CalcQuest — Flashcards

A Spark & Anvil printable flashcard deck. Quiz yourself on the CalcQuest cast's curriculum — one card at a time.

How to use these cards

Fold each sheet down the middle line so the answers are hidden. Read the **question** on the left, say your answer out loud, then **unfold** to check the right side. Testing yourself — then checking — is one of the most powerful ways to remember. Get one wrong? That's the best kind of practice: try it again tomorrow.

Want real flashcards? Cut along the fold line and the rows into cards — question on one side, answer on the other.

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1. Functions & average rate of change — precalculus foundation

✂ Question (fold →)	Answer
If f(3) = 7 and the average rate of change on [3, 5] is 4, then f(5) equals:	**15** — (f(5) − 7)/(5 − 3) = 4 ⟹ f(5) − 7 = 8 ⟹ f(5) = 15.
Which best describes what a function does?	**assigns exactly one output to each input** — By definition a function pairs each input with exactly one output — the vertical-line test.
A linear function has the special property that its average rate of change is:	**constant over every interval** — A line has one slope, so its average rate of change is the same no matter which interval you pick.
The 'net change' of f over [a, b] is f(b) − f(a); the average rate of change is that net change:	**divided by the interval length (b − a)** — Average rate of change = net change ÷ interval length — the two differ by the factor (b − a).
For $f(x) = 2^x$, the average rate of change on [0, 1] is:	**1** — $(2^1 - 2^0)/(1 - 0) = (2 - 1)/1 = 1$.
The difference quotient (f(x + h) − f(x))/h represents:	**the average rate of change of f over a step of size h** — The difference quotient is the average rate of change over [x, x+h] — the secant slope. Its limit (h→0) is the derivative.
On a distance–time graph, a STEEPER secant line indicates:	**a greater average speed** — Steeper secant = larger rise per run = greater average speed over that interval.
Why is average rate of change the natural launching point for calculus?	**shrinking the interval turns it into the instantaneous rate — the derivative** — The derivative is defined as the limit of the average rate of change as the interval shrinks to zero — so average rate is the concept the whole course builds on.
The domain of f(x) = 1/(x − 2) excludes:	**x = 2** — The denominator is zero at x = 2, so the function is undefined there; every other real number is allowed.
On a graph of f, the average rate of change over [a, b] is represented by the slope of:	**the secant line through (a, f(a)) and (b, f(b))** — A secant line connects two points on the curve; its slope is the average rate of change between them.
A function that repeats its values at regular intervals is:	**periodic** — A periodic function (like sine) repeats on a fixed period; its rate of change also repeats.
Unlike a linear function, an exponential function's average rate of change:	**grows larger over equal-length intervals farther right** — Exponential growth accelerates: the average rate of change increases as x increases, unlike a line's constant rate.
A population grows from 200 to 260 over 3 years. The average rate of change is:	**20 people per year** — (260 − 200)/3 = 60/3 = 20 people per year.
As the interval [a, b] shrinks toward a single point, the average rate of change approaches:	**the instantaneous rate of change at that point** — Shrinking the interval turns the average (secant) rate into the instantaneous (tangent) rate — the derivative.

For the function values $f(0) = 5$, $f(4) = 13$, the average rate of change on [0, 4] is:	**2** — $(13 - 5)/(4 - 0) = 8/4 = 2$.
For $f(x) = x^2$, the average rate of change on [1, 1+h] simplifies to:	**2 + h** — $((1+h)^2 - 1^2)/h = (1 + 2h + h^2 - 1)/h = (2h + h^2)/h = 2 + h$.
If $f(x) = 3x + 1$, its average rate of change on ANY interval equals:	**3** — For a line $y = mx + b$ the average rate of change is the slope $m = 3$ on every interval.
A function is increasing on an interval when, as x increases, the output values:	**increase** — Increasing means larger inputs give larger outputs; the graph rises left to right.
For $f(x) = x^2$, the average rate of change on [1, 3] equals:	**4** — $(f(3) - f(1))/(3 - 1) = (9 - 1)/2 = 8/2 = 4$.
A table shows $f(2) = 10$, $f(6) = 2$. The function's average rate of change on [2, 6] is:	**−2** — $(2 - 10)/(6 - 2) = (-8)/4 = -2$ — negative because the output decreased.
A NEGATIVE average rate of change over an interval means the function is, on average:	**decreasing on that interval** — A negative rate means output fell as input rose — the function decreased on average.
A car travels 150 miles in 3 hours. Its average speed is:	**50 miles per hour** — Average speed = distance ÷ time = 150/3 = 50 mph — a rate of change of position with respect to time.
The average rate of change of a function f on the interval [a, b] is:	**(f(b) − f(a)) / (b − a)** — Average rate of change is the change in output over the change in input: the slope of the secant line between the two points.
The units of the average rate of change of position (meters) with respect to time (seconds) are:	**meters per second** — A rate divides output units by input units: meters ÷ seconds = meters per second.
A function's average rate of change over [a, b] is zero. This means:	**f(a) = f(b) (the outputs at the endpoints are equal)** — Zero average rate only forces equal endpoint values; f could still rise and fall in between (e.g. a parabola over a symmetric interval).

2. Limits — the value a function approaches

✂ Question (fold →)	Answer
If $\lim_{x\to 1^-} f(x) = 2$ and $\lim_{x\to 1^+} f(x) = 5$, then $\lim_{x\to 1} f(x)$:	**does not exist** — The two-sided limit fails to exist because the left and right limits disagree (a jump).
The two-sided limit $\lim_{x\to a} f(x)$ exists only if:	**the left-hand and right-hand limits are equal** — A two-sided limit exists exactly when both one-sided limits exist and agree.
The notation $\lim_{x\to a} f(x) = L$ means:	**f(x) gets arbitrarily close to L as x gets close to a** — A limit describes the value f approaches near a — it says nothing on its own about f(a).
If f is continuous at a, then $\lim_{x\to a} f(x)$ equals:	**f(a)** — Continuity at a means exactly that the limit equals the function value f(a).
$\lim_{x\to 3} (2x + 1)$ equals:	**7** — The function is continuous, so substitute: 2(3) + 1 = 7.
A common misconception says $\lim_{x\to a} f(x)$ is 'the value the function can never reach.' The correct idea is that the limit:	**is the value f approaches, which f may or may not actually attain** — The limit is simply the approached value; for a continuous function f actually equals it. 'Unreachable' is a documented misconception.
The Squeeze (Sandwich) Theorem lets you find a limit of g when:	**g is trapped between two functions with the same limit at that point** — If $h(x) \le g(x) \le f(x)$ near a and lim h = lim f = L, then lim g = L too.
Why are limits the foundation the whole calculus course rests on?	**derivatives and integrals are both defined as limits** — The derivative is a limit of average rates; the definite integral is a limit of Riemann sums — every core idea is built on the limit.
$\lim_{x\to 4} \sqrt{x}$ equals:	**2** — √ is continuous for x > 0, so substitute: $\sqrt{4} = 2$.
To evaluate $\lim_{x\to 0} (\sqrt{x+1} - 1)/x$, a useful technique is to:	**multiply by the conjugate $(\sqrt{x+1} + 1)/(\sqrt{x+1} + 1)$** — Rationalizing clears the 0/0 form: the numerator becomes x, cancelling the denominator to give a limit of 1/2.
For a rational function, a limit of the form (nonzero)/0 typically indicates:	**a vertical asymptote (an infinite limit)** — A nonzero numerator over a zero denominator blows up — a vertical asymptote / infinite limit, not a removable hole.
Direct substitution gives 0/0. This form is called:	**an indeterminate form** — 0/0 is indeterminate — it doesn't settle the limit; you must simplify (factor, rationalize, etc.) first.
The formal (ε–δ) definition of a limit captures the idea that:	**you can make f(x) as close to L as desired by keeping x close enough to a** — For every tolerance ε on the output there is a closeness δ on the input that guarantees it — the rigorous 'approaches' idea.
$\lim_{x\to 2} (x^3)$ equals:	**8** — Polynomials are continuous, so substitute: $2^3 = 8$.
$\lim_{x\to\infty} (5x^2 + 3)/(2x^2 - x)$ equals:	**5/2** — For equal top/bottom degrees the limit is the ratio of leading coefficients: 5/2.
$\lim_{x\to 0} (x^2 - x)/x$ equals:	**−1** — Factor: $x(x - 1)/x = x - 1$ for $x \ne 0$, so the limit is 0 − 1 = −1 (even though direct substitution gives 0/0).
For $f(x) = (x^2 - 4)/(x - 2)$, $\lim_{x\to 2} f(x)$ equals:	**4** — Factor: $(x-2)(x+2)/(x-2) = x + 2$ for $x \ne 2$, so the limit is 2 + 2 = 4 — even though f(2) is 0/0.
A function can have a limit of 3 at x = 2	**f(2) is undefined or f(2) ≠ 3** — The limit only depends on nearby values;

even if:	the function can have a hole or a different value at the point itself.
A limit that grows without bound is written = ∞. Does the limit 'exist' as a finite number?	**no — it does not exist as a finite value (it is an infinite limit)** — '= ∞' describes unbounded behavior; there is no finite limiting value, so the limit does not exist in the finite sense.
lim(x→a) [f(x) + g(x)] equals lim f(x) + lim g(x) provided:	**both individual limits exist** — The limit of a sum is the sum of the limits — when each limit exists.
For a piecewise function that equals x for x < 1 and x + 3 for x ≥ 1, lim(x→1) f(x):	**does not exist (left limit 1, right limit 4)** — Left: x → 1 gives 1. Right: x + 3 → 4. They disagree, so the two-sided limit does not exist.
As x → ∞, lim (1/x) equals:	**0** — As x grows without bound, 1/x shrinks toward 0.
lim(x→0^+) (1/x) equals:	**+∞** — As x approaches 0 from the right, 1/x grows without bound toward +∞.
lim(x→0) (sin x)/x equals:	**1** — This foundational limit equals 1 (provable by the Squeeze Theorem); it underlies the derivative of sine.
A one-sided limit lim(x→a^+) f(x) considers x approaching a:	**only from values greater than a (the right side)** — The '+' superscript means approach from the right (x slightly bigger than a).

3. Continuity — where a function is unbroken

✂ Question (fold →)	Answer
$f(x) = (x^2 - 9)/(x - 3)$ has, at $x = 3$:	**a removable discontinuity (a hole)** — It simplifies to $x + 3$ ($x \neq 3$), so the limit is 6 but f(3) is 0/0 — a removable hole.
A 'jump' discontinuity occurs when:	**the left and right limits exist but are not equal** — In a jump, both one-sided limits exist but disagree — the graph leaps between two levels.
Continuity is a REQUIREMENT for which major theorem?	**the Intermediate Value Theorem** — The IVT guarantees a function continuous on [a, b] takes every value between f(a) and f(b) — continuity is essential.
Is every continuous function differentiable?	**no — a continuous function can have a sharp corner where it isn't differentiable** — $\lvert x\rvert$ is continuous everywhere but not differentiable at 0 (a corner). Continuity is necessary but not sufficient for differentiability.
The rational function $1/(x - 5)$ is discontinuous at:	**$x = 5$** — The denominator is zero at $x = 5$, giving a vertical asymptote (infinite discontinuity) there.
Intuitively, a function is continuous on an interval if you can draw its graph:	**without lifting your pencil** — Continuous = no breaks, holes, or jumps — drawable in one unbroken stroke.
Why can't the Extreme Value Theorem be applied to $f(x) = 1/x$ on $[-1, 1]$?	**f is discontinuous at $x = 0$, so the interval isn't one of continuity** — 1/x has an infinite discontinuity at 0 inside $[-1, 1]$, so the EVT's continuity hypothesis fails.
A step function (like the greatest-integer function) has, at each integer:	**a jump discontinuity** — A step function leaps to a new level at each integer — jump discontinuities.
Which situation makes a function DIScontinuous at a point?	**the limit and the function value disagree** — Discontinuity happens when any of the three conditions fails — commonly limit ≠ value.
A function f is continuous at $x = a$ when all three hold: f(a) exists, $\lim_{x\to a} f(x)$ exists, and:	**$\lim_{x\to a} f(x) = f(a)$** — Continuity at a requires the limit to exist AND equal the function's value there.
The Intermediate Value Theorem guarantees that a continuous f on [a, b] with $f(a) = -1$ and $f(b) = 4$:	**takes the value 0 somewhere on (a, b)** — Since 0 lies between −1 and 4, the IVT guarantees some c in (a, b) with $f(c) = 0$.
tan(x) is discontinuous at $x = \pi/2$ because:	**cos(x) = 0 there, giving a vertical asymptote** — tan = sin/cos; where cos = 0 (like $\pi/2$) the denominator vanishes with nonzero numerator — an infinite discontinuity.
If f and g are both continuous at a, then f + g is:	**continuous at a** — Sums, differences, products, and (nonzero-denominator) quotients of continuous functions are continuous.
Why does continuity matter so much in calculus?	**the core theorems (IVT, EVT) and easy limit evaluation all rely on it** — Continuity is the hypothesis behind the IVT and EVT and is what makes limits computable by substitution — a foundational property.
A function has $f(2) = 3$ but $\lim_{x\to 2} f(x) = 5$. At $x = 2$ the	**discontinuous (the limit and the value disagree)** — Continuity requires lim =

function is:	f(a); here 5 ≠ 3, so f is discontinuous at 2 (a removable discontinuity).
The Extreme Value Theorem says a function continuous on a CLOSED interval [a, b]:	**attains both an absolute maximum and an absolute minimum** — Continuity on a closed, bounded interval guarantees the function reaches a highest and lowest value.
A piecewise function equals 2x for x < 1 and x + 1 for x ≥ 1. At x = 1 it is:	**continuous (both pieces give 2)** — Left: 2(1) = 2; right: 1 + 1 = 2; f(1) = 2. All three agree, so it's continuous.
A function is continuous 'from the right' at a if:	**$\lim(x \to a^+) f(x) = f(a)$** — Right-continuity requires the right-hand limit to equal the function value — relevant at interval endpoints.
If a function is continuous at a, then lim(x→a) f(x):	**can be found by direct substitution of a** — Continuity is precisely what lets you substitute the point to evaluate the limit.
A function with a hole at x = 2 can be MADE continuous there by:	**defining f(2) to equal the limit at 2** — A removable discontinuity is repaired by assigning the limiting value to the point.
Polynomials are continuous:	**for all real numbers** — Every polynomial is continuous on the entire real line — no breaks anywhere.
A 'removable' discontinuity is a point where:	**the limit exists but f(a) is missing or the wrong value (a hole)** — A removable discontinuity is a hole: the limit exists, so redefining f(a) would repair it.
An 'infinite' discontinuity occurs at:	**a vertical asymptote where the function grows without bound** — At a vertical asymptote the one-sided limits are ±∞ — an infinite (non-removable) discontinuity.
sin(x) and cos(x) are continuous:	**for all real numbers** — Sine and cosine are continuous everywhere on the real line.
For the piecewise function above but with x + 3 (x ≥ 1) instead, x = 1 is:	**a jump discontinuity (left 2, right 4)** — Left gives 2, right gives 4 — the pieces disagree, producing a jump.

4. The derivative — instantaneous rate of change

✂ Question (fold →)	Answer
A horizontal tangent line at x = a means:	**f'(a) = 0** — A horizontal tangent has slope 0, so the derivative is 0 there — a candidate max or min.
A key misconception is confusing f and f' on a graph. At a point where f is at a maximum, f' is:	**zero** — At a smooth maximum the tangent is horizontal, so f' = 0 — even though f itself is largest there.
A function is differentiable at a point when:	**its derivative (the limit of the difference quotient) exists there** — Differentiability means the difference-quotient limit exists — a well-defined tangent slope.
If f'(x) > 0 on an interval, the function is:	**increasing there** — A positive derivative means positive slope — the function rises.
For $f(x) = x^3$, using the definition, f'(x) equals:	**$3x^2$** — $\lim(h\to0)[(x+h)^3 - x^3]/h$ expands to $3x^2 + 3xh + h^2 \to 3x^2$ as $h \to 0$ (matching the power rule).
For f(x) = 3x + 2, f'(x) equals:	**3** — The derivative of a line is its slope; here that is 3 (constant).
If s(t) is position, then s'(t) represents:	**velocity** — The derivative of position with respect to time is velocity — the instantaneous rate of change of position.
The second derivative f'' is:	**the derivative of the derivative** — f'' is found by differentiating f'; it measures how the rate of change is itself changing (e.g. acceleration, concavity).
Reading a graph of f, where f is DECREASING, the derivative f' is:	**negative** — Decreasing f means negative slope, so f' < 0 there. (Confusing f with f' is a common error.)
The derivative f'(a) is defined as:	**$\lim(h\to0)$ [f(a + h) − f(a)] / h** — The derivative is the LIMIT of the difference quotient as $h \to 0$ — the instantaneous rate of change.
Differentiability at a point IMPLIES:	**continuity at that point** — If a derivative exists at a, the function must be continuous at a (the converse is false).
At a sharp CORNER (like |x| at 0), the function is:	**continuous but NOT differentiable** — The left and right difference quotients disagree at a corner, so no single tangent slope exists — yet the graph is unbroken.
A velocity v(t) that is negative means the object is:	**moving in the negative direction** — Velocity is signed: negative velocity means motion in the negative direction (position decreasing).
If s(t) is position, then s''(t) is:	**acceleration** — The first derivative is velocity; the second derivative (rate of change of velocity) is acceleration.
The difference quotient uses (a + h); as $h \to 0^-$ vs $h \to 0^+$, for a differentiable function these:	**agree (both give the same derivative)** — Differentiability requires the left and right difference-quotient limits to agree — a single tangent slope.
The 'instantaneous' in 'instantaneous rate of change' comes from:	**taking the limit of average rates over shrinking intervals** — Shrinking the interval to zero via a limit produces a rate at a single instant.
	it turns 'how fast is this changing right now?' into a computable slope —

Why is the derivative the central object of differential calculus?	The derivative makes instantaneous rate of change precise and computable — the engine of optimization, motion, and analysis.
For $f(x) = x^2$, the tangent line at $x = 3$ has slope:	**6** — $f'(x) = 2x$, so $f'(3) = 6$.
The secant slope becomes the tangent slope as the two points:	**merge into a single point ($h \to 0$)** — Sliding the second point toward the first shrinks h to 0, turning the secant into the tangent.
For $f(x) = x^2$, using the definition, $f'(x)$ equals:	**2x** — $\lim(h \to 0)[(x+h)^2 - x^2]/h = \lim(2x + h) = 2x$.
The derivative measures:	**the instantaneous rate of change of the function** — f' gives the rate at which f is changing at each instant — the tangent slope.
A function that is defined but has a VERTICAL tangent at a point is:	**not differentiable there (the slope is infinite)** — A vertical tangent means the difference quotient grows without bound — no finite derivative exists.
Geometrically, $f'(a)$ is the slope of:	**the tangent line to the curve at x = a** — The derivative at a point is the slope of the tangent line there — the limiting position of secant lines.
The notation dy/dx means:	**the derivative of y with respect to x** — dy/dx is Leibniz notation for the derivative — the instantaneous rate of change of y as x changes.
The derivative of a CONSTANT function $f(x) = 7$ is:	**0** — A constant never changes, so its rate of change is 0. (A common error is to leave the constant.)

5. Differentiation rules — power, product, quotient, chain

✄ Question (fold →)	Answer
The chain rule is needed whenever a function is:	**composed — one function plugged inside another** — Composition f(g(x)) triggers the chain rule; sums/products/quotients have their own rules.
The Sum Rule: d/dx [f(x) + g(x)] equals:	**f'(x) + g'(x)** — Differentiate term by term: the derivative of a sum is the sum of the derivatives.
The Constant Multiple Rule: d/dx [c·f(x)] equals:	**c·f'(x)** — Constants factor straight through the derivative: d/dx[c·f] = c·f'.
d/dx [e^(2x)] equals:	**2e^(2x)** — Chain rule: e^(2x)·(derivative of 2x) = e^(2x)·2 = 2e^(2x).
The Product Rule: d/dx [f·g] equals:	**f'·g + f·g'** — The derivative of a product is f'g + fg' — NOT the product of the derivatives.
The Power Rule says d/dx [x^n] equals:	**$n \cdot x^{n-1}$** — Bring the exponent down as a coefficient and reduce it by one: d/dx[x^n] = $n \cdot x^{n-1}$.
The Quotient Rule: d/dx [f/g] equals:	**(f'·g − f·g') / g^2** — Low·d-high minus high·d-low, over the square of the bottom: (g·f' − f·g')/g^2.
d/dx [cos x] equals:	**−sin x** — The derivative of cos x is −sin x (note the minus sign).
d/dx [$7x^2 - 2x + 4$] equals:	**14x − 2** — Term by term: 14x − 2 + 0 (the constant 4 differentiates to 0).
d/dx [$(3x + 1)^4$] equals:	**$12(3x + 1)^3$** — $4(3x+1)^3 \cdot 3 = 12(3x+1)^3$ (outer power rule times inner derivative 3).
d/dx [1/x] (i.e. x^{-1}) equals:	**$-1/x^2$** — $(-1)x^{-2} = -1/x^2$.
d/dx [x] equals:	**1** — x^1 has derivative $1 \cdot x^0 = 1$.
d/dx [$x^2 + 3x - 5$] equals:	**2x + 3** — Term by term: 2x + 3 + 0 = 2x + 3 (the constant −5 has derivative 0).
d/dx [√x] (i.e. x^(1/2)) equals:	**1/(2√x)** — (1/2)x^(−1/2) = 1/(2√x).
To differentiate y = $(x^2 + 1) \cdot (x - 3)$, the appropriate rule is the:	**product rule (or expand first)** — It's a product of two functions, so use the product rule (expanding to a polynomial also works).
d/dx [$4x^3$] equals:	**$12x^2$** — $4 \cdot (3x^2) = 12x^2$.
The Chain Rule: d/dx [f(g(x))] equals:	**f'(g(x))·g'(x)** — Differentiate the outer function at the inner, then multiply by the inner's derivative.
d/dx [e^x] equals:	**e^x** — e^x is its own derivative — a defining property of the natural exponential.
d/dx [ln x] equals:	**1/x** — The derivative of the natural log is 1/x (for x > 0).
d/dx [x^5] equals:	**$5x^4$** — $5 \cdot x^{5-1} = 5x^4$. (A common error is forgetting to reduce the exponent.)
d/dx [$x^2 \cdot \sin x$] equals:	**$2x \cdot \sin x + x^2 \cdot \cos x$** — f'g + fg' = (2x)(sin x) + (x^2)(cos x).
To differentiate y = sin(x^2), the appropriate rule is the:	**chain rule** — sin(x^2) is a composition (outer sine, inner x^2), so use the chain rule: cos(x^2)·2x.
A frequent power-rule error is	**$3x^2$** — You must REDUCE the exponent by 1: $3x^2$. Forgetting to subtract 1 is a

writing d/dx[x^3] as $3x^3$. The correct answer is:	documented common error.
Why do we memorize differentiation rules instead of always using the limit definition?	**the rules compute the same limit-based derivative far more efficiently** — Each rule is proven FROM the limit definition; they package that limit into fast, reliable procedures.
d/dx [sin x] equals:	**cos x** — The derivative of sin x is cos x.

6. Interpreting f' and f" — slope, monotonicity, concavity

✂ Question (fold →)	Answer
A common error is thinking a maximum of f occurs where f" = 0. Actually a local max requires:	**f' = 0 (a horizontal tangent), with f" < 0 or a First-Derivative-Test sign change** — Extrema come from f' = 0/undefined; f" = 0 marks possible inflection, not extrema.
On the graph of f', a local MAXIMUM of f' corresponds to, on f, a:	**point of steepest increase (an inflection point of f)** — Where the slope f' is largest, f is increasing fastest — an inflection point of f.
If a particle's position graph is concave up, its:	**velocity is increasing** — Concave-up position means acceleration is positive, so velocity is increasing.
The connection 'f increasing ⟺ f' > 0' is useful because it lets you analyze a function's shape by:	**studying the sign of its derivative** — A sign chart of f' reveals where f rises and falls without plotting every point.
A positive velocity with a positive acceleration means the object is:	**moving forward and speeding up** — Same-sign velocity and acceleration means speed is increasing in the direction of motion.
If f'(c) = 0 and f"(c) < 0, then c is:	**a local maximum** — A horizontal tangent on a concave-down curve sits at the top of a cap — a local maximum.
An inflection point is where:	**concavity changes (f" changes sign)** — At an inflection point the curve switches between concave up and down — f" changes sign.
Why is interpreting derivatives (not just computing them) the heart of applied calculus?	**the sign and size of f' and f" describe how a real quantity behaves and turns** — Reading f'/f" translates calculus into meaning — when a quantity rises, peaks, accelerates, or bends — which is what applications need.
If f' changes from negative to positive at c, then c is:	**a local minimum** — Falling then rising (f': − → +) makes a valley — a local minimum.
The First Derivative Test: if f' changes from positive to negative at c, then c is:	**a local maximum** — Rising then falling (f': + → −) makes a peak — a local maximum.
Where f"(x) > 0, the graph is:	**concave up (opens upward, like a cup)** — Positive second derivative means the slope is increasing — the curve bends upward.
If f' is CONSTANT (nonzero), then f is:	**a line (linear)** — A constant slope everywhere describes a straight line.
A function increasing on all of ℝ has a derivative that is:	**positive (or zero) everywhere** — An increasing function has non-negative slope throughout.
The full 'curve sketching' toolkit uses f' for __ and f" for __:	**increasing/decreasing (and extrema); concavity (and inflection)** — f' governs monotonicity + extrema; f" governs concavity + inflection — together they determine the shape.
The Second Derivative Test: if f'(c) = 0 and f"(c) > 0, then c is:	**a local minimum** — A horizontal tangent on a concave-up curve sits at the bottom of a cup — a local minimum.
Where f'(x) < 0, the graph of f is:	**decreasing** — A negative derivative means negative slope — the function falls.

At a point where f''(x) = 0 but f'' does NOT change sign, the point is:	**not an inflection point** — An inflection point requires f'' to CHANGE sign; f'' = 0 alone (e.g. x^4 at 0) does not guarantee one.
The second derivative f'' measures:	**concavity (how the slope is changing)** — f'' is the rate of change of the slope; its sign gives concavity.
If f''(x) = 0 for all x, then f is:	**linear (at most degree 1)** — Zero concavity everywhere means the slope never changes — a line.
On an interval where f is concave up and increasing, the graph is:	**rising at an increasing rate** — Increasing (f'>0) and concave up (f''>0) together mean the slope is positive and getting steeper.
A 'critical point' of f is where:	**f'(x) = 0 or f'(x) is undefined** — Critical points — where the derivative is zero or undefined — are the candidates for local extrema.
If velocity is positive but acceleration is negative, the object is:	**moving forward but slowing down** — Opposite-sign velocity and acceleration means the object decelerates while still moving forward.
A graph of f' (the derivative) crossing from positive to negative tells you f has a:	**local maximum there** — Where f' crosses zero going + to −, the original f peaks — a local maximum.
Where f''(x) < 0, the graph is:	**concave down (opens downward, like a cap)** — Negative second derivative means the slope is decreasing — the curve bends downward.
Where f'(x) > 0, the graph of f is:	**increasing** — A positive derivative means positive slope — the function rises.

7. Optimization — finding maxima and minima

✂ Question (fold →)	Answer
If the profit function is concave down with $P'(x_0) = 0$, then x_0 gives:	**the maximum profit** — Concave down ($P'' < 0$) at a critical point confirms a maximum.
A local minimum that is NOT the absolute minimum can happen when:	**a different critical point or endpoint has a smaller value** — A local min is only smallest nearby; another point may dip lower, making it not the global min.
The distinction between a LOCAL and ABSOLUTE extremum matters because a solution must be:	**the best over the ENTIRE feasible region, not just nearby** — Optimization seeks the global best; a merely local extremum can be beaten elsewhere in the domain.
A function continuous on a closed interval is GUARANTEED to have an absolute max and min by the:	**Extreme Value Theorem** — The EVT guarantees a continuous function on $[a, b]$ attains both extremes — which is why the closed-interval method works.
A rectangle has a fixed perimeter of 20. Its area is maximized when the rectangle is:	**a square (5 by 5)** — With perimeter fixed, area $A = x(10 - x)$ is maximized at $x = 5$ — a square.
In a fence problem with 100 m of fencing against a wall (three sides), the enclosed area is maximized when the side parallel to the wall is:	**50 m (with 25 m sides)** — Area $A = x(100 - 2x)/\ldots$ maximizing $A(x) = x(100 - 2x)$ gives $x = 25$ for the sides and 50 m parallel to the wall.
The Closed-Interval (Candidates) Test finds absolute extrema by checking:	**the critical points AND the endpoints** — On a closed interval, compare f at every critical point and at both endpoints; the largest/smallest wins.
To maximize the area of a rectangle inscribed under a downward parabola, you would:	**express area as a function of x and set its derivative to zero** — Write $A(x)$ using the parabola's height, then optimize A with $A'(x) = 0$.
To confirm a critical point is a MAXIMUM using the second derivative, you check that:	**$f'' < 0$ there** — A concave-down critical point ($f'' < 0$) is a local maximum.
For revenue $R(x)$, profit is maximized (in the standard model) where marginal profit is zero, i.e. where:	**marginal revenue equals marginal cost** — Profit $P = R - C$ is maximized where $P' = 0$, i.e. $R' = C'$ — marginal revenue = marginal cost.
When you reduce an optimization problem to one variable, the CONSTRAINT equation is used to:	**eliminate the extra variable by substitution** — The constraint lets you express one variable in terms of another, so the objective becomes single-variable and differentiable.
For $f(x) = x^2$ on $[-1, 3]$, the absolute minimum occurs at:	**$x = 0$ (value 0)** — The critical point $x = 0$ gives the smallest value, 0.
The maximum height of a projectile $s(t)$ occurs when:	**its velocity $s'(t) = 0$** — At the top, the object momentarily stops rising — velocity is zero.
If $f'(x) = 0$ at $x = c$ but f' does NOT change sign there, then c is:	**neither a local max nor a local min** — Without a sign change in f', there is no peak or valley — e.g. $f(x) = x^3$ at $x = 0$.
	physical limits may put the optimum at an endpoint — Real

The domain of the variable in an optimization problem matters because:	quantities (length ≥ 0, etc.) restrict the domain, and the optimum can land at a boundary.
Fermat's principle in optimization says an interior extremum of a differentiable function must occur where:	**the derivative is zero** — At an interior local extremum of a differentiable function, the tangent is horizontal (f' = 0).
An ABSOLUTE maximum of f on [a, b] is:	**the largest value f attains anywhere on [a, b]** — The absolute (global) max is the single largest output over the whole interval.
The first step in a word-problem optimization is usually to:	**write the quantity to optimize as a function of ONE variable** — Use the constraint to reduce to a single-variable function, then optimize with calculus.
A soup can (cylinder) of fixed volume is optimized to use the least material by minimizing its:	**surface area as a function of the radius** — Express surface area in terms of radius (using the volume constraint) and minimize it.
On an OPEN interval or all of ℝ, a function may have:	**no absolute maximum at all** — Without closed endpoints (or with unbounded behavior), a function like f(x) = x has no absolute max.
Why is optimization one of calculus's most powerful applications?	**it finds the best possible outcome by locating where the rate of change turns** — Optimization turns 'find the best design/price/path' into 'find where f' = 0 and classify it' — a universal engineering and economics tool.
A common optimization error is to stop after finding f' = 0 without:	**verifying it's a max/min and checking endpoints** — A critical point is only a CANDIDATE; you must classify it (1st/2nd derivative test) and, on a closed interval, compare endpoints.
To find candidates for a function's extreme values, you first find where:	**f'(x) = 0 or f'(x) does not exist (critical points)** — Extrema of a differentiable function occur at critical points — where the derivative is zero or undefined.
For $f(x) = x^2$ on [−1, 3], the absolute maximum occurs at:	**x = 3 (value 9)** — Critical point x = 0 gives 0 (the min); endpoints give f(−1)=1, f(3)=9. The max is 9 at x = 3.
A key modeling habit before optimizing is to:	**state the constraint and the objective explicitly** — Naming what's fixed (constraint) and what's optimized (objective) prevents the most common setup errors.

8. Related rates — linked changing quantities

✂ Question (fold →)	Answer
A spherical snowball melts so its volume decreases; dV/dt is:	**negative** — Melting shrinks the volume, so its rate of change dV/dt is negative.
The central technique in related rates is to differentiate the relating equation with respect to:	**time (t)** — Differentiate both sides implicitly with respect to t so every variable contributes a d/dt term.
A related-rates problem asks how the rate of change of one quantity relates to:	**the rate of change of another quantity linked by an equation** — Related rates link the time-rates of two (or more) quantities tied together by an equation.
The general problem-solving order for related rates is: draw a picture, name variables, write an equation relating them, differentiate in t, then:	**substitute the known instantaneous values and solve for the unknown rate** — After differentiating, plug in the given quantities and rates at the instant of interest and solve.
The relationship equation in related rates should contain:	**only quantities whose rates you know or want, plus constants** — Pick a relation linking the known and unknown quantities; extra variables should be eliminated via constraints.
A 13-ft ladder leans on a wall; $x^2 + y^2 = 169$. Differentiating gives:	**$2x\cdot(dx/dt) + 2y\cdot(dy/dt) = 0$** — $d/dt[x^2 + y^2 = 169]$: $2x\cdot dx/dt + 2y\cdot dy/dt = 0$ (the constant 169 differentiates to 0).
For a circle, $A = \pi r^2$. Differentiating with respect to time gives:	**$dA/dt = 2\pi r \cdot dr/dt$** — $d/dt[\pi r^2] = \pi\cdot 2r\cdot(dr/dt) = 2\pi r\cdot dr/dt$ by the chain rule.
Differentiating $z^2 = x^2 + y^2$ in time gives:	**$z\cdot(dz/dt) = x\cdot(dx/dt) + y\cdot(dy/dt)$** — $2z\cdot dz/dt = 2x\cdot dx/dt + 2y\cdot dy/dt$; dividing by 2 gives $z\cdot dz/dt = x\cdot dx/dt + y\cdot dy/dt$.
In a related-rates problem, you should substitute specific numerical values:	**AFTER differentiating, not before** — If you plug in a value that is changing before differentiating, its rate disappears. Differentiate first, then substitute.
If the radius of a circle grows at 2 cm/s, when r = 5 cm the area grows at:	**20π cm²/s** — $dA/dt = 2\pi r\cdot dr/dt = 2\pi(5)(2) = 20\pi$ cm²/s.
A spherical balloon's radius grows at 3 cm/s. When r = 2 cm, its volume grows at:	**48π cm³/s** — $dV/dt = 4\pi r^2\cdot dr/dt = 4\pi(2^2)(3) = 4\pi\cdot 4\cdot 3 = 48\pi$ cm³/s.
In the ladder problem, if the base slides away ($dx/dt > 0$), the top of the ladder:	**slides down ($dy/dt < 0$)** — From $2x\cdot dx/dt + 2y\cdot dy/dt = 0$, a positive dx/dt forces dy/dt negative — the top descends.
Related rates are essentially an application of:	**implicit differentiation with respect to time** — Every quantity is an implicit function of t; differentiating the relating equation implicitly in t is the whole method.
If two cars leave an intersection (one north, one east), the distance z between them satisfies:	**$z^2 = x^2 + y^2$** — The straight-line distance is the hypotenuse: $z^2 = x^2 + y^2$ (Pythagorean).
A common related-rates mistake is plugging in the changing value too early. The fix is to:	**keep variables symbolic through differentiation, then substitute at the end** — Substituting a moving value before differentiating erases its rate; stay symbolic, differentiate, then substitute.

Water fills a cone; you relate dV/dt to dh/dt. A useful first step is often to:	**use similar triangles to write r in terms of h** — Expressing r as a multiple of h reduces V to a single variable before differentiating.
The units of dr/dt in a growing-circle problem (r in cm, t in s) are:	**cm per second** — dr/dt is a length rate: centimeters per second.
When a problem says 'how fast is the angle changing,' you should relate the sides to the angle using:	**a trigonometric relation (e.g. tan θ = opposite/adjacent), then differentiate** — Trig relations connect an angle to side lengths; differentiate in time to link dθ/dt to the side rates.
In a related-rates problem, a quantity that is truly CONSTANT (like a fixed ladder length):	**has a rate of change of zero** — A constant doesn't change, so its time-derivative is 0 — often eliminating a term.
A conical tank fills at a constant dV/dt, yet dh/dt (the rate the height rises) is:	**not constant — it changes as the water level rises** — Because the cross-section widens with height, a constant volume rate produces a changing height rate.
Why are related rates a signature calculus skill?	**they turn a geometric or physical constraint into a live relationship between rates of change** — Related rates let you predict how one changing quantity drives another in real time — pervasive in physics, engineering, and biology.
A shadow-length problem (person walking from a lamppost) is solved with:	**similar triangles to relate the shadow length and the distance** — Similar triangles relate the person's distance and shadow length; differentiate that relation in time.
The chain rule is essential in related rates because each variable is:	**an implicit function of time** — Each geometric variable changes with time, so differentiating it produces a d(var)/dt factor via the chain rule.
A balloon's volume $V = (4/3)\pi r^3$ changes as r changes. dV/dt equals:	**$4\pi r^2 \cdot dr/dt$** — $d/dt[(4/3)\pi r^3] = (4/3)\pi \cdot 3r^2 \cdot dr/dt = 4\pi r^2 \cdot dr/dt$.
The instant chosen for substitution in a related-rates problem is the moment when:	**the given quantities take their specified values** — You evaluate the differentiated equation at the specific instant described (given lengths, angles, rates).

9. Antiderivatives — reversing differentiation

✂ Question (fold →)	Answer
Antidifferentiation and differentiation are:	**inverse operations** — One undoes the other — the core relationship the Fundamental Theorem makes precise.
$\int x^2\,dx$ equals:	**$x^3/3 + C$** — $x^{(2+1)}/(2+1) + C = x^3/3 + C$.
$\int (1/x)\,dx$ equals:	**$\ln\|x\| + C$** — Since $d/dx[\ln\|x\|] = 1/x$, the antiderivative is $\ln\|x\| + C$ (absolute value covers $x < 0$).
An antiderivative of a function f is a function F such that:	**$F'(x) = f(x)$** — F is an antiderivative of f when differentiating F gives back f — antidifferentiation reverses the derivative.
$\int 6x\,dx$ equals:	**$3x^2 + C$** — $6\cdot(x^2/2) = 3x^2$; add C.
$\int 3\,dx$ equals:	**$3x + C$** — The antiderivative of a constant k is $kx + C$ (since $d/dx[3x] = 3$).
To pin down the constant C, you need:	**an initial condition (a known value of F at some point)** — One known point of F lets you solve for C, selecting one member of the antiderivative family.
$\int e^x\,dx$ equals:	**$e^x + C$** — e^x is its own antiderivative, so $\int e^x\,dx = e^x + C$.
Why are antiderivatives essential to the whole 'accumulation' half of calculus?	**the Fundamental Theorem computes accumulated change using an antiderivative** — The FTC turns a hard limit-of-sums (a definite integral) into a simple antiderivative evaluation — antiderivatives are the key.
Recognizing $\int 2x\cdot\cos(x^2)\,dx$ calls for:	**u-substitution ($u = x^2$, $du = 2x\,dx$)** — The integrand has an inner function and its derivative present — the signature of u-substitution.
$\int \sin x\,dx$ equals:	**$-\cos x + C$** — Since $d/dx[-\cos x] = \sin x$, $\int \sin x\,dx = -\cos x + C$.
An object with acceleration $a(t) = -9.8$ (gravity) has velocity found by:	**antidifferentiating: $v(t) = -9.8t + v_0$** — Velocity is the antiderivative of acceleration; the constant v_0 is the initial velocity.
$\int \cos x\,dx$ equals:	**$\sin x + C$** — Since $d/dx[\sin x] = \cos x$, $\int \cos x\,dx = \sin x + C$.
If $F'(x) = 2x$ and $F(0) = 5$, then $F(x)$ equals:	**$x^2 + 5$** — $\int 2x\,dx = x^2 + C$; $F(0) = 0 + C = 5 \Longrightarrow C = 5$, so $F(x) = x^2 + 5$.
$\int \sec^2 x\,dx$ equals:	**$\tan x + C$** — Since $d/dx[\tan x] = \sec^2 x$, $\int \sec^2 x\,dx = \tan x + C$.
The general antiderivative of f includes a '+ C' because:	**any constant differentiates to 0, so many functions share the same derivative** — Adding any constant doesn't change the derivative, so antiderivatives form a family differing by C.
A slope field (direction field) is a picture that displays:	**the slope f(x, y) at many points, hinting at the antiderivative family** — A slope field shows little tangent segments; solution curves (antiderivatives) follow the flow, differing by C.
Two antiderivatives of the same function differ by:	**a constant** — If $F' = G' = f$, then $(F - G)' = 0$, so $F - G$ is constant.
$\int x^n\,dx$ (for $n \neq -1$) equals:	**$x^{n+1}/(n + 1) + C$** — Reverse the power rule: raise the exponent by 1 and divide by the new exponent.

The '+C' matters in indefinite integrals but DROPS OUT in a definite integral because:	**the constants cancel when you subtract $F(b) - F(a)$** — $F(b) + C - (F(a) + C) = F(b) - F(a)$; the C's cancel, so any antiderivative works for a definite integral.
u-substitution is the integration counterpart of which differentiation rule?	**the chain rule** — u-substitution reverses the chain rule, undoing the inner-function factor.
$\int 4x^3\,dx$ equals:	**$x^4 + C$** — $4 \cdot x^4/4 = x^4$; add C. ($12x^2$ is the DERIVATIVE, not the antiderivative — a common reversal.)
The antiderivative of velocity $v(t)$ gives:	**position (up to a constant)** — Since position's derivative is velocity, antidifferentiating velocity recovers position (plus a constant set by initial data).
$\int (f(x) + g(x))\,dx$ equals:	**$\int f(x)\,dx + \int g(x)\,dx$** — Integration is linear: the integral of a sum is the sum of the integrals.
$\int (x^2 + 4x + 1)\,dx$ equals:	**$x^3/3 + 2x^2 + x + C$** — Integrate term by term: $x^3/3 + 2x^2 + x + C$.

10. Riemann sums — accumulation as adding up pieces

✂ Question (fold →)	Answer
A MIDPOINT Riemann sum samples f at:	**the midpoint of each subinterval** — Midpoint sums use the center of each strip and are usually more accurate than left/right sums.
A LEFT Riemann sum uses, for each rectangle's height:	**the function value at the left endpoint of the subinterval** — Left sums sample f at each subinterval's left edge.
The exact value of $\int_0^2 x\,dx$ (area of a triangle) is:	**2** — The region is a triangle with base 2 and height 2: area = ½·2·2 = 2.
The Trapezoidal Rule improves on left/right sums by using:	**the average of the left and right heights (trapezoids)** — Trapezoids connect the endpoint heights with a slanted top, averaging left and right estimates.
The Σ (sigma) notation $\Sigma_{i=1}^{n} f(x_i)\Delta x$ means:	**add $f(x_i)\Delta x$ for i from 1 to n** — Sigma notation compactly writes the sum of the rectangle areas.
A midpoint sum is often better than left OR right because errors on the two sides of each strip:	**tend to cancel** — The midpoint rectangle over/under-shoots on opposite halves of each strip, so the errors largely cancel.
Concavity affects the trapezoidal rule: for a CONCAVE UP function, trapezoids:	**overestimate the area (they sit above the curve)** — On a concave-up curve the straight trapezoid tops lie above the curve, overestimating.
A Riemann sum approximates a definite integral by:	**adding up the areas of many thin rectangles (f(x)·Δx pieces)** — Chop the interval into strips, take f(x)·Δx for each, and sum — the 'adding up pieces' conception of the integral.
For an INCREASING function, a LEFT Riemann sum will:	**underestimate the true area** — On an increasing function, left endpoints are the lowest points, so left rectangles fall short — an underestimate.
Why do Riemann sums matter even though the FTC gives a shortcut?	**they DEFINE the integral and reveal its meaning as accumulated change** — The FTC computes integrals, but Riemann sums are what an integral MEANS — the conceptual foundation and the tool when no antiderivative exists.
The key insight linking a rate and its accumulation is:	**∫ (rate) dt = total accumulated change** — Integrating a rate of change over time yields the total change — the accumulation idea at the heart of the integral.
For a DECREASING function, which sum OVERestimates the area?	**the left Riemann sum** — On a decreasing function, left endpoints are the highest points, so the left sum overshoots.
If f dips BELOW the x-axis on part of [a, b], the Riemann sum counts that region's area as:	**negative (signed area)** — Where f < 0, f(x)·Δx is negative, so the definite integral measures signed (net) area.
Interpreting ∫ v(t) dt (velocity over time), the Riemann sum Σ v·Δt accumulates:	**displacement (net change in position)** — Each v·Δt is a small displacement; summing them gives the net change in position.

As the number of rectangles n increases ($\Delta x \to 0$), the Riemann sum approaches:	**the exact value of the definite integral** — The definite integral is defined as the limit of Riemann sums as the strips become infinitely thin.
For an INCREASING function, a RIGHT Riemann sum will:	**overestimate the true area** — On an increasing function, right endpoints are the highest points, so right rectangles overshoot.
The definite integral $\int_a^b f(x)\,dx$ is formally DEFINED as:	**the limit of Riemann sums as $n \to \infty$** — By definition the definite integral is $\lim(n\to\infty)\ \Sigma\ f(x^*)\cdot\Delta x$ — a limit of sums.
In a Riemann sum, the width of each rectangle is:	**$\Delta x = (b - a)/n$ for n equal subintervals** — Splitting [a, b] into n equal pieces gives each width $\Delta x = (b - a)/n$.
A Riemann sum with rectangles of UNEQUAL width is:	**still valid — the pieces just need not be equal** — The general Riemann sum allows any partition; equal widths are just a common convenient choice.
Doubling the number of rectangles in a Riemann sum (halving Δx) generally makes the approximation:	**more accurate** — Thinner strips hug the curve more closely, reducing the error — the estimate improves toward the exact integral.
$\int_0^2 3\,dx$ (a constant function) equals:	**6** — A constant 3 over width 2 gives a rectangle of area $3\cdot2 = 6$.
The 'adding up pieces' view is more powerful than 'area only' because it:	**makes sense of accumulated quantities like distance, work, and volume — not just area** — Research shows the multiplicatively-based summation conception transfers to any accumulated quantity (Σ rate$\cdot\Delta t$ = total), unlike the narrow 'area under a curve' idea.
The HEIGHT of each rectangle in a Riemann sum is:	**a function value $f(x^*)$ at a sample point in the strip** — Each rectangle's height is the function's value at a chosen sample point (left, right, or midpoint).
Overestimate vs underestimate depends on BOTH the sampling (left/right) and:	**whether the function is increasing or decreasing** — Left/right choice interacts with monotonicity to decide over- vs under-estimation.
Using 2 right-endpoint rectangles for $\int_0^2 x\,dx$ ($\Delta x = 1$) gives the estimate:	**3** — Right endpoints x = 1, 2: heights 1 and 2; sum = $1\cdot1 + 2\cdot1 = 3$ (an overestimate of the true area 2).

11. The definite integral — accumulation as a total

✂ Question (fold →)	Answer
If f is ODD (symmetric about the origin), then $\int_{-a}^{a} f(x)\,dx$ equals:	**0** — For an odd function, the accumulation on [−a, 0] cancels that on [0, a], giving 0.
If a car's velocity is integrated over [0, 5] and the result is negative, the car:	**ended up behind where it started (negative net displacement)** — A negative velocity integral means the net displacement is backward — the car finished behind its start.
$\int_a^a f(x)\,dx$ (equal limits) equals:	**0** — An interval of zero width accumulates nothing, so the integral is 0.
$\int_b^a f(x)\,dx$ (limits reversed) equals:	**$-\int_a^b f(x)\,dx$** — Swapping the limits flips the sign of the integral.
To find TOTAL DISTANCE traveled (not displacement), integrate:	**the absolute value of velocity, $\|v(t)\|$** — Total distance sums the SIZE of motion regardless of direction, so integrate $\|v\|$.
The average value of f on [a, b] equals:	**$(1/(b - a)) \cdot \int_a^b f(x)\,dx$** — The average value is the integral divided by the interval length — the constant height with the same accumulated total.
The units of $\int v(t)\,dt$ where v is in m/s and t in s are:	**meters** — Integrating a rate (m/s) over time (s) gives the accumulated quantity in meters — the units multiply.
If f is EVEN (symmetric about the y-axis), then $\int_{-a}^{a} f(x)\,dx$ equals:	**$2\int_0^a f(x)\,dx$** — An even function accumulates equally on both halves, so the integral doubles the [0, a] part.
The 'accumulation function' $A(x) = \int_a^x f(t)\,dt$ gives:	**the accumulated amount of f from a up to x** — A(x) accumulates f as its upper limit moves; it's a function of x, and A'(x) = f(x) (FTC).
$\int_a^b k \cdot f(x)\,dx$ (constant k) equals:	**$k \cdot \int_a^b f(x)\,dx$** — Constants factor out of a definite integral.
If $\int_a^b f\,dx = 10$ and $\int_a^b g\,dx = 3$, then $\int_a^b (2f - g)\,dx$ equals:	**17** — 2·10 − 3 = 20 − 3 = 17 by linearity.
The Mean Value Theorem for Integrals guarantees a continuous f on [a, b] attains its average value:	**at some point c in (a, b)** — A continuous function must actually reach its average value somewhere inside the interval.
The definite integral $\int_a^b f(x)\,dx$ represents:	**the net (signed) accumulation of f from a to b** — It accumulates f·dx across the interval, counting area below the axis as negative — net accumulation.
Interpreting ∫ (rate of water flow) dt gives:	**the total volume of water** — Integrating a flow rate over time accumulates the total volume — a general accumulation, not 'area.'
The 'area interpretation' of $\int_a^b f\,dx$ holds exactly when:	**f(x) ≥ 0 on [a, b]** — The integral equals geometric area only where f is non-negative; below the axis it subtracts.
The definite integral is a NUMBER, whereas the indefinite integral is:	**a family of functions (with + C)** — A definite integral evaluates to a number; an indefinite integral is the antiderivative family F + C.
A definite integral that comes out negative means the accumulated quantity:	**decreased on net over the interval** — A negative net accumulation means the quantity ended lower than it started.
$\int_a^c f\,dx + \int_c^b f\,dx$ equals:	**$\int_a^b f\,dx$** — Accumulation from a to c plus c to b equals accumulation from a to b.

Why is the definite integral, understood as accumulation, one of the two pillars of calculus?	**it turns a rate of change back into a total, complementing the derivative** — The derivative breaks a total into a rate; the definite integral rebuilds the total from the rate — inverse pillars linked by the FTC.
$\int_a^b [f(x) + g(x)]\,dx$ equals:	**$\int_a^b f(x)\,dx + \int_a^b g(x)\,dx$** — The integral of a sum is the sum of the integrals (linearity).
A quantity's total change over [a, b] equals the integral of its:	**rate of change over [a, b]** — The Net Change Theorem: $\int_a^b F'(x)\,dx = F(b) - F(a)$ — integrate the rate to get the total change.
A definite integral can be estimated from a TABLE of values using:	**a Riemann or trapezoidal sum** — With only sampled values, approximate the accumulation with a Riemann or trapezoidal sum.
The properties of definite integrals (linearity, additivity, symmetry) are powerful because they let you:	**evaluate and combine integrals without always finding an antiderivative** — These properties simplify and combine integrals — sometimes yielding an answer geometrically or by symmetry alone.
If $\int_0^3 f\,dx = 12$, the average value of f on [0, 3] is:	**4** — Average = (1/3)·12 = 4.
$\int_0^4 2\,dx$ equals:	**8** — A constant 2 over width 4 gives 2·4 = 8.

12. The Fundamental Theorem of Calculus — the derivative–integral link

✂ Question (fold →)	Answer
The Net Change Theorem, a corollary of the FTC, says $\int_a^b F'(x)\,dx$ equals:	**F(b) − F(a) (the total change in F)** — Integrating a rate of change over [a, b] gives the net change in the quantity — a direct FTC consequence.
A predict-first habit for FTC problems: before computing $\int_0^a f$, you might predict its sign by asking:	**is f mostly positive or negative on [0, a]?** — Predicting the sign/size from the graph before computing (POE) turns the FTC evaluation into understanding, not just procedure.
When the upper limit is a FUNCTION, $d/dx \int_a$^(g(x)) f(t) dt equals:	**f(g(x))·g'(x) (chain rule)** — FTC Part 1 combined with the chain rule multiplies by the derivative of the upper limit.
The FTC is 'fundamental' because it:	**converts a hard limit-of-sums into a simple antiderivative evaluation** — It reduces computing a definite integral (a limit of Riemann sums) to finding and evaluating an antiderivative.
FTC Part 1 shows that the accumulation function $A(x) = \int_a^x f(t)\,dt$ is:	**an antiderivative of f** — Since A'(x) = f(x), the accumulation function IS an antiderivative of f — connecting the two big ideas.
$\int_0^1 3x^2\,dx$ equals:	**1** — Antiderivative x^3; $1^3 - 0^3 = 1$.
The FTC is what makes the Phase-1 'geometry-first' approach (areas before formal limits) pay off, because:	**the area you built by summing pieces can now be computed exactly via an antiderivative** — Riemann sums give the meaning (accumulated area); the FTC gives the exact value through an antiderivative — meaning plus method.
For $d/dx \int_x^b f(t)\,dt$ (variable LOWER limit), the result is:	**−f(x)** — A variable lower limit is a reversed integral, contributing a minus sign: −f(x).
FTC Part 2 (evaluation): $\int_a^b f(x)\,dx$ equals:	**F(b) − F(a), where F is any antiderivative of f** — Evaluate an antiderivative at the endpoints and subtract: F(b) − F(a).
FTC Part 1 (derivative of an accumulation): $d/dx \int_a^x f(t)\,dt$ equals:	**f(x)** — Differentiating the accumulation function returns the integrand: $d/dx \int_a^x f(t)\,dt = f(x)$.
$\int_0$^(π/2) cos x dx equals:	**1** — Antiderivative sin x; sin(π/2) − sin(0) = 1 − 0 = 1.
The two parts of the FTC together assert that:	**integration and differentiation undo each other** — Part 1 (derivative of an integral) and Part 2 (integral via antiderivative) express the same inverse relationship.
$d/dx \int_2^x \sin(t)\,dt$ equals:	**sin(x)** — By FTC Part 1, substitute the upper limit into the integrand: sin(x). (The lower constant limit contributes nothing.)
$\int_0^2 2x\,dx$, using FTC with $F(x) = x^2$, equals:	**4** — $F(x) = x^2$; F(2) − F(0) = 4 − 0 = 4.
The Fundamental Theorem of	**differentiation and integration as inverse processes** — The FTC shows that

Calculus links:	antidifferentiation and definite integration are two sides of the same coin.
$\int_0^3 x^2$ dx equals:	**9** — Antiderivative $x^3/3$; $(27/3) - 0 = 9$.
Before the FTC, evaluating $\int_a^b f$ dx exactly required:	**computing the limit of Riemann sums directly** — Without the FTC you'd compute the integral from its definition — a tedious limit of sums.
Why is the FTC often called the crowning result of introductory calculus?	**it unites the course's two big ideas — the derivative and the integral — into one relationship** — Differential and integral calculus are developed separately, then the FTC reveals they are inverse operations — the unifying theorem of the subject.
$\int_1^e (1/x)$ dx equals:	**1** — An antiderivative is $\ln\lvert x\rvert$; $\ln(e) - \ln(1) = 1 - 0 = 1$.
$\int_1^4 (1/\sqrt{x})$ dx, with antiderivative $2\sqrt{x}$, equals:	**2** — $2\sqrt{x}$ evaluated: $2\sqrt{4} - 2\sqrt{1} = 4 - 2 = 2$.
If F is an antiderivative of f and G is another, then $F(b) - F(a)$ and $G(b) - G(a)$:	**are equal (the +C cancels)** — Because the two antiderivatives differ by a constant that cancels in the subtraction, any antiderivative gives the same definite integral.
$\int_{-1}^1 x^3$ dx equals (using symmetry or FTC):	**0** — x^3 is odd; $F(x) = x^4/4$ gives $1/4 - 1/4 = 0$.
d/dx $\int_0^x (t^2 + 1)$ dt equals:	**$x^2 + 1$** — By FTC Part 1, just substitute the upper limit into the integrand: $x^2 + 1$.
A quantity accumulates at rate r(t); to find how much accumulates from t = 2 to t = 5 you compute:	**$\int_2^5 r(t)$ dt** — The total accumulated is the definite integral of the rate over the interval.
The FTC guarantees that EVERY continuous function has:	**an antiderivative (namely its accumulation function)** — FTC Part 1 constructs $A(x) = \int_a^x f$ dt as an antiderivative — so continuity guarantees one exists (even if not elementary).

13. Function transformations — shifting and scaling graphs

✂ Question (fold →)	Answer
A common transformation error: students think $f(x-2)$ shifts LEFT. It actually shifts:	**right (inside changes reverse the expected direction)** — Horizontal shifts run opposite to the sign inside: $x-2$ gives a rightward shift.
The parent function for $g(x) = (x-1)^2 + 4$ is:	**$f(x) = x^2$** — g is x^2 transformed (right 1, up 4); the parent is the basic square function.
Applying $f(x) \to 2f(x-1) - 3$ to a point (a, b) on f gives the new point:	**(a + 1, 2b − 3)** — Inside $(x-1)$: x-coordinate goes to a + 1. Outside $2(\cdot) - 3$: y-coordinate goes to 2b − 3.
The graph of $f(x/2)$ is the graph of f:	**stretched horizontally by a factor of 2** — Dividing x by 2 inside stretches the graph horizontally to twice the width.
The graph of $\lvert f(x)\rvert$ takes any part of f BELOW the x-axis and:	**reflects it above the axis** — Taking absolute value forces every output non-negative, flipping below-axis pieces up.
Order matters when combining a stretch and a shift because:	**applying them in different orders can produce different graphs** — A vertical stretch then a shift differs from a shift then a stretch — the operations don't always commute.
The graph of $g(x) = -x^2$ compared to $f(x) = x^2$ is:	**reflected across the x-axis (it opens downward)** — The outside negative flips the outputs, turning the upward cup into a downward cap.
The graph of $f(2x)$ is the graph of f:	**compressed horizontally by a factor of 1/2** — A factor inside multiplies x, compressing horizontally by 1/(that factor) = 1/2.
Why is mastering transformations a foundation for the whole precalc→calc arc?	**predicting a graph from its formula builds the symbol↔picture fluency calculus depends on** — Fluidly moving between a formula and its graph (the CRA bridge) underlies reading derivatives, integrals, and models graphically.
An EVEN function satisfies $f(-x) = f(x)$, so its graph is symmetric about:	**the y-axis** — Even functions are unchanged by reflecting across the y-axis (e.g. x^2, cos x).
Vertical changes (outside f) affect the __ and horizontal changes (inside f) affect the __:	**y-values; x-values** — Outside operations transform outputs (y); inside operations transform inputs (x).
The graph of $f(x) - 3$ is the graph of f shifted:	**down by 3 units** — Subtracting outside lowers every output — down 3.
The graph of $f(-x)$ is the graph of f:	**reflected across the y-axis** — Negating the input mirrors the graph horizontally — a reflection over the y-axis.
The graph of $f(x) + k$ ($k > 0$) is the graph of f shifted:	**up by k units** — Adding k OUTSIDE the function raises every output — a vertical shift up.
For $g(x) = f(x-3) + 2$, the transformation is:	**right 3, up 2** — Inside $(x-3)$ shifts right 3; outside (+2) shifts up 2.

The graph of sin(2x) versus sin(x) has:	**half the period (compressed horizontally)** — The inside factor 2 compresses horizontally, halving the period.
The graph of (1/2)·f(x) is the graph of f:	**compressed vertically by a factor of 1/2** — Multiplying outputs by 1/2 makes the graph shorter — a vertical compression.
The graph of f(x − 2) is the graph of f shifted:	**right by 2 units** — Subtracting INSIDE shifts the graph RIGHT — the opposite of what many expect.
The vertex of $g(x) = (x − 1)^2 + 4$ is at:	**(1, 4)** — Right 1, up 4 moves the vertex from (0,0) to (1, 4).
The graph of f(x + 4) is the graph of f shifted:	**left by 4 units** — Adding inside shifts LEFT (the counterintuitive direction).
An ODD function satisfies f(−x) = −f(x), so its graph is symmetric about:	**the origin** — Odd functions have 180° rotational symmetry about the origin (e.g. x^3, sin x).
Predicting a transformed graph WITHOUT plotting points is possible because:	**each algebraic change corresponds to a known geometric move** — Reading shift/stretch/reflect straight from the formula lets you predict the graph — the CRA symbol↔picture bridge.
The amplitude change in g(x) = 3·sin(x) versus sin(x) is a:	**vertical stretch (amplitude 3)** — The outside factor 3 stretches sine vertically, making its amplitude 3.
The graph of −f(x) is the graph of f:	**reflected across the x-axis** — Negating outputs flips the graph vertically — a reflection over the x-axis.
The graph of 3·f(x) is the graph of f:	**stretched vertically by a factor of 3** — Multiplying outputs by 3 stretches the graph vertically (taller).

14. Exponential and logarithmic functions — growth, decay, e and ln

✂ Question (fold →)	Answer
A key exponent/log misconception is writing log(x + y) = log x + log y. This is:	**FALSE — logs turn products (not sums) into sums** — There is no simple rule for log of a SUM; the sum rule applies to log of a PRODUCT.
log and exponential functions are:	**inverses of each other** — Logarithms undo exponentials: b^(log_b y) = y and log_b(b^x) = x.
The power rule for logs: log(x^n) equals:	**n·log(x)** — An exponent inside a log comes out front as a multiplier: log(x^n) = n·log x.
The derivative of e^x is e^x, which means an exponential's rate of change is:	**proportional to its current value** — d/dx[e^x] = e^x means the bigger the quantity, the faster it grows — proportional growth, the essence of exponential change.
The number e ≈ 2.718 is special because:	**the function e^x is its own derivative** — e is defined so that d/dx[e^x] = e^x — the natural base of calculus.
Exponential functions grow by EQUAL FACTORS over equal intervals, whereas linear functions grow by:	**equal DIFFERENCES over equal intervals** — Linear = constant added amount; exponential = constant multiplied factor. This is the defining contrast (F-LE).
An exponential function with base b > 1 models:	**growth (increasing without bound)** — A base greater than 1 grows: outputs multiply by b each step.
Exponential growth eventually outpaces ANY polynomial because:	**its growth factor compounds while a polynomial's slows relatively** — For large x, b^x (b > 1) exceeds any power x^n — exponential growth dominates polynomial growth.
Continuous compound interest uses the model A = P·e^(rt) because:	**e arises as the limit of compounding infinitely often** — Compounding more and more frequently approaches the base e — hence e^(rt) for continuous growth.
A population doubling every 3 years is modeled by an exponential function because:	**it multiplies by a constant factor over equal time intervals** — Constant multiplicative growth over equal intervals is the hallmark of exponential behavior.
An exponential function has the form f(x) = a·b^x where the variable is:	**in the exponent** — Exponential functions put the variable in the exponent — the source of their explosive growth.
The quotient rule for logs: log(x/y) equals:	**log(x) − log(y)** — The log of a quotient is the DIFFERENCE of the logs (exponents subtract when you divide).
The logarithm log_b(y) = x answers the question:	**'to what power must b be raised to get y?'** — A logarithm is an exponent: log_b(y) is the power on b that yields y.
Log scales (like pH or decibels) are useful because they:	**compress a huge range of values into a manageable scale** — Logarithms turn multiplicative (order-of-magnitude) differences into additive steps — taming enormous ranges.

The product rule for logs: log(xy) equals:	**log(x) + log(y)** — The log of a product is the SUM of the logs — because exponents add when you multiply.
The horizontal asymptote of f(x) = 2^x is:	**y = 0 (as x → −∞)** — As x → −∞, $2^x \to 0$, so y = 0 is a horizontal asymptote (approached, never reached).
An exponential function with base 0 < b < 1 models:	**decay (decreasing toward zero)** — A base between 0 and 1 shrinks outputs toward zero — decay.
Radioactive decay with a fixed half-life is modeled by:	**an exponential decay function** — Halving over equal time intervals is exponential decay (base 1/2 per half-life).
The natural log ln(x) is the logarithm with base:	**e** — ln means log base e — the inverse of e^x.
To SOLVE $2^x = 16$, you can:	**take a logarithm of both sides (or recognize $2^4 = 16$)** — Logarithms bring the variable out of the exponent; here $x = \log_2 16 = 4$.
ln(e) equals:	**1** — log base e of e is 1 ($e^1 = e$).
Solving ln(x) = 0 gives:	**x = 1** — ln(x) = 0 means $e^0 = x$, so x = 1.
The domain of ln(x) is:	**x > 0 (positive reals only)** — You can only take the log of a positive number, so ln's domain is x > 0.
ln(1) equals:	**0** — Any base to the 0 power is 1, so log of 1 is 0.
Why are exponential and log functions essential across calculus and science?	**they model proportional growth/decay and are inverse tools that appear throughout applications** — Populations, radioactivity, cooling, interest, and information all follow exponential/log laws — and e^x/ln are the natural calculus of change.

15. Sequences and series — convergence and divergence

✂ Question (fold →)	Answer
An ALTERNATING series has terms that:	**switch sign from term to term** — Alternating series alternate + and − signs, e.g. 1 − 1/2 + 1/3 − ….
A p-series Σ $1/n^p$ converges if and only if:	**p > 1** — The p-series converges exactly when p > 1 (e.g. Σ $1/n^2$ converges; Σ 1/n does not).
The series Σ $1/n^2$:	**converges (p = 2 > 1)** — With p = 2 > 1, the p-series test guarantees convergence (it sums to $\pi^2/6$).
The Ratio Test examines the limit of $\lvert a_{n+1} / a_n \rvert$; the series converges when this limit is:	**less than 1** — If $\lim\lvert a_{n+1}/a_n\rvert$ < 1 the series converges absolutely; > 1 diverges; = 1 is inconclusive.
Convergence tests matter because an infinite sum:	**can settle on a finite value OR blow up, and you must determine which** — Some infinite sums total a finite number; others grow forever — a convergence test decides which.
Terms shrinking to 0 is NECESSARY but NOT SUFFICIENT for convergence, as shown by:	**the harmonic series Σ 1/n, which diverges** — Σ 1/n has terms → 0 yet diverges — proof that shrinking terms alone don't guarantee convergence.
The distinction between a sequence converging and a SERIES converging is:	**a series converges only if its PARTIAL SUMS approach a finite limit** — Series convergence is about the partial sums settling on a value — a stricter condition than the terms alone converging.
Why do infinite series matter in calculus and its applications?	**they let us represent and approximate functions (like e^x, sin x) and sum infinitely many contributions** — Power/Taylor series approximate complicated functions with polynomials, powering computation, physics, and engineering.
The sequence a_n = 1/n as n → ∞:	**converges to 0** — As n grows, 1/n shrinks toward 0 — it converges to 0.
A GEOMETRIC series Σ ar^n converges if and only if:	**|r| < 1** — A geometric series converges exactly when the common ratio satisfies |r| < 1.
The harmonic series Σ 1/n:	**diverges (even though 1/n → 0)** — The harmonic series famously diverges despite its terms shrinking to zero.
The nth-term (divergence) test says: if the terms a_n do NOT approach 0, the series:	**diverges** — If lim a_n ≠ 0, the series must diverge — the terms must shrink to 0 for any hope of convergence.
A sequence CONVERGES if:	**its terms approach a single finite limit** — Convergence means the terms settle toward one finite value as n → ∞.
A convergent geometric series a + ar + ar^2 + … (|r| < 1) sums to:	**a / (1 − r)** — The closed form for a convergent geometric series is first term over (1 − ratio): a/(1 − r).
A sequence is:	**an ordered list of numbers (terms)** — A sequence is an ordered list a_1, a_2, a_3, … — a function whose inputs are the positive integers.

The geometric series 1 + 2 + 4 + 8 + ... (ratio $r = 2$):	**diverges ($\lvert r\rvert \geq 1$)** — With $\lvert r\rvert = 2 \geq 1$ the terms grow, so the series diverges — the $a/(1-r)$ formula does NOT apply.
A predict-first habit for series: before applying a test, you might guess convergence by asking:	**how fast do the terms shrink?** — Estimating the shrink rate (compared to $1/n$ or $1/n^2$) predicts convergence before formal testing — POE for series.
The series 1 + 1/2 + 1/4 + 1/8 + ... converges to:	**2** — $a = 1$, $r = 1/2$: sum = $1/(1 - 1/2) = 2$.
The sequence $a_n = n$ as $n \to \infty$:	**diverges (grows without bound)** — The terms 1, 2, 3, ... grow without bound, so the sequence diverges.
The Maclaurin series for sin x begins:	**$x - x^3/3! + x^5/5! - \ldots$** — $\sin x = x - x^3/3! + x^5/5! - \ldots$ (odd powers, alternating signs). The all-plus or even-power versions are cosine/exponential look-alikes.
A power series $\Sigma\, c_n(x - a)^n$ converges on:	**an interval of convergence centered at a** — A power series converges on an interval (radius R) around its center a; behavior at the endpoints is checked separately.
The Maclaurin series for e^x is:	**$1 + x + x^2/2! + x^3/3! + \ldots$** — $e^x = \Sigma\, x^n/n! = 1 + x + x^2/2! + x^3/3! + \ldots$ (a Taylor series centered at 0).
A series is:	**the SUM of the terms of a sequence** — A series adds up a sequence's terms: $a_1 + a_2 + a_3 + \ldots$.
The Alternating Series Test guarantees convergence when the terms' sizes:	**decrease monotonically to 0** — An alternating series converges if the absolute terms shrink monotonically to 0.
A Taylor series represents a function as:	**an infinite sum of power terms built from its derivatives at a point** — A Taylor series expands f as $\Sigma\, f^{(n)}(a)/n! \cdot (x - a)^n$ — an infinite polynomial matching f near a.

16. Capstone — integrating rates and accumulation on real models

✂ Question (fold →)	Answer
A model where the rate of change is proportional to the current amount (Q' = kQ) produces:	**exponential growth or decay** — Q' = kQ is the differential equation of exponential change — $Q(t) = Q_0e$^(kt).
Interpreting a NEGATIVE area between a rate curve and the axis in a model means:	**the quantity decreased during that interval** — Below-axis rate contributes negative accumulation — the quantity dropped.
The average value of a rate r(t) over [a, b] gives:	**the constant rate that would accumulate the same total** — The average value (1/(b−a))∫r dt is the steady rate producing the same net accumulation.
If a car's velocity v(t) is given, the net displacement over [0, T] is:	**$\int_0^T$ v(t) dt** — Net displacement is the (signed) integral of velocity.
The single biggest idea CalcQuest builds toward is:	**change can be measured instantaneously (derivative) and accumulated totally (integral), and the two are inverse** — The whole course is the interplay of instantaneous rate and accumulated total, unified by the FTC.
The metacognitive habit Adaeze coaches throughout CalcQuest is to:	**predict before you compute, then reflect on what the answer means** — Predict-then-reflect (Polya-style monitoring) is what turns calculus procedures into genuine understanding.
If given a graph of a RATE and asked for the total change, you should:	**find the (signed) area between the rate curve and the axis** — Total change is the signed area under the rate graph (the definite integral of the rate).
To find WHEN a modeled quantity is largest, you:	**set its derivative to zero and classify the critical points** — Maxima occur where the rate of change turns — find and classify critical points of the model.
For that same car, the TOTAL DISTANCE traveled over [0, T] is:	**$\int_0^T$ |v(t)| dt** — Total distance ignores direction, so integrate the absolute value of velocity.
A capstone problem often requires you to switch between the rate and the total, using:	**differentiation one way and integration the other** — Real models demand fluent movement between a quantity and its rate — the two calculus operations, applied as needed.
When a model gives a rate as a TABLE (not a formula), you estimate the accumulated total with:	**a Riemann or trapezoidal sum** — Without a formula to antidifferentiate, approximate the accumulation numerically (Riemann/trapezoid).
Given an accumulated quantity Q(t), the RATE at which it changes is:	**Q'(t)** — Differentiate the accumulated quantity to recover its instantaneous rate.
Why does calculus, more than any earlier math, describe the real world so powerfully?	**it is the mathematics of change — rates and accumulation — which nature runs on** — Motion, growth, flow, and optimization are all change; calculus's rate-and-accumulation toolkit is the language nature is written in.
The 'predict–observe–explain'	**predict the behavior, compute/observe it, then reconcile any surprise** —

habit applied to a model means:	Predicting before computing (and explaining discrepancies) is the reasoning loop that turns a model into understanding.
To decide whether an accumulated quantity Q is increasing at t = 3, you evaluate:	**its rate Q'(3) and check the sign** — A quantity increases where its rate of change (derivative) is positive — check the sign of Q'(3).
The derivative and the definite integral are related by the FTC as:	**inverse operations (rate ↔ total)** — The derivative extracts a rate; the integral rebuilds the total — inverse pillars joined by the FTC.
Concavity of an accumulated quantity Q(t) tells you whether its rate is:	**increasing (concave up) or decreasing (concave down)** — Q" = (Q')' governs whether the rate itself is speeding up or slowing down.
A quantity whose rate of change is CONSTANT grows:	**linearly** — A constant rate of change adds the same amount each unit of time — straight-line (linear) growth.
A drug's concentration C(t) peaks where:	**C'(t) = 0 changing from positive to negative** — The peak concentration occurs at a maximum — where the rate C' turns from rising to falling.
Before trusting a calculus model's answer, a good habit is to check that the answer's:	**units and sign make physical sense** — Verifying units and sign against the situation catches setup errors — part of reasoning about, not just computing, a model.
A population grows at rate P'(t). To find the population at t = 10 given P(0), you compute:	**P(0) + $\int_0^{10}$ P'(t) dt** — Add the accumulated change (integral of the rate) to the initial value.
A tank's water level L(t) has L'(t) = inflow − outflow. The level is RISING when:	**inflow exceeds outflow (L' > 0)** — The sign of the net rate L' tells you whether the level rises or falls; positive means rising.
A quantity is at a POINT OF INFLECTION in its accumulation graph exactly when its rate is:	**at a local maximum or minimum** — An inflection of Q(t) is where Q" = 0, i.e. where the rate Q' peaks or bottoms out.
Given a rate of change r(t), the TOTAL accumulated change over [a, b] is:	**$\int_a^b$ r(t) dt** — Integrate the rate over the interval to get the total accumulated change (Net Change Theorem).
Reasoning responsibly about a model means acknowledging that a model:	**is a useful simplification, not perfect reality** — Good quantitative reasoning states a model's assumptions and limits rather than treating its output as certain truth.

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