RatioRealm

Meet the Cast

Advanced Edition

Spark & Anvil

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This advanced edition collects 6 chapter books from the RatioRealm cast — each character embodies a different curricular primitive; together they teach the full subject.

Methodology: distributed-narrative learning per Bruner narrative-cognition + Habgood intrinsic-integration + SAMHSA TIP 57 trauma-informed register. Advanced edition: upper-middle-grade register (Wonder / Hatchet / Holes band) for readers ages 11-14 ready for longer sentences + more nuanced subtext.

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##

For everyone who learns by reading between the lines.

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Introduction

The RatioRealm cast was authored to embody the curriculum, not decorate around it.
Each of the 6 characters you'll meet in this book teaches a specific primitive —
a particular tactic, a particular technique, a particular way of seeing.
Together they form an ensemble: the cast IS the curriculum.

Read in any order. Each chapter stands alone.
Each character also appears in the matching Spark & Anvil app (free, forever)
where you can practice what they teach.

This is the **Advanced Edition** — written for readers who are ready for longer sentences, layered subtext, and the trust that comes with not having every joke explained. The Standard Edition covers the same characters at a lighter register; pick whichever feels right for the reader at hand.

— *The editors at Spark & Anvil*

Scale the Doubler (also serves as mentor)

- "salt"
    - "yeast"

Chapter 1 — Scale and the Bread That Fed Forty

The scent of flour and warm yeast was the first thing **Scale** remembered. She grew up in *Hearth and Loaves*, a bakery on the main square of the town of *Measure*. Four generations of women in her family had kneaded life into that building. Her great-grandmother had founded it, a sturdy woman with strong hands. Her grandmother had expanded its ovens and its reputation. Scale's mother had modernized the shop, adding new pastries and a faster delivery route.

By the time she was twelve, Scale was already an essential part of the bakery. She rose before dawn, her small hands surprisingly strong as she helped knead the first batches of dough. At fourteen, she was already managing the younger apprentices, showing them the precise way to shape baguettes. By sixteen, while her mother handled the intense heat of the kitchen, Scale ran the bustling daytime shop, her voice calm and clear even during the busiest rushes.

The bakery's master recipe was a treasure. It was written in her great-grandmother's elegant, looping hand on a piece of parchment, yellowed and fragile with age. This precious document was kept safe in a small, wooden box, tucked away in the deepest pantry. The recipe was simple, a guide for baking just *one loaf*:

> *Two cups of flour. One cup of water. One spoonful of salt. One spoonful of yeast. Knead until smooth. Bake one hour at hot-as-the-oven-can-be.*

This single recipe had nourished the town for a hundred years. It was *the* recipe, the foundation of everything Hearth and Loaves created.

But, of course, the bakery never baked just one loaf a day. On a typical weekday, the ovens produced forty loaves. Market day might see sixty golden-brown loaves cooling on the racks. For a feast-day, the bakers prepared a hundred and twenty. The bakery thrived because it could take that single, perfect recipe and **scale** it up, making more without losing any of its delicious quality.

"If we want forty loaves," her mother had explained, her voice steady and patient, "we multiply every single ingredient by forty. Two cups of flour becomes eighty cups. One cup of water becomes forty cups. One spoonful of salt turns into forty spoonfuls. And one spoonful of yeast? That also becomes forty spoonfuls." Her mother gestured with a flour-dusted hand. "Every ingredient grows by the same exact factor. The ratio of flour to water is still two-to-one. The ratio of salt to yeast is still one-to-one. The bread is still the same incredible bread. There's just *more* of it."

Scale had understood immediately. At eight, the principle seemed obvious. If you wanted more of something, you increased all its parts equally. The bread didn't change its essence; it simply multiplied.

However, she soon noticed something important, a subtle detail that would later shape her path as a teacher. The bakery's *apprentices* didn't always grasp this fundamental idea.

The first apprentice Scale trained was a boy named Brod. He was thirteen, hired to help with the morning rush. On his second day, Scale asked him to triple the master recipe, to make three loaves instead of one. She left him in the kitchen, trusting him with the task, and went to manage the busy shop.

Brod made only one mistake. A single, small error.

When Scale returned to the kitchen, the three loaves were cooling on the counter. They were dense, heavy, and flat. The small amount of yeast hadn't been enough to lift such a large mass of dough. They looked like bricks, not bread.

Scale sat down next to Brod on the worn wooden bench. She didn't scold him. Instead, she picked up one of the heavy loaves. "The ratio is the recipe, Brod," she said gently. "Our recipe is two parts flour to one part water, with one spoonful of salt and one spoonful of yeast, all for one loaf. To make three loaves, *every* part of that recipe must triple. The yeast is part of that ratio. It's essential to the recipe. If you don't scale the yeast, you change the ratio. Then the bread changes too. It’s no longer the same bread."

Brod nodded slowly, his face flushed. He understood. He never made that mistake again. The bakery had no more brick-bread.

But Scale had seen a deeper problem. People who hadn't grown up surrounded by the precise measurements of a bakery didn't instinctively know that every part of a ratio had to grow together. They sometimes scaled some parts and not others. They might even keep certain ingredients the same because they were *afraid of adding too much* of that particular item. And the bread, or whatever they were making, would come out wrong.

When she was twenty, Scale decided she would teach this vital principle. She studied for three years with the RatioRealm academy, continuing to run the bakery on weekends. At twenty-three, she joined the faculty. For nine years, she has taught equivalent ratios—the principle that multiplying all parts of a recipe by the same factor preserves the original ratio, keeping the bread, or the dish, or the pattern, exactly the same.

In her classroom, she begins every first-day lesson the same way. She brings the original master recipe parchment from the bakery, which her younger brother now runs. She holds it up carefully, its ancient paper rustling softly. "This recipe makes one loaf," she announces. "To make ten loaves, what do we do?"

The children, always, call out, "Multiply everything by ten!"

Scale smiles, a warm, floury smile. "Yes," she confirms. "*Everything*. Two cups of flour becomes twenty. One cup of water becomes ten. One spoonful of salt becomes ten. One spoonful of yeast becomes ten. The ratio of flour to water is still two-to-one. The bread is still the same bread, just *more* of it. Not different."

She pauses, letting the idea settle. "If you change one ingredient and not the others, the ratio is no longer the recipe. The bread becomes something else entirely. Apprentice Brod learned this the hard way. We, however, will not."

When children ask if ratios and proportions are difficult, Scale always gives the same answer:

Listen along + meet more of the cast at:

https://spark-and-anvil.com/cast/ratiorealm/scale-the-doubler

Pair the Ratio-Speaker

From her very first breath, Pair knew the world came in twos. She was born with a twin sister, Echo, on a crisp spring morning outside the village of *Couplet*. Their tiny farmhouse, usually quiet, now hummed with double the joy, double the fuss. They shared everything: a cradle, then a cot, then a small bed where their heads nearly touched. They learned to walk together, their small hands clasped. They learned to speak in a shared language of whispers and giggles. Even by the standards of twins, who were famously inseparable, Pair and Echo were *unusually close*.

When they were seven, a bitter winter fever swept through Couplet. Many children fell ill; most eventually recovered. But Echo, bright and quick, did not. The village healer, a kind woman with knowing eyes, did what she could. One morning, Echo was a small, laughing child, chasing snowflakes. By afternoon, her eyes were dim with sickness. By evening, she slept, and by the second dawn, she was gone.

Couplet, as Pair was then known, grieved with her family through a long, silent winter. She grieved for a year, a hollow ache in her chest. For the rest of her life, a smaller, quieter sadness remained, a shadow at the edges of her joy. Her parents, lost in their own profound sorrow, were patient with her. They understood the depth of a loss that felt like half of oneself.

But Echo did not simply vanish from Pair's world. In her absence, Echo *taught Pair about pairs*.

In the quiet years that followed, a strange new awareness began to settle over young Couplet. She had always known a partner. Every memory from her earliest days, before the fever, held two figures: two small girls, side-by-side. There had been two cradles, two identical dresses, two sets of small footprints marking the muddy path to the market. When her family sang, two voices had always mingled, one clear, one slightly higher. Her mother's hand had always held two small hands, not one. There had always been *two*.

After Echo died, there was *one*. Pair held only one hand now. She sang in a single voice. She walked alone, a solitary figure where once there had been two.

Gradually, almost imperceptibly at first, Pair began to notice the world around her *in twos*. It was a quiet observation, a habit of mind that grew from her loss. Shoes, for instance, always came in twos. Her own eyes, when she looked in the polished surface of a bucket, were two. Her hands, her lungs, even the two church bells that called the villagers to worship – one north, one south – all existed in a fundamental duality. Wheels on a cart came in twos. Horses, when harnessed for work, were always paired. Even the villagers' winter mittens came in twos, one for each hand. The world, Pair realized when she was nine, seemed to be *organized into pairs*. It felt, to her young mind, as though this was *somehow because of Echo*. The world was teaching her about pairs because she had lost one.

The true revelation came years later, during a long, sun-drenched summer when she was thirteen. She found an old, dusty arithmetic book in the village market, its pages filled with strange symbols and equations. As she slowly deciphered the text, a new kind of understanding clicked into place. The numbers on the page began to mirror the world she had observed for so long. The book spoke of **ratios** – a word that felt both new and profoundly familiar.

She saw how the village blacksmith, crafting horseshoes, always made them in sets of four. Each horse had two front feet and two back feet, she knew. This meant a 2-and-2 arrangement of front-to-back, or a 4-to-1 ratio of shoes to horse. The potter, shaping clay, made cups and saucers in equal measure, a perfect 1-to-1 ratio. Every cup needed its saucer. The weaver, at her loom, ensured that every warp-thread had a corresponding weft-thread, another 1-to-1 ratio of crossings. The world, she understood, was not just full of pairs. It was full of *fixed pairings*, and these fixed pairings were, in essence, *ratios*.

Pair, at thirteen, grasped this concept with a clarity most children her age would never possess. She had spent six years thinking about pairs, because Echo had taught her to.

She extended the principle further, beyond simple 1-to-1 pairings. She noticed *non-1-to-1* ratios. Two wheels for every cart meant a 2:1 ratio of wheels to carts. Four sturdy legs held up every chair, a 4:1 ratio of legs to chairs. Even the eight notes that made up an octave, a concept she did not yet fully understand, represented an 8:1 ratio in some deeper, musical sense. The world, she realized, was full of *fixed proportions*. Every fixed proportion, she understood, was a *recurring ratio*.

When Pair was nineteen, she walked into the RatioRealm academy. In her hand, she carried a small wooden carving of two clasped hands. Her mother had carved it for her after Echo's death; Pair had kept it in her pocket for twelve years, its smooth surface worn by her thumb. She placed the carving carefully on the academy master's polished desk.

"I would like to teach ratios," she stated simply.

The academy master, a man accustomed to grand pronouncements and complex theories, raised an eyebrow. He knew nothing of Pair's childhood, or the quiet journey that had brought her to his door. "And why, young woman," he asked, "do you believe you are qualified?"

Pair met his gaze. "Because I have been thinking about pairs for twelve years," she replied. Her voice was steady. "I believe I understand them well enough now to teach them."

Intrigued, the master invited her to demonstrate. Pair picked up the wooden carving. She held it up for him to see. "Each hand," she explained, "has five fingers. So, for every one hand, there are five fingers. That is a ratio: 1:5." She paused, letting the simple truth sink in. "These two clasped hands together have ten fingers. The ratio of hands to fingers here is 2:10. Both ratios describe the same world. Both ratios are true. They are simply different *expressions* of the same fundamental pairing."

The academy master, by his own later admission, was profoundly moved. He had heard countless lectures on the intricacies of ratios, but never had one begun with such a humble, yet powerful, object. He saw the depth of understanding in her eyes, the quiet conviction in her voice. He invited Pair to join the faculty. She accepted.

That was eleven years ago.

In her classroom, Pair begins every first-day lesson the same way. She places the wooden carving of clasped hands on her desk. She points at it. "For every one hand," she says, her voice gentle but clear, "there are five fingers. That is a ratio. It is the same as saying 'one to five' or '1:5'. The colon is just a way of writing 'for every'. *For every X, there are Y*. That is the foundation of ratio."

She lets the children pass the carving around, feeling its smooth, worn wood. They notice that the two clasped hands together have ten fingers. They see that the ratio of one hand to its fingers (1:5) and the ratio of two hands to their fingers (2:10) are *equivalent*. Pair watches, a small smile playing on her lips, as they discover this connection for themselves.

"The pair holds the whole world together," she tells them. "Once you see that two things come in a fixed proportion, you can scale them up or down – but the ratio always stays the same. That is the secret of ratios. Two-to-five, four-to-ten, ten-to-twenty-five – all describe the same relationship. All reflect the same world."

When children ask whether ratios are hard, Pair always says the same thing. Her gaze drifts to the carving on her desk.

"They are not hard," she assures them. "They are *pairings*. For every X, there are Y. The pair is the world. The ratio is the world's way of telling you how the pair stays together."

Sometimes, when the classroom is quiet, she adds, almost to herself: "My sister taught me about pairs."

She does not, usually, explain who her sister was. But the children, holding the carving, eventually figure it out.

Listen along + meet more of the cast at:

`https://spark-and-anvil.com/cast/ratiorealm/pair-the-ratio-speaker`

Unit the Per-One-Counter

- "Pinforth"
  - "Saltwell"
  - "vs"
  - "vs."
  - "for"
  - "apple"
  - "apples"
  - "VS"
  - "3 for 2"

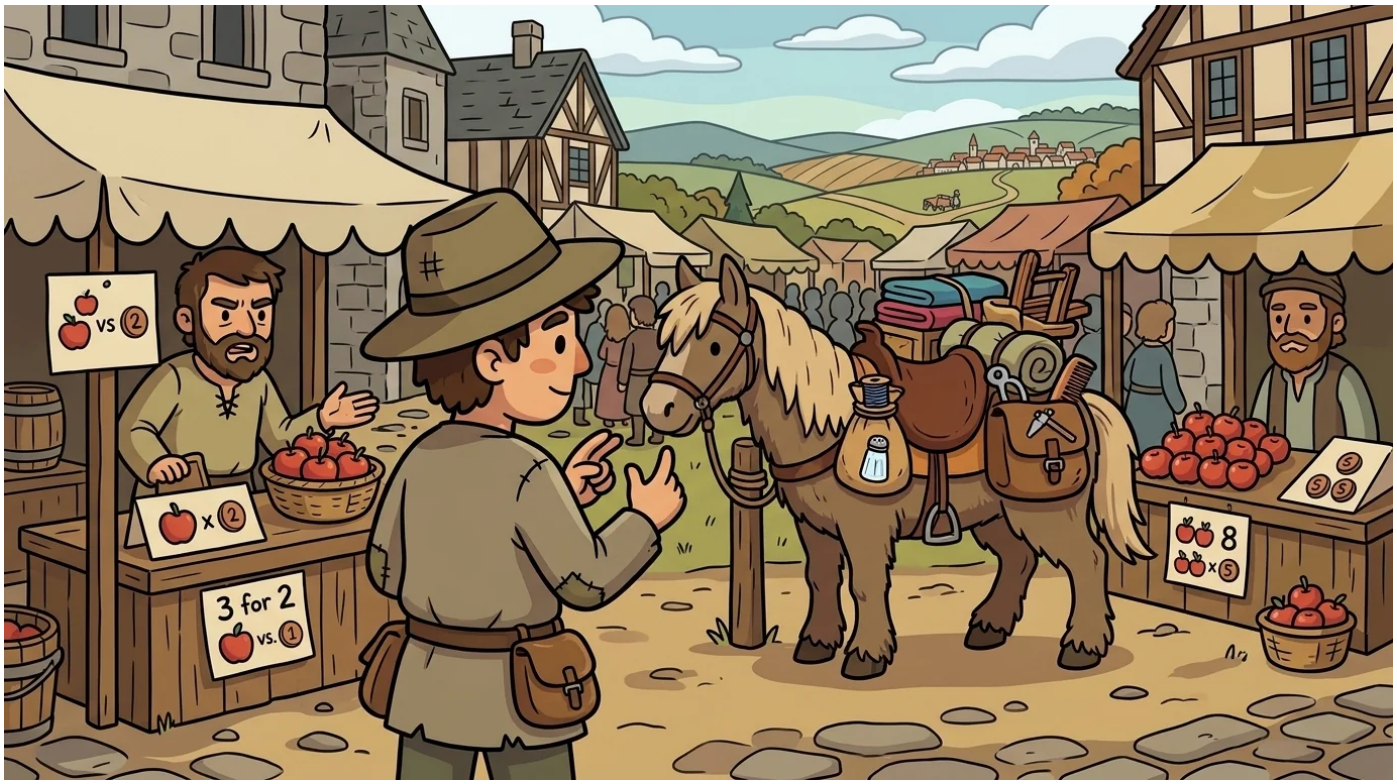
- "2"
  - "3"
  - "5"
  - "8"
  - "S"
  - "11"
    gate-allow-text-pattern: '^[0-9]{1,2}$'

Unit walked for *twelve years.*

This is, by most standards, *a lot of walking.* Most people walk a great deal in the course of their lives — to school, to market, to work, to a friend's house — but they do not, ordinarily, do all of their walking *with the same packhorse and the same canvas pack.* Unit did. He spent twelve years as a *travelling pedlar*, walking from market to market across the kingdom's three central provinces, selling cloth, salt, small metalwork, and (occasionally) wooden combs.

He had been apprenticed to a master pedlar — a gruff old man named *Tenstride* — at the age of seventeen. He had inherited the packhorse and the canvas pack at the age of twenty-five when Tenstride retired. He had walked the routes himself for twelve years after that, between twenty-five and thirty-seven, before joining the academy.

What Unit eventually understood — and what made him the teacher he became — was that *every market measured things differently.*



He would walk into the market town of *Loomley* and see a bolt of woolen cloth quoted at *eight coppers per bolt.* He would walk into the next market town of *Pinforth* (about half a day east) and see a bolt of woolen cloth quoted at *one copper per yard.* He would walk into the third market town of *Saltwell* (another half-day) and see *armspan* of cloth quoted at *eleven coppers.*

He had three prices. In three different units. For what was, essentially, the same product.

Was Loomley's cloth a better deal than Pinforth's? Was Pinforth's better than Saltwell's? *He could not tell* — not directly. The numbers were not comparable.

What he learned to do — what Tenstride had taught him in the first year of apprenticeship, in the master pedlar's gruff voice — was *reduce everything to per-one.*

A bolt of cloth, Tenstride had said, was *eight yards.* An armspan was *roughly a yard and a half.* So Loomley's eight-coppers-per-bolt was *one copper per yard.* Pinforth's one-copper-per-yard was, well, *one copper per yard.* Saltwell's eleven-coppers-per-armspan was *roughly seven-and-a-half coppers per yard.*

The key was *per yard.* The key was *per one of a common thing.*

Unit, who was nineteen at the time, sat with this for several weeks. He realized it was *general.* Any time you had a rate quoted in different units, you could reduce it to *per-one-of-a-common-thing* and then the rates were directly comparable.

He started doing this in his head. Cost per yard. Distance per day. Calories per serving (when he was negotiating for food at inns). Wages per hour. Salt per pound. He could not look at any rate without reducing it to per-one.

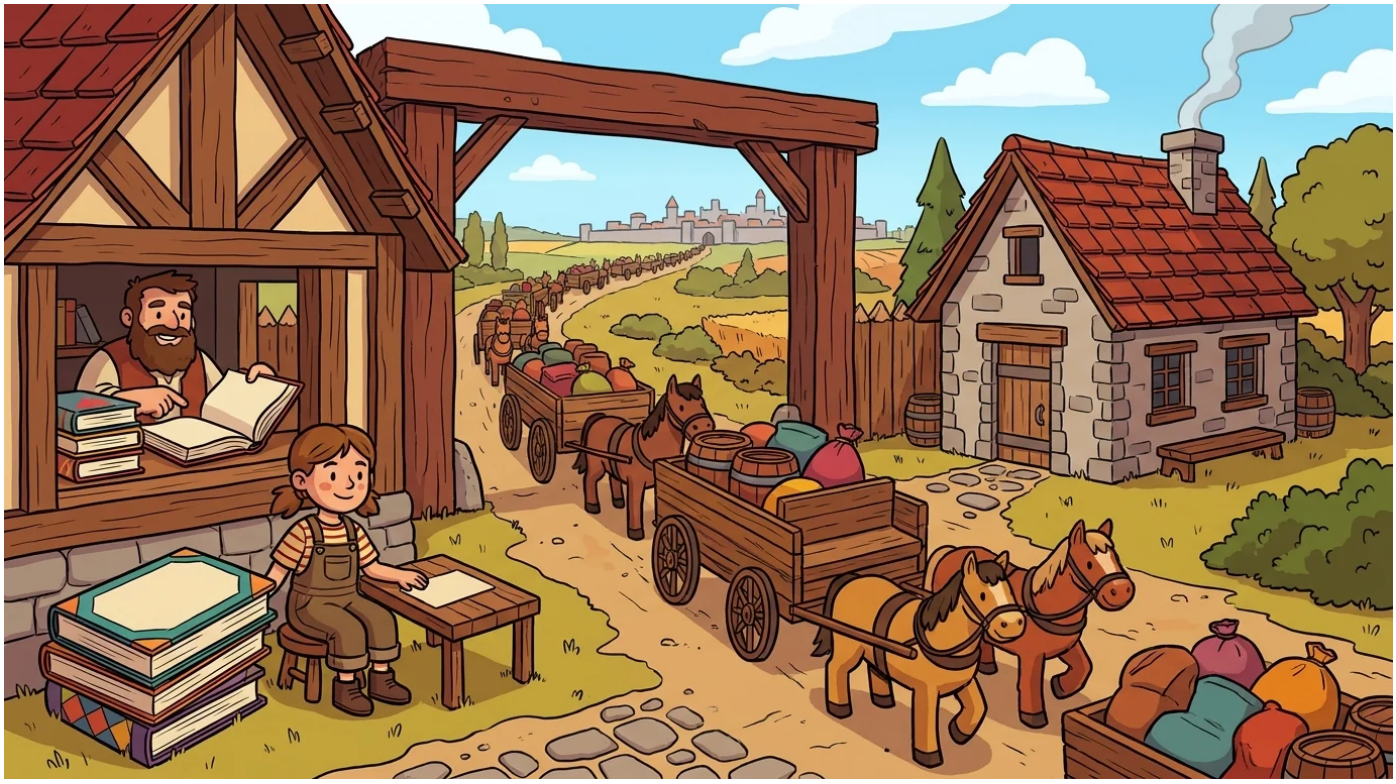
By the end of his twelve years on the road, he could do it instantly. Quote him any price in any unit and he would, within seconds, have a per-one comparison ready.

When the RatioRealm academy was looking for someone to teach rates and unit rates to children, the academy master had heard about Unit from a Loomley merchant who said: *"He is the only pedlar I have ever met who treats price-comparison as an arithmetic discipline rather than an instinct."*

Listen along + meet more of the cast at:

`https://spark-and-anvil.com/cast/ratiorealm/unit-the-per-one-counter`

Centa the Percent-Translator



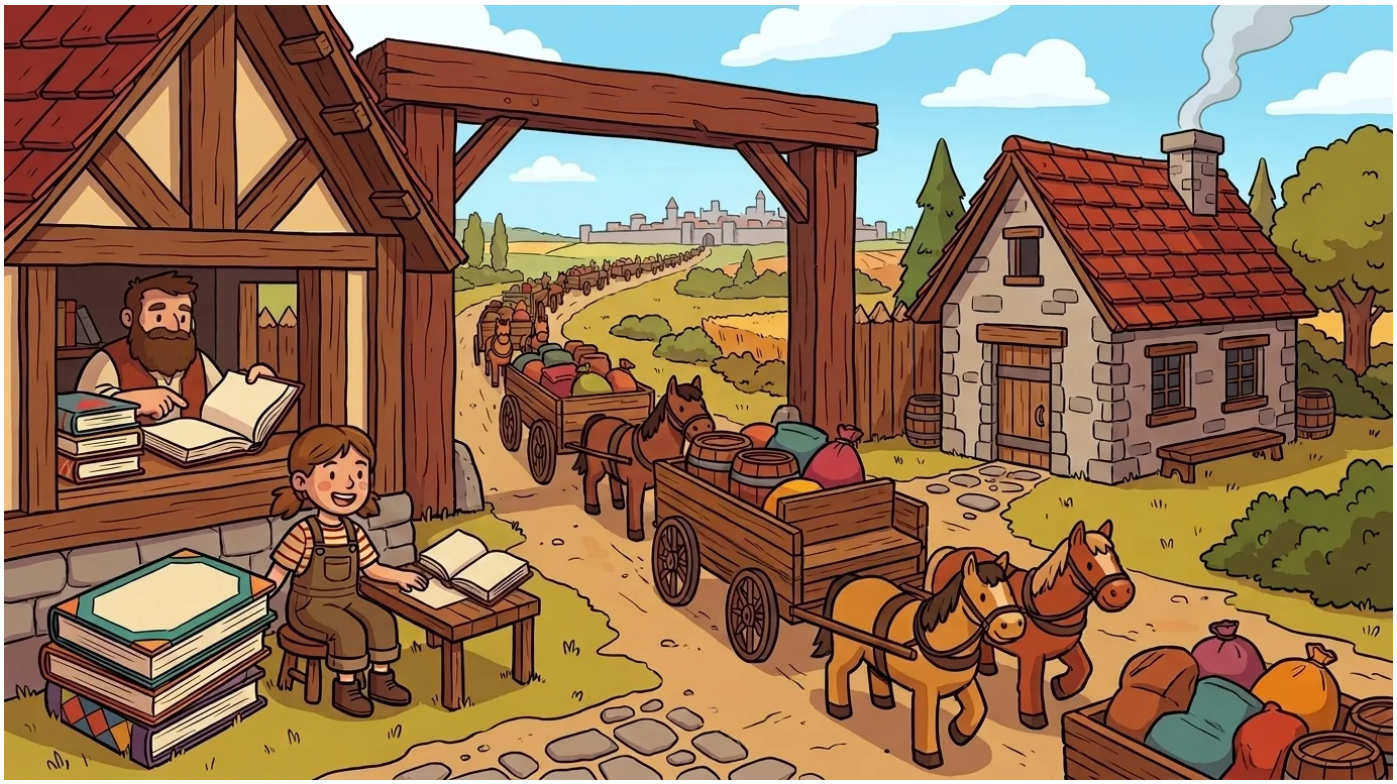
Centa grew up at *a toll-gate.*

Specifically: at the *Northgate Toll* of the kingdom's main north-south trade road, about ten miles outside the capital. The Northgate Toll was, in the kingdom, *busy.* On a normal weekday, two or three hundred carts came through the gate. On market-days, six or seven hundred. The toll-collectors at Northgate handled enormous volumes of small-cart-by-small-cart tax assessment, and the kingdom's chief assessor lived in a stone house attached to the gate's south side.

The chief assessor — and this is the essential fact of the chapter — was *Centa's father.*

His name was *Centesimal.* Everyone called him *Cent.* He had inherited the chief-assessor's position from his father, who had inherited it from his father, who had inherited it from his uncle. The family had been chief assessors at Northgate Toll for *four generations.*

Centa — whose given name was *Mira*, though by the time she was twelve everyone called her Centa, which her father had nicknamed her — was the eldest of three siblings. She grew up at the toll-gate. She climbed the gate's wooden cross-beams as a small child. She ate her suppers at a small table in the chief-assessor's office. She fell asleep, more nights than not, listening to her father read aloud from the kingdom's grain-tax rolls.



What Centa understood, from a very early age, was that *the kingdom's tax system was built on percentages.*

The kingdom did not tax things in fixed amounts. (Fixed amounts would be unfair: a wealthy merchant and a poor farmer would pay the same, which was not the point of taxation.) The kingdom taxed things *as percentages of the cart's value.* Grain was taxed at ten percent. Copper was taxed at five percent. Textiles were taxed at two percent. Salt was taxed at twelve percent (salt was a luxury). Imported spices were taxed at twenty percent (spices were a great luxury).

Cent calculated these percentages for every cart that came through the gate, for fourteen years of Centa's childhood.

He calculated them *very fast.*

A merchant would arrive with a cart of, say, forty bushels of grain. Cent would assess the grain's value (the going market rate, which he kept in a leather-bound ledger). He would multiply by ten percent. He would announce the tax. The merchant would pay. The cart would pass. The next cart would arrive.

By the time Centa was eight, she could compute ten percent of any number in her head. (Move the decimal one place to the left.) By the time she was ten, she could compute one percent of any number. (Move it two places to the left.) By the time she was twelve, she could compute any percentage by combining the moves: *to compute twelve percent of three hundred, compute ten percent (thirty), compute one percent (three), multiply the one-percent by two (six), add thirty plus six (thirty-six).*

She was, by twelve, *as fast as her father.*

What Centa eventually understood — and what made her the teacher she became — was that *percentages were a translation language.* They were *the universal way to express a ratio at the per-hundred scale.* Any ratio — three-to-five, one-to-eight, seventeen-to-twenty — could be translated into a percentage, and once translated, every percentage was directly comparable. *Three-to-five became sixty percent. One-to-eight became twelve-and-a-half percent. Seventeen-to-twenty became eighty-five percent.* The kingdom used per-hundred as its tax denominator because *per-hundred is the common scale that lets you compare any two rates immediately.*

She thought about this for years.

When she was twenty (and had been helping her father at the toll-gate for several years), a wandering scholar passed through Northgate. The scholar was, by Centa's father's instructions, not charged a toll (scholars were exempt; the kingdom valued learning), but the scholar spent a long afternoon at the toll-gate watching Centa compute percentages. At the end of the afternoon, the scholar approached Centa and said:

"You are the fastest percentage-calculator I have met. Have you considered teaching?"

Centa had not. She thought about it. She talked to her father. Cent (who had been a tax-collector for thirty years and was, in his own quiet way, glad his daughter had options other than the toll-gate) said: *"Go. Teach. The kingdom has many tax-collectors. It has few teachers."*

Centa went to the RatioRealm academy when she was twenty-one. She studied for three years. She joined the faculty when she was twenty-four. She has been teaching percentages ever since.

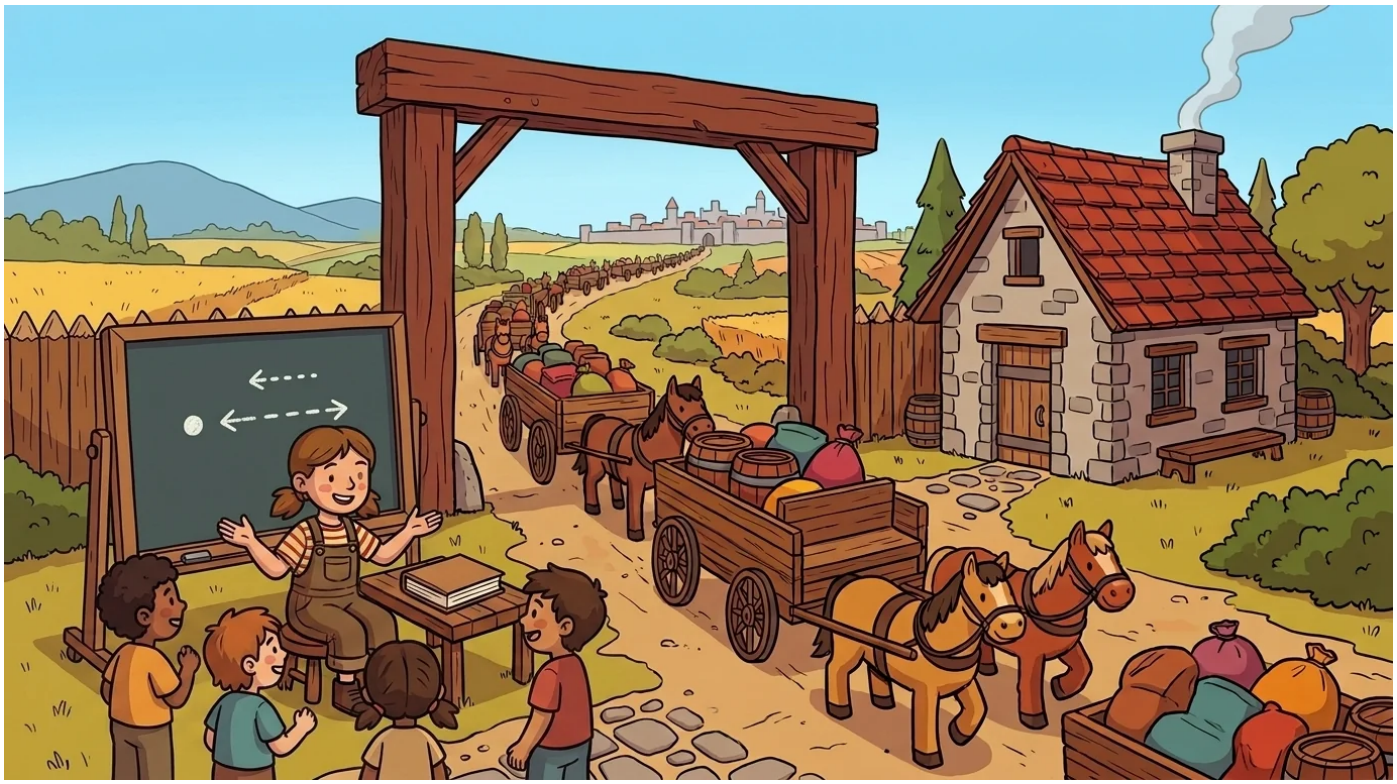
In her classroom, she begins every first-day lesson the same way. She brings, from her father's office, *a small wooden ledger* — a child-sized version of the kingdom's tax-ledger her father had made for her when she was six. The ledger has tiny columns for cart-value and tax-percent. She places it on the desk. She turns to the class. She says: *"What is ten percent of one hundred?"*

The children — always — say ten.

Centa says: *"What is ten percent of two hundred?"*

The children say twenty.

Centa says: *"What is ten percent of seventy?"*



The children — most of them — say seven.

Centa smiles. She says: *"You already know how to compute percentages. You have been doing it. *Ten percent is moving the decimal one place to the left.* That is the whole trick of ten-percent. The rest of percentages are built on top of that trick."*

She then teaches them one-percent (move two places left), and how to combine them (twelve percent = ten percent + one percent + one percent), and how to compute any percentage by combining the simple ones.

The children — always — find it easier than they expected. They had been told percentages were hard. They had been imagining complicated multiplication. They had not been imagining *moving the decimal* and *adding small percentages together.* Centa's approach is, in her own quiet way, *a revelation.*

When children ask whether percentages are hard, Centa always says the same thing:

*"They are not hard. They are *per hundred.* Once you translate any ratio to per hundred, every percentage becomes comparable. To compute ten percent: move the decimal one place to the left. To compute one percent: move it two places. The rest is combining these moves."*

She still keeps the small wooden ledger. The children sometimes ask to use it (it has rows for them to write practice problems). She always lets them.

Listen along + meet more of the cast at:

`https://spark-and-anvil.com/cast/ratiorealm/centa-the-percent-translator`

Cross the Proportion-Solver

- "B"
    - "C"
    - "D"

Chapter 4 — Cross and the Loom That Checked Itself

Cross grew up in *a weaving family.*

Cross — whose given name was *Marlee*, though everyone called her Cross from the time she was twelve — was the second of four children. She grew up among the looms.

The family's loom-tradition had *a specific, peculiar habit* that Cross's grandmother had inherited from her grandmother and had passed on to Cross's mother and then to Cross. The habit was this:

After every yard of cloth was woven, the weaver checked the diagonals of the cloth's rectangular border.

This was, even in the weaving trade, *unusual.* Most weavers, when they wanted to check that their cloth was woven *straight* (rather than skewed into a parallelogram), would measure the cloth's *width* in three places — top, middle, bottom — and confirm the widths were equal. Cross's family did not do this. Cross's family checked *the two diagonals.*

This was Cross's grandmother's diagnostic. It was an elegant test. Width-checking required three measurements. Diagonal-checking required two. The diagonals were also more sensitive — a small skew that did not show in widths would show in diagonals.

Cross learned to check diagonals before she could read. She was eight years old before she had ever heard of mathematics. She was ten years old before she had ever encountered a proportion. She was thirteen years old when she walked into the small village school for the first time and heard the schoolteacher say:

"If a/b = c/d, then ad = bc. This is called cross-multiplication. You multiply diagonally."

The schoolteacher wrote it on the board. Cross looked at it. Cross said, slowly: *"That is the same as checking the diagonals on a loom."*

The schoolteacher said: *"What?"*

The schoolteacher set down his chalk. He had been teaching the cross-multiplication rule for twenty years. He had never heard a child compare it to weaving.

He said: *"Yes. That is correct. The geometric basis of cross-multiplication is exactly the same idea — equal diagonals indicate a closed proportional relationship. You have, frankly, *understood the rule better than I ever did,* and you are thirteen."*

Cross was a little embarrassed. She had not meant to outdo the schoolteacher. She had just been thinking out loud.

But the comparison stuck with her. Over the next several years, every time she encountered a proportion problem at school, she pictured the loom. *Does the proportion hold? Check the diagonals.* The cross-products either matched (the proportion was true) or they did not (the proportion was false). The rule was elegant and visible and *the same rule she had been applying to cloth for ten years.*

When she was eighteen she walked away from the family's looms (her younger brother had taken to weaving naturally and was eager to inherit the workshop) and went to the RatioRealm academy. She studied for four years. She joined the faculty when she was twenty-two. She has been teaching cross-multiplication ever since.

The children — always — agree.

Then she writes on the board: *2/3 = 4/6.* She points at the two diagonals of the proportion. She says: *"Now the proportion. Two-times-six equals twelve. Three-times-four equals twelve. The diagonals are equal. The proportion holds."*

The children stare. They see the rectangle. They see the proportion. They see that the rectangle's diagonals and the proportion's diagonals are *doing the same job.*

Cross says: *"This is cross-multiplication. The rectangle's diagonals tell you whether the cloth is straight. The proportion's diagonals tell you whether the proportion is true. Same diagnostic. Same diagonals. Different scale."*

When children ask whether proportions are hard, Cross always says the same thing:

Listen along + meet more of the cast at:

https://spark-and-anvil.com/cast/ratiorealm/cross-the-proportion-solver

Pair and Unit

The harvest market in *Loomley* always drew a bustling crowd, with vendors arriving from three different provinces. A familiar pattern emerged each market day: three cloth stalls, two butter stalls, and five vegetable stalls. Loomley's mayor insisted on these fixed numbers, believing the marketplace became *easier to look at if you knew what you were looking at.* This particular morning, the familiar order was about to be tested.

Pair had come to the market with a simple errand: buying butter for her aunt. Unit, however, visited Loomley for a different reason. He hadn't stepped foot in the town for eleven years and felt curious about how the prices might have shifted. Their paths crossed unexpectedly near the cloth stalls. Pair was still navigating the market, trying to locate the butter vendors. Unit, a former pedlar of woolens, habitually checked prices wherever he went, and today he was comparing woolen bolts. They arrived at the same moment, recognized each other from the academy, and paused for a quick chat.

As they spoke, an argument erupted nearby. A young farmer stood at the middle cloth stall, clutching a length of brown wool against his chest. His finger jabbed furiously toward the other two stalls.

"This is *eleven coppers* for an *armspan!*" he declared to the middle vendor. "That one over there is *eight coppers a bolt.* And the one further down is *one copper a yard.* You're clearly cheating me. I can buy the same cloth for less money anywhere else."

The middle vendor, an old woman with remarkably calm hands, simply shook her head. "I don't believe that's true," she said softly. "My cloth is the same as theirs."

"Then why do you charge more?" the farmer demanded.

The old woman's expression remained serene. "I don't think I do," she replied.

Frustrated, the farmer tossed the brown wool back onto the stall and turned to leave. Unit, stepping forward, spoke with quiet politeness. "Excuse me. Perhaps I can assist?"

The farmer hesitated, his shoulders still tense.

Pair, who had been listening intently, offered a reassuring smile. "Please," she said. "We both teach mathematics. We untangle problems like this every day."

The farmer looked from Pair to Unit, then to the old woman. He let out a long sigh. "Untangle it then," he conceded, though his voice still held a note of suspicion.

Unit moved methodically toward the three cloth stalls, his gaze sweeping over the displayed prices.

- Stall one (Mira's): 8 coppers per bolt.
- Stall two (the old woman's): 11 coppers per armspan.
- Stall three (Bran's): 1 copper per yard.

He pulled a small, worn notebook from his pocket and carefully recorded each price. "We have three different prices," he observed, "and three different units: *bolt, armspan, yard.* These numbers aren't directly comparable as they are written."

"They are **fixed-pairings**," Pair explained in a quiet voice. "Each price represents a ratio. Stall one offers *eight coppers for one bolt.* Stall two sells *eleven coppers for one armspan.* And stall three provides *one copper for one yard.* We have three ratios, but we need to find a *common second-side* for them."

Unit nodded, understanding immediately. "Per-one," he clarified. "Per *one of the same thing.*"

He turned back to the farmer. "Tell me, how many yards are in a bolt?"

The farmer paused, thinking. "A bolt is eight yards," he stated. "Everyone knows that."

"And an armspan?" Unit prompted.

"An armspan is, ah, about a yard and a half," the farmer replied. "Mine is a hair more than my brother's, but we usually round it to a yard and a half."

Unit jotted down the conversions in his notebook:

- 1 bolt = 8 yards.
- 1 armspan = 1.5 yards.

He drew a neat line beneath the conversions and began to calculate each price based on a common unit.

- Stall one: 8 coppers per bolt = 8 coppers per 8 yards = 1 copper per yard.
- Stall two: 11 coppers per armspan = 11 coppers per 1.5 yards = 7.33 coppers per yard.
- Stall three: 1 copper per yard = 1 copper per yard.

He underlined the final per-yard prices. "Stalls one and three," he announced, "offer the same price: one copper per yard. Stall two, however, charges more than seven coppers per yard. *Stall two is more than seven times as expensive.*"

The farmer's eyes widened, his jaw dropping slightly. "I knew it!" he exclaimed, a triumphant note returning to his voice. "She *is* cheating me!"

"Wait a moment," Pair interjected. While Unit had been absorbed in his calculations, she had been closely examining the cloth on the old woman's stall. "Look at this fabric."

She picked up a length of the brown wool, rubbing it gently between her fingers. Then she held it up, letting the sunlight filter through the weave. "This isn't the same cloth as the others," she observed. "This wool is *finer.* The weave is noticeably tighter, and the wool itself has been combed twice, not just once. It's a different product entirely. You're not paying for the same yardage; you're paying for a different *grade* of cloth."

The old woman, who had watched the entire process with her calm hands folded in her lap, nodded slowly. "It's three-pass wool," she confirmed. "It costs more to make. The sheep are a different breed, and the carding process is more intensive. That's why the price is higher."

The anger drained from the farmer's face, replaced by a profound confusion. He had shifted from feeling *cheated* to utterly bewildered in a matter of seconds. "Then how do I compare them?" he asked, his voice subdued. "If the cloth isn't the same, I can't compare prices, can I?"

"You compare the *same things,*" Unit explained gently. "Per yard, yes, but also per same grade. *Stalls one and three offer the same grade of wool. Stall two offers a different grade.* You can honestly compare stall one to stall three; both are one copper per yard. But you cannot directly compare stall two to either of them."

"So what is stall two for?" the farmer wondered.

"It's for people who specifically want finer cloth," Pair answered. "People who might prefer a 1-to-1 ratio of stitches per inch, rather than a 1-to-2. The **fixed-pairing** here is different, and the price naturally reflects that particular pairing."

The farmer looked at all three stalls again, then at the brown wool he had so dramatically thrown back. He picked it up once more, rubbing it between his fingers just as Pair had done. "It *is* finer," he admitted, a note of surprise in his voice.

"Yes," Pair confirmed.

"But I don't need fine wool," he said. "I need wool that won't tear easily."

"Then stall one or stall three would suit your needs," Unit suggested. "They are the same price. You can pick whichever vendor seems friendlier."

The farmer considered this, scanning the three vendors. Mira at stall one was engrossed in a small book, not looking up. Bran at stall three was loudly chatting with another customer. The old woman at stall two had simply resumed her knitting, her needles clicking softly.

"Stall one," the farmer decided.

He walked over to Mira’s stall and quietly purchased eight yards of brown wool for eight coppers.

The old woman set down her knitting, offering Pair and Unit a warm smile. "You two are very skilled at this," she said.

"It's simply our profession," Pair replied.

"All three prices were fair, in the end," Unit mused. "Once you reduce them to a common per-yard unit, the comparison becomes honest. Two of the per-yard prices happened to be identical. The third, however, clearly reflected a different grade of material. The farmer didn't see the *grade*; he only saw three different numbers."

"Most people only see the numbers," the old woman agreed, her gaze thoughtful.

"That's precisely why the academy exists," Pair said gently. "We teach children to look *past* the raw numbers. We help them see *what the numbers are actually about.* A ratio remains meaningless until you truly understand its second side. A rate, similarly, is meaningless until you reduce it to a common per-one."

"We had to teach that farmer in three minutes what we typically teach our students over an entire year," Unit added with a wry smile.

"He'll grasp the per-yard concept now," Pair predicted. "The idea of differing grades will take him longer to understand. But that's perfectly acceptable."

The old woman picked up her knitting once more. "Thank you," she said. "I make this finer wool because my daughter's hands cramp at the loom now. She works more slowly. The wool is better, but she sells less of it. People often don't truly understand what *finer* means."

Pair and Unit lingered for a few more minutes. Before leaving, they purchased a small length of the fine wool from the old woman, ensuring her morning wasn't a complete loss. They split the cost between them.

Only then did Pair remember the butter she had originally come for.

That evening, as they walked back to the academy along the quiet road, Pair broke the silence. "I was thinking about Echo today," she said.

"Echo, your sister?" Unit asked softly.

"Yes."

Unit offered no further words, simply continuing their steady pace.

"She would have *seen* the three stalls just as we did," Pair continued, her voice tinged with melancholy. "She always saw the **fixed-pairing** behind everything. She understood the *what-goes-with-what*."

"Mm," Unit murmured.

"I think Echo would have truly liked the academy."

"I think she would have," Unit agreed, his tone gentle. "I believe she would have become a wonderful Per-One-Counter."

Pair offered a small, wistful smile. They walked on, the silence comfortable between them.

"You know what the academy doesn't have yet?" Pair said after a while. "An ensemble lesson specifically about this. About the three-stalls scenario. About the *grade-of-the-cloth* problem. About how reducing to per-one only works when the *thing* on each side of the ratio is genuinely comparable."

"Then we shall teach it next month," Unit decided. "For Kit Four. We'll write it together."

"All right," Pair said, a hint of renewed energy in her voice.

They completed the rest of their journey without speaking. The hills outside Loomley grew very quiet as the sun dipped below the horizon.

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