show it for all cases.
A POSTULATE (axiom) is:	**a basic statement accepted without proof** — Postulates are foundational assumptions accepted without proof, on which theorems are built.
The drag-invariant approach (dynamic geometry) helps proof by:	**revealing what STAYS true as a figure changes — a conjecture to then prove** — Dragging a figure shows which properties are invariant (always true), yielding a conjecture that a proof then establishes.
The difference between 'it LOOKS true' and 'it IS true' is:	**a proof (justification), which appearance alone cannot provide** — Appearance can mislead; only a proof establishes that a statement is always true.
Reasoning that carefully justifies every step reflects the van Hiele level of:	**formal deduction** — Justifying every step is the hallmark of the deductive van Hiele level — the goal of formal geometry.
In a proof, every statement must be supported by a:	**reason (a given, definition, postulate, or theorem)** — Each step in a proof needs a justification: a given, definition, postulate, or previously proven theorem.
A two-column proof lists:	**statements paired with their reasons** — A two-column proof pairs each statement with the reason that justifies it.
A CONJECTURE is:	**a statement believed true but not yet proven** — A conjecture is an educated guess awaiting proof.
The contrapositive of a true statement is:	**always also true (logically equivalent)** — A statement and its contrapositive are logically equivalent — both true or both false.
A PROOF BY CONTRADICTION assumes:	**the opposite of what you want to prove, then derives a contradiction** — You assume the negation, reach an impossibility, and conclude the original must be true.
A common proof error is assuming a statement's CONVERSE is automatically true. Actually the	**must be proven separately (it doesn't follow automatically)** — A true statement doesn't guarantee its converse; the converse requires its own

converse:	proof.
A valid proof must be free of:	**logical gaps and unjustified steps** — A rigorous proof justifies every step and contains no logical gaps (a documented student difficulty).
Why is proof the heart of formal geometry?	**it turns observations into certain, general truths through logical justification** — Proof elevates geometry from 'it looks true' to 'it is certainly, always true' — the essence of mathematical reasoning.
Proof gives mathematics its:	**certainty (truths established beyond doubt)** — Proof is what makes mathematical truths certain rather than merely likely.
The CONTRAPOSITIVE of 'if P then Q' is:	**'if not Q then not P'** — The contrapositive negates and swaps: 'if not Q then not P' — always logically equivalent to the original.
A flow proof organizes a proof as:	**a diagram of statements connected by arrows to their reasons** — A flow proof shows the logical flow of statements and reasons with connecting arrows.
A THEOREM is:	**a statement proven true from postulates and definitions** — A theorem is a proven result derived from accepted postulates and definitions.
Inductive reasoning (noticing a pattern) is useful for:	**forming a conjecture, which then needs a proof** — Inductive observation suggests conjectures; deductive proof is still needed to establish them.
A geometric PROOF is:	**a logical argument showing a statement is always true** — A proof uses deductive reasoning to show a claim holds in every case, not just one.
A single COUNTEREXAMPLE:	**disproves a general statement** — One counterexample is enough to show a general claim is false.

16. Capstone — conjecture and prove a locus or construction

✂ Question (fold →)	Answer
Reasoning responsibly about a geometric result means:	**stating the conditions under which it holds** — A rigorous result specifies the conditions (givens) under which the conclusion is guaranteed.
The first step in solving a locus problem is often to:	**explore/drag to conjecture what the locus looks like** — Exploring (dragging in dynamic geometry) reveals the likely shape of the locus — a conjecture.
Predicting a locus BEFORE proving it (POE) helps because:	**the prediction guides and checks the proof** — Predicting the locus focuses the proof and provides a check on the result — the POE reasoning habit.
Why does formal geometry, with proof, matter beyond the classroom?	**it trains rigorous logical reasoning that underlies mathematics, science, and law** — The disciplined, justify-every-step reasoning of geometric proof is a transferable model of rigorous logical thinking.
The locus of points a fixed distance from a single point is a:	**circle** — All points at a fixed distance from a center form a circle.
The deepest lesson of formal geometry is that:	**careful reasoning turns appearances into certain, general truths** — Formal geometry teaches that disciplined reasoning — not appearance — establishes certain, general truth.
After conjecturing the locus, the next step is to:	**prove that the conjectured set is exactly the locus** — A conjecture must be proven to establish the locus with certainty.
The drag-to-conjecture-then-prove method embodies the reasoning cycle:	**explore → conjecture → justify (prove)** — Dynamic geometry supports explore → conjecture → prove — the full geometric reasoning loop.
The capstone combines skills from the whole course, including:	**construction, transformations, coordinates, and proof** — A locus problem draws on constructions, transformations, coordinate methods, and proof together.
The habit of asking 'is this ALWAYS true or does it just LOOK true here?' is central to:	**geometric reasoning and proof** — Distinguishing 'always true' from 'looks true here' is the core discipline of proof.
A coordinate approach to a locus problem translates the condition into:	**an equation whose graph is the locus** — Expressing the locus condition algebraically gives an equation whose graph is the locus.
The locus of points a fixed distance from a LINE is:	**two parallel lines (one on each side)** — Points a fixed distance from a line form two lines parallel to it, one on each side.
A locus proof that only checks a few points is:	**incomplete (it must cover all cases)** — Checking sample points suggests a conjecture but doesn't prove the full locus.
A dynamic-geometry construction that stays valid as you DRAG the figure demonstrates:	**that the relationship is an invariant (always true), not a coincidence** — If a property survives dragging, it's an invariant — strong evidence for a conjecture to prove.
The locus of points equidistant from two intersecting lines is:	**the pair of angle bisectors** — Points equidistant from two lines lie on the bisectors of the angles they form.

A well-posed locus problem specifies:	**the exact condition the points must satisfy** — A clear locus problem states precisely the condition defining the point set.
Proving a set IS the locus requires showing two things: every point in the set satisfies the condition, AND:	**every point satisfying the condition is in the set** — A complete locus proof shows the condition and the set coincide in both directions (set ⊆ condition and condition ⊆ set).
The locus of points equidistant from two points is the:	**perpendicular bisector of the segment joining them** — Points equidistant from two points form the perpendicular bisector of the segment between them.
A LOCUS is:	**the set of all points satisfying a given condition** — A locus is the collection of all points meeting a specified geometric condition.
Constructing the locus of points equidistant from two points gives a construction of the:	**perpendicular bisector** — The equidistant-from-two-points locus is exactly the perpendicular bisector construction.
Dynamic geometry software helps with locus problems by:	**tracing a point's path as a figure moves, revealing the locus** — Tracing a driven point as the figure varies reveals the locus shape to conjecture.
To be sure a conjectured locus is complete, you must check it captures:	**ALL points meeting the condition and NO extra points** — A correct locus includes every point satisfying the condition and excludes all others.
Combining coordinate geometry with a locus condition lets you find:	**the equation of the locus** — Translating the locus condition into coordinates yields its equation.
Solving a locus problem well demonstrates mastery because it requires:	**exploring, conjecturing, AND rigorously proving** — A full locus solution integrates exploration, conjecture, and rigorous proof — the whole reasoning arc.
The single most important reasoning habit LocusForge builds is to:	**conjecture what's true, then prove WHY it's always true** — Moving from a drag-invariant conjecture to a rigorous proof — 'it looks true' to 'here's why it's always true' — is the core habit.

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