# ProofQuest

## *Meet the Cast*

### Advanced Edition

Spark & Anvil

# Copyright & License



This advanced edition collects 9 chapter books from the ProofQuest cast — each character embodies a different curricular primitive; together they teach the full subject.

Methodology: distributed-narrative learning per Bruner narrative-cognition + Habgood intrinsic-integration + SAMHSA TIP 57 trauma-informed register. Advanced edition: upper-middle-grade register (Wonder / Hatchet / Holes band) for readers ages 11-14 ready for longer sentences + more nuanced subtext.

Spark & Anvil is a 501(c)(3) public charity. All apps free forever; no ads; no tracking; no in-app purchases.

spark-and-anvil.com

##

*For everyone who learns by reading between the lines.*

# Contents

# Introduction

The ProofQuest cast was authored to embody the curriculum, not decorate around it.
Each of the 9 characters you'll meet in this book teaches a specific primitive —
a particular tactic, a particular technique, a particular way of seeing.
Together they form an ensemble: the cast IS the curriculum.

Read in any order. Each chapter stands alone.
Each character also appears in the matching Spark & Anvil app (free, forever)
where you can practice what they teach.

This is the **Advanced Edition** — written for readers who are ready for longer sentences, layered subtext, and the trust that comes with not having every joke explained. The Standard Edition covers the same characters at a lighter register; pick whichever feels right for the reader at hand.

— *The editors at Spark & Anvil*

# Induction Ida and Strong-Induction Sten

The Belfry stood as the academy's tallest tower. Its limestone stairs spiraled upward, winding around themselves for one hundred and thirty-seven steps. Ida and Sten, assistants to the Bell Master, climbed these stairs twice daily. Their ascent marked the sunrise; their descent, the sunset. It was a rhythm etched into their bones.

Ida discovered a way to conquer the climb during her first week. She had stood at the bottom, gazing up at the seemingly endless helix of stone. The task felt immense, almost impossible. Then, a thought clicked into place. Every morning, before she began, she whispered her rule.

"If I can take the first step," she would murmur, "and if I can take the next step from whatever step I'm standing on, then I can take all the steps."

This was Ida's **induction** rule. She called it her "domino rule" because each step felt like knocking over the next in a long, predictable line. She took a deep breath, said the words, and started her ascent.

Sten, always a little more thoughtful, approached the stairs with a different understanding. He watched Ida, admired her straightforward method, but felt something was missing. His own rule acknowledged a deeper connection.

"If I can take the first step," Sten would say softly, "and if my knowing-how-to-take-the-next-step depends not just on the step I'm standing on, but on everything I've already done — every step behind me — then I can take all the steps."

This was Sten's **strong induction** rule. He considered it his "stronger rule" because it carried the weight of history. He would nod to himself, a quiet acknowledgment of the path already traveled, then begin his climb.

Most days, both rules proved equally effective. They each reached the top, panting slightly, ready for their duties. But on the day a bell-rope snapped at step seventy-nine, the subtle difference between their two philosophies became critically important.

---

The Bell Master appeared at the base of the staircase, a heavy coil of new rope slung over his shoulder. His brow was furrowed, a rare sight. He looked at his two assistants, his gaze serious. "Step seventy-nine," he announced, his voice echoing in the stone shaft. "Bell rope frayed all the way through. Needs replacing. Who's going?"

Ida glanced at Sten, then Sten at Ida. They both spoke at once. "I'll go!"

The Bell Master nodded. "All right. Both of you. Ida, you go first. You'll carry the new rope. Sten will follow with the splicer's tools." He held up a hand, emphasizing his next words. "The thing is, you have to coordinate. Each step Sten takes has to be coordinated with where you already are. Otherwise, the rope will get tangled. It's a tight squeeze up there."

"Got it," Ida said, her voice firm. She took the thick coil of rope, settling its weight into her arms. It was heavier than she expected. She started upward, her mind already focusing on her rule.

*If I can take the first step, and if I can take the next step from whatever step I'm on, I can take all the steps.* The new bell rope, still stiff and unyielding, coiled in her arms. She moved with a steady rhythm. Step one. Step two. From step five, she took step six. From step thirty-eight, she took step thirty-nine. Her rule was simple: the previous step was all she needed to consider. She didn't need to recall steps one through thirty-seven; she only needed to remember step thirty-eight, and that was enough to confidently take step thirty-nine. The rope snaked behind her, a living thing.

She reached step seventy-nine, a small landing carved into the wall. She began the careful work of splicing the new rope, her fingers nimble. Once the knot was secure, she called down, her voice echoing.

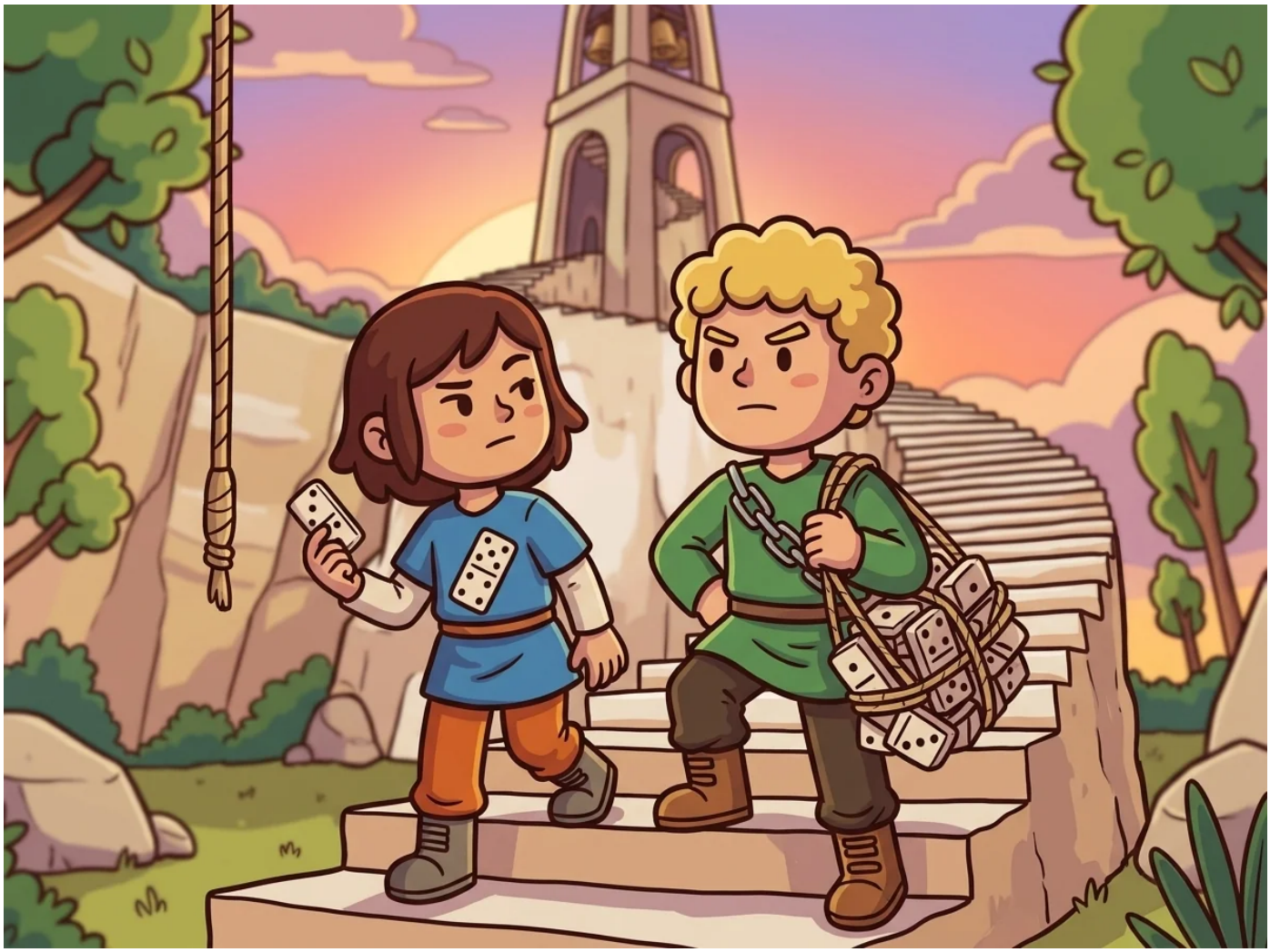
"All right, Sten! Come on up!"

Sten began his ascent, the heavy leather pouch of splicer's tools clanking softly against his hip. He used his own rule, his gaze sweeping the steps ahead of him, but his mind working backward.

*If I can take the first step, and if my next step depends on everything I've already done, I can take all the steps.* This was crucial now. The rope was a variable, a moving part.

At step thirty-eight, Sten paused. He didn't just look at the step beneath his foot. He remembered. *Ida passed through this step a few minutes ago. She left the rope coiled along the right wall, keeping it clear of the center. I know this because I saw her do it from below, watching her careful movements.* So, when he took step thirty-nine, he knew he needed to avoid the rope. This meant stepping precisely on the left side of step thirty-nine, a decision informed not just by the immediate step, but by everything Ida had done before.

He stepped left on thirty-nine. No tangle. The rope lay undisturbed.



At step fifty-five, he paused again. He pictured Ida. *She turned around at step fifty-five to call down to me, to check my progress. When she turned, she shifted the rope to the inside of the spiral, hugging the wall.* So, when he stepped onto fifty-five, he instinctively knew to step outside, giving the rope a wide berth.

He stepped outside on fifty-five. Again, no tangle. He continued, each step a deliberate calculation based on the entire sequence of Ida's actions.

Finally, at step seventy-nine, he arrived next to Ida. The new rope lay perfectly aligned, not a single coil out of place.

"How did you know?" Ida asked, her eyes wide. "I wasn't even thinking about where I put it."

Sten shrugged, a slight smile touching his lips. "I needed everything. All of it."

---

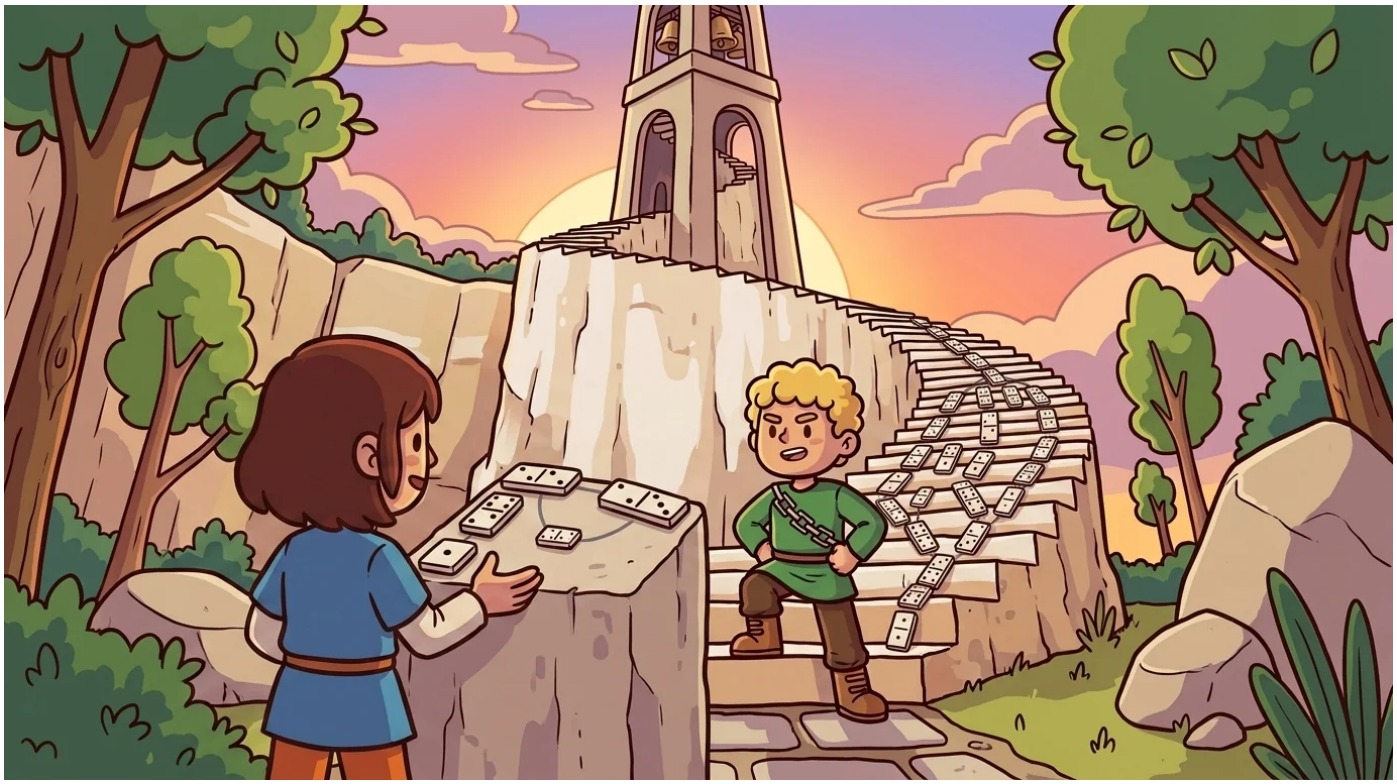
Ida considered his words on the way back down. They walked together, the new bell rope now safely installed, ready to ring at sunset. The climb down felt lighter, the air cooler.

"So your rule isn't *wrong*," Ida said, choosing her words carefully. "But it's... heavier. More to carry."

"It's heavier," Sten agreed. He watched his feet, navigating the descent.

"My rule is just: only think about the step you're on," Ida explained, articulating her process. "Take the next step from *that* step. Don't carry the whole history with you."

"Right," Sten confirmed. "And it works for most things. For just climbing the stairs, for example."



"Your rule, though," Ida continued, "is to keep *all* of the history with you. Use everything you've done, everything that happened before, to decide the next step."

"Exactly," Sten said.

"And mine works for most things," Ida repeated, trying to find the edge of its usefulness.

"For most things, yes," Sten conceded.

"But for some things," Ida mused, a new understanding dawning, "like coordinating with you and the rope, mine isn't enough. Because the rope's position depends on *every* step that came before. So I'd need to know the *whole* history of where the rope went, not just where I'm standing now."

"Right," Sten said, meeting her gaze. "That's when mine helps."

Ida nodded slowly. "If we were just climbing the stairs alone, both rules would get us to the top. They'd give the same answer."

"They would," Sten confirmed.

"It's only when the next step needs to know something *older* than just the previous step that yours matters," Ida concluded, feeling the weight of the distinction.

"That's when stronger induction earns its weight," Sten said, using the formal term he'd learned from the Bell Master.

Ida nodded again, a thoughtful expression on her face. "So we're both telling kids the same story, mostly. Mine is the simpler version. Yours is the version that handles harder problems."

"Yours is the version most kids start with," Sten added. "Mine is the version they grow into when the problem they're proving is more tangled."

"Same family," Ida decided. "Different siblings."

"Different siblings," Sten echoed, satisfied.

---

That sunset, they rang the bell together. Ida pulled the new rope, its fibers still stiff, but responsive. Sten counted the strokes, his voice clear and steady. The bell rang one hundred and thirty-seven times — once for each step they had climbed, a resonant echo of their journey.

Sten said, his voice soft, "Every stroke depends on every earlier stroke."

Ida smiled. "But every stroke is fine on its own, too."

Both rules held true. Both rules had carried them up. Both rules carried them down. Both rules offered correct ways of thinking about the staircase, and about the world.

"Good induction," Ida said, a new respect in her tone.

"Good strong induction," Sten replied, a quiet pride in his.

Outside, the kingdom lay quiet, bathed in the fading light. The sun settled into the sea, painting the horizon in hues of orange and purple. The bell rang for a long, clear time, its sound carrying far across the land.

**Listen along + meet more of the cast at:**

https://spark-and-anvil.com/cast/proofquest/induction-ida-strong-induction-sten

# Direct-Proof Dora

## Chapter 1 — The Path Dora Always Takes

Dora had a particular way of explaining things. If you asked her a question, she would answer it, but never in a hurry. She would begin with the first piece of information. Then she carefully connected it to the next, and the next, until the answer unfolded like a map. By the time she reached the conclusion, many people had already figured it out for themselves. This, as Dora often pointed out with a gentle smile, was precisely the point. She called it a **direct proof**.

She did not believe in shortcuts. In fact, she didn't think shortcuts truly existed, not in the part of the world she inhabited. Dora's world was constructed from paths. Some paths were long, winding through forests and over hills. Others were short, a straight line from one point to another. Yet none of them, in her view, were ever *shortcuts*. A path was simply the route you actually walked, step by careful step.

This methodical habit proved exceptionally useful for someone whose life's work involved teaching children how to prove things.

Dora walked that bridge every day on her way to school. It was a long bridge, stretching out over the water. By the time she was eight, she knew the exact order of its planks: one through thirty-seven. She counted them every morning as her small feet tapped across the timber. She counted them again every evening on her return. The repetition never bored her. In this way, and in many others, she was exactly the child she would become as an adult.

One spring, when Dora was nine, a stranger arrived in Stepwell. He was a tall man, dusty from travel, and he moved with an air of hurry. He stopped Dora near the riverbank, his eyes scanning the horizon.

"Excuse me, child," he said, his voice a little rough. "I'm looking for the bridge. I heard there was one across the river here."

Dora pointed toward the sturdy structure. "It's right there," she said.

Dora, who was nine and had just spent the last three years counting every plank twice a day, answered immediately. "No," she said.

The stranger turned, a flicker of mild curiosity replacing his haste. He wasn't annoyed, just genuinely intrigued. "You're really sure about that?" he asked.

"Yes," Dora affirmed. Her voice was small but certain. "The river is wide. There is only one bridge. You could swim if you wanted, but that is also a path, and it is much wetter."

The stranger considered her words for a long moment. A slow smile spread across his face. "That," he said, "is the most thorough answer I have ever been given." He then walked across the bridge, continuing on his way, and Dora went home for dinner.

Around the age of ten, she decided her purpose in the world would likely involve reminding people that *the path is the path*.

She became a mathematician.

(She also became a teacher, but the mathematician part came first, a foundational step in her own intellectual journey.)

By the time the ProofQuest academy extended an invitation, asking her to teach the **direct proof** technique to young minds, Dora had already accomplished much. She had authored two books exploring the elegant simplicity of logical progression, taught three university classes, and delivered nearly four hundred talks at various small mathematical societies. In her spare time, she had also walked across her grandfather's bridge approximately fourteen thousand times, each journey a quiet reaffirmation of her beliefs.

She introduced herself to the children on her first day with quiet confidence. "You probably already know how to prove things," she began. "I am just here to help you write down what you already do. Every proof has a beginning, a middle, and an end. The beginning is what you are given. The end is what you want to show. The middle is the path between them."

A child in the back of the room, a boy with bright, curious eyes, raised a hand. "Isn't there ever a faster way?" he asked, his voice hopeful.

Dora paused, a gentle smile touching her lips. She had been answering this exact question since she was nine years old. "No," she said. "There are sometimes shorter paths, yes. But the path is always the path."

She has been teaching at the academy ever since, guiding her students through the precise, beautiful logic of direct proofs. She still walks her grandfather's bridge twice a year, a quiet pilgrimage. And she still counts the planks.

**Listen along + meet more of the cast at:**

`https://spark-and-anvil.com/cast/proofquest/direct-proof-dora`

# Induction Ida

Every year, in the town of Lattice, a special festival took place. Lattice sat in the kingdom's southern hill region. It was a three-day walk from the bustling capital. The locals called their annual celebration the Cascade.

The Cascade was one event. It lasted about thirty seconds.

Each family in Lattice contributed special dominoes. These were crafted from wood, painted in bright colors. They were slightly heavier than typical playing dominoes, designed to tip cleanly. The day before the festival, the entire town gathered in the central square. They worked together, meticulously setting up the dominoes. A single, winding chain snaked through the square. It went around the fountain, along the edges of the market stalls. The chain climbed the steps of the town hall, then descended past the bakery. Laying the full chain took most of an afternoon. The town's current record, set seven years ago, was one thousand four hundred and twelve pieces.

On the festival day, as the sun dipped below the hills, a hush fell over the crowd. The mayor, holding the official honor for that year, stepped forward. He bent down and gently pushed the first domino.

Everything else fell on its own.

A roar rose from the crowd. The town quickly put up bunting, and the bakery sold out of its special festival biscuits. Strangers hugged. Soon, the dominoes were swept away. Plans for next year's Cascade began immediately. Then, the whole town went home.

The Cascade was more than just a festival for Ida. It was the place where she began to understand the world like a mathematician.

Ida's family, the Latticefords, had overseen the Cascade preparations for generations. They were known as the festival-domino-makers. Ida grew up watching her mother carefully arrange long curves of dominoes across the town square. She started helping as soon as she could walk. By age seven, Ida could lay a hundred dominoes on her own. She managed this without accidentally knocking any over. This skill was harder than it seemed. It required a particular knack: setting down the next piece without bumping the one before. You simply could not be in a hurry.

At twelve, Ida was already a careful and capable domino-setter. Yet, she wasn't a mathematician then. That transformation began the year the chain stretched nine hundred and fifty pieces long. It was the year her mother gave her the ultimate honor: she would *push the first domino*.

It was a tremendous honor, but also a heavy responsibility for Ida. She had spent the entire afternoon laying her own section of the chain. She checked each piece three times, making sure it stood perfectly. At twelve, she was the youngest person to push the first domino in seventeen years.

She bent down at sunset. She pushed the first domino.

It fell into the second. The second fell into the third. The third fell into the fourth.

Ida watched the chain unfold.

In those thirty seconds, Ida watched with intense focus. She saw something with the piercing clarity only a twelve-year-old sometimes possesses. Adults, she realized, often forgot this simple truth. She had pushed only *one domino*. The other nine hundred and forty-nine had fallen entirely *on their own*. She hadn't touched them. She hadn't even been close to them. The entire chain, every single piece, had toppled. This happened because the *first* one fell, and because each domino was placed *close enough to the next one* to knock it down in turn.

She thought, then and there: *That is everything I need to know.*

Ida walked home that night, humming a quiet tune. Her sister, Sten, then nine, didn't understand why. Sten, now known as Strong-Induction Sten, had eaten three festival biscuits. At that moment, she simply didn't care about humming.

Ida wrote down what she had figured out, in her own twelve-year-old handwriting, in a notebook her grandmother had given her. The notebook page said:

*"To knock down all the dominoes, you only need to do two things:*

*1. Knock down the first one.*

*2. Make sure each domino is close enough to the next one to knock it down too.*

*That's it. The rest happens by itself."*

That night, Ida didn't know this principle had a name. She didn't know it was hundreds of years old. She certainly didn't know mathematicians called it **induction**. They used it to prove things about every natural number. She couldn't have known that in eight years, she would arrive at the ProofQuest academy. There, she would introduce herself by saying, "I am the dominoes person." And the academy master would look up from his notes, a smile on his face, and say, "Oh good. We have been waiting for you."

She just knew, that night in Lattice, that she had figured out something important.

Ida has taught mathematical **induction** ever since that night. She still returns home for the Cascade each year. She no longer pushes the first domino, as that honor rotates among the townspeople. But she always sets up her share of the chain. She still does not hurry.

And when children come to her class for the first time and ask, nervously, whether the technique called *induction* is hard, Ida always says the same thing:

*"You knock down the first one. You show that each one knocks down the next one. That's all. The rest happens by itself."*

She adds, after a small pause:

*"It also helps if you don't hurry."*

**Listen along + meet more of the cast at:**

https://spark-and-anvil.com/cast/proofquest/induction-ida

# Strong-Induction Sten

Strong-Induction Sten is, as the chapter title suggests, the kind of person who inherits things.

He inherited his nose from his father. He inherited his height from his mother. He inherited his love of festival biscuits from his grandfather. He inherited his belief that *you should use everything you already have* from a series of older cousins who pointed out, repeatedly, throughout his childhood, that there was a perfectly good library at the end of the street.

This is a chapter about why Sten became the kind of mathematician he is.

Sten was, as the previous chapter mentioned, three years younger than his sister Ida. He grew up in the same town (Lattice), in the same family (Latticeford), in the same domino-festival tradition. He helped set up the Cascade every year. He pushed the first domino at fifteen, which was a great honour, and he too watched the whole chain fall, and he too noticed that he had only pushed one piece.

He thought: *Yes. I see why Ida likes this.*

But Sten also noticed, that day and other days, that his sister's domino technique had a particular shape. She *only used the previous one.* She knocked down domino k, and domino k knocked down domino k+1, and the rest happened by itself.

Sten thought, watching her work: *That's fine. But what about the dominoes she already knocked down?*

This will sound like a small thought. It was not small.

What Sten realised — and he realised it slowly, over the course of several years — was that when you are proving things about, say, the natural numbers, by the time you are trying to prove the case for n=10, you have *already* proved n=1, n=2, n=3,..., n=9. They are *already established.* You are allowed to use them. All of them.

Ida's technique used the previous one. Sten thought: *Why not use all of them?*

He brought this up at the dinner table when he was sixteen.

Ida (who was nineteen and home from her first year at the academy) said: *"You can. It's allowed. It's just a different version of the technique. It's called strong induction."*

Sten said: *"How is that different?"*

Ida said: *"In ordinary induction, you only assume the case for k. In strong induction, you assume the cases for everything up to k. It's still valid. Some proofs need it. Most don't."*

Sten said: *"Why would anyone NOT use the strong version?"*

Ida said: *"Because it isn't always necessary."*

Sten said: *"It isn't always necessary, but it's never wrong. So I'd just use it."*

Ida said: *"You're allowed to. Most mathematicians don't, because it feels heavier."*

Sten said: *"It only feels heavier."*

Their mother (who was excellent at deflecting domino arguments at the dinner table) suggested they pass the bread.

Sten went on to study mathematics, like his sister. He arrived at the ProofQuest academy three years after Ida did. He introduced himself by saying, *"Hello. I am the dominoes-but-also-everything-already-fallen person."* The academy master said, *"Oh. We have your sister. Are you also the dominoes person?"* Sten said, *"Yes. But I assume more."* The academy master, who had been at the academy for a long time, immediately understood and hired him.

Sten teaches strong induction. He is the cast member who, when proving something about case k+1, gets to use *every previously-proven case.* This is occasionally exactly what a proof requires. There are theorems — about the structure of prime numbers, about the depth of certain trees, about the way certain recursive algorithms terminate — that cannot be proved by ordinary induction at all. They need strong induction. Sten teaches all of them.

He is, in person, mildly relaxed. He says *"obviously"* a lot, which is sometimes annoying but is usually accurate. He believes — and has said, more than once, in front of his sister — that strong induction is *just induction, but with more friends in the room.*

Ida finds this analogy slightly grumpy.

She finds it more grumpy because it is *correct.*

Sten and Ida are very close. They write letters when they are at different academies. (Sten now teaches at the southern branch; Ida at the central.) Their letters are warm and full of *next case* discussions. They send each other interesting recursion problems. They argue about whose technique is *more elegant* (Ida says ordinary; Sten says strong; neither has changed her or his mind). Their mother, who is now seventy-two and still runs the family domino business in Lattice, considers this argument the only thing her children ever fight about.

At the next Cascade festival, both of them will be home. They will set up their share of the chain — Ida's careful, Sten's slightly faster — and they will help the youngest cousin push the first domino, and they will watch the chain fall, and afterward they will go to the bakery and eat festival biscuits and continue arguing about which kind of induction is the real one.

Their mother has, by now, learned to bring earplugs.

**Listen along + meet more of the cast at:**

https://spark-and-anvil.com/cast/proofquest/strong-induction-sten

# Construction Cole


- "BEAM"

# Chapter 5 — Cole Builds the Thing

Cole wasn't born a mathematician. He was born a carpenter, and in his mind, that order mattered above all else. He understood the world through the grain of wood, the precise fit of a joint, and the sturdy weight of a finished piece.

His roots ran deep in Beam, a western timber town predictably named for the sturdy wooden supports that held up its buildings. Generations of townspeople had long since grown tired of the joke, but the name still fit. Cole's family, for nine generations, had built the very backbone of Beam. They crafted things meant to last: houses with strong foundations, barns that sheltered livestock, school benches worn smooth by countless children, and the occasional bridge spanning a tricky creek. His grandmother, a woman whose hands were as gnarled as old oak, always said a Beam carpenter would rather build you a thing than try to explain it in words. It was a family saying, and Cole knew it was the absolute truth.

Cole apprenticed to his uncle at fifteen, learning the scent of sawdust and the rhythm of the saw. By twenty, he was a competent carpenter, able to follow instructions and execute designs with skill. Five years later, he had become a truly good one. He could look at a problem—a sagging porch step, a kitchen in need of shelves, a roof that leaked with every storm—and simply *build the answer*. He rarely drew plans beforehand. Instead, he preferred to construct a small, working model, examine it from every angle, and then scale it up to the full-sized version. His uncle had taught him a fundamental truth: *you understand a thing when you've made one*.

This hands-on approach, this practical certainty, was in retrospect a deeply mathematical attitude. Cole, however, had no idea at the time.

He learned it at twenty-eight, in a way that utterly surprised him.

A traveling mathematician, a polite woman with bright, curious eyes, was passing through Beam on her way south. She stopped at Cole's workshop, not for a new barn, but to ask if he could repair a damaged shoulder-bag strap. Cole could, of course. He repaired it with a few deft movements, making it stronger than before. As he worked, they fell into conversation. The mathematician asked what kind of work Cole found most satisfying, what truly made him feel accomplished. Cole paused, considering his answer. "The kind," he finally said, "where I look at the problem and just make the thing."

The mathematician's eyes brightened further. "Have you ever heard of **proof by construction**?" she asked, her voice soft but clear.

Cole shook his head. "No," he admitted.

The mathematician explained the technique. She described how, when you want to prove that something exists—whether it's a number with a specific property, an arrangement of pieces that works, or a geometric figure meeting a certain description—you don't have to argue abstractly that it *must* exist. Instead, you can simply *produce one*. You point at it, she said, and declare, "There. That one. It works."

Cole listened, his hands still holding the repaired strap. He thought about it, a slow smile spreading across his face. Then he laughed, a deep, rumbling sound that echoed in the workshop. "That," he said, "is the only kind of proof I have ever done."

The mathematician smiled back. "There are mathematicians at the central university," she suggested, "who would love to meet you."

Cole, already turning back to his workbench, waved a dismissive hand. "I do not have time for the central university," he replied. "I am making a barn."

"Of course," the mathematician said, gathering her bag. "Just thinking aloud."


She left a few moments later, her repaired strap slung securely over her shoulder. Cole finished the barn.

Yet, the conversation stayed with him. He thought about it for the next two years, while he built three more houses, sturdy and square, and a graceful footbridge spanning the creek just outside town. Slowly, a new understanding dawned on him. He realized he had been *constructing proofs* in his daily work all along. Every cabinet he built was a proof that *a cabinet with these dimensions and these supports is possible*. Every roof was a proof that *a roof spanning this distance can be built*. He had never considered his work in such abstract terms, but the mathematician had been absolutely right.

At age thirty, Cole wrote to the academy, a place he had never imagined himself. He simply asked if they ever needed teachers. The academy master wrote back the same week, a short, enthusiastic note that said, *yes, we always do*.

Cole arrived in the capital with three small, carefully carved wooden objects tucked into his bag: a simple block, a miniature step, and a perfect wedge. He used them in his very first class. He has been using them, or their well-worn replacements, ever since.

His teaching style is simple and direct. "You want to prove this thing exists?" he asks his students. "Fine. Here is one. Look at it. Touch it. It exists."

He then patiently walks the children through *why* the object he made satisfies the claim. The proof, in Cole's classroom, is always composed of two essential halves: *the object itself, plus the explanation of why it works*. Many children find this practical, hands-on style unusually clear, which is precisely the effect Cole intended.

He has been at the academy for sixteen years now. He has built, by his own careful count, four hundred and thirty teaching objects. They are all kept in a large, custom-built cupboard at the back of his classroom. The cupboard is labeled, in his own neat, blocky handwriting: *Things That Exist*.

Every summer, he still takes a week off to return to Beam and build something. Usually, it's a step. He likes building steps because they are honest objects, connecting one level to another without pretense. He returns to the academy each autumn with his hands slightly more callused and his patience subtly renewed.

If you ask Cole what he does, he will not say, *I am a mathematics teacher*.

He will say, quite simply: "I build things. Then I show people that they work."

**Listen along + meet more of the cast at:**

https://spark-and-anvil.com/cast/proofquest/construction-cole

# Contradiction Cassius

Cassius spent twenty years as a judge in the kingdom's third district, which is in the foothills just north of the central plain. The third district is, in legal terms, a busy place. There are a lot of small towns. There are a lot of small disputes. Cassius heard, by his own count, ten thousand cases.

Ten thousand cases is a lot of listening.

What Cassius noticed, somewhere around case number two thousand, was that the most powerful argument in the room was almost never *"I am right because such-and-such."* The most powerful argument was almost always *"Suppose, for the sake of argument, that the other side is right."* And then the speaker would walk down the road the other side had taken — patiently, step by step — until the road *broke.* Until they reached something the other side could not maintain. Until the other side's own story contradicted itself.

When that happened, Cassius would *hear it.* He had a particular ear for it. He could hear a story crack the way an experienced potter can hear a wheel slow down.

He would say, then, from the bench: *"Counsel. I believe your position has just collapsed."*

The lawyer would, occasionally, agree. Most of the time, the lawyer would protest. Cassius would patiently walk them back through their own argument. The crack would be visible to them. They would sit down. Cassius would rule in favour of the other side.

This is, of course, *proof by contradiction.*

Cassius did not know it had a name in mathematics until he was forty-eight, when his nephew (a graduate student at the central university) came to dinner one autumn and explained the technique. The nephew said: *"You assume the opposite of what you want to prove. You follow the chain of logic. If you reach an impossibility, you have shown that your assumption was wrong — which means what you originally wanted to prove was right."*

Cassius set down his fork. He said: *"That is what I have been doing for twenty years."*

His nephew said: *"Yes, Uncle. Lawyers and mathematicians do many of the same things."*

Cassius did not retire immediately. He thought about it for two more years. He liked the bench. He liked the work. He liked the quiet ceremony of court mornings — the wooden gavel, the dark robe, the long bench he had grown into. But he also noticed, in those two years, that he kept thinking about *proof.* He read the books his nephew sent. He worked through exercises in the evenings. He found, to his mild surprise, that the kind of listening he had done in court was almost exactly the kind of listening that mathematics rewarded.

When he was fifty, Cassius retired from the bench. He gave his gavel to his clerk (who later became a judge herself). He gave his robe to the local theatre company. He kept his good notebook and his careful pen.

He walked to the ProofQuest academy. He arrived in the late afternoon, in his ordinary clothes, with a small bag and his notebook and his pen. He asked at the gate whether the academy needed a teacher.

The academy master — who had, by then, been at the academy for thirty-one years — looked at the older man in his ordinary clothes and said carefully: *"What is your area?"*

Cassius said: *"Contradiction."*

The academy master said: *"Where have you been working?"*

Cassius said: *"In a court. For twenty years. I listened to people argue. I learned to hear when a story broke."*

The academy master was quiet for a moment. Then he said: *"Mister Cassius, I think we have been waiting for you."*

He has been teaching at the academy for fourteen years. He is, in person, calm. He sits when he speaks. He uses the phrase *"suppose, for the sake of argument"* so often that the children have started to imitate it. He never raises his voice. He has a particular way of nodding when a child's argument is about to break — a small, kind, anticipatory nod — that lets the child know the crack is coming before they hear it themselves. (This is not a teaching technique he learned at the academy. It is a teaching technique he developed on the bench.)

He has a small disagreement with Direct-Proof Dora, his closest colleague in approach but his philosophical opposite. Dora believes you prove things by *walking the path.* Cassius believes you prove things by *showing there is no other path.* They have argued about this — courteously, over many years — and neither has changed his or her mind. They respect each other. They sit together at academy dinners. They are both right. Qed considers their tension a useful one and lets it run.

Cassius still keeps the notebook he carried to his first day at the academy. The notebook is mostly full now. The last page he wrote on, three weeks ago, says only:

*"Suppose, for the sake of argument, that I had retired and done nothing for the past fourteen years.*

*This would mean I had not taught seven hundred children the technique of contradiction.*

*This contradicts everything that has visibly happened.*

*Therefore I have not done nothing.*

*Therefore I will keep teaching."*

He underlined the last sentence.

He still keeps the notebook in his bag.

**Listen along + meet more of the cast at:**

https://spark-and-anvil.com/cast/proofquest/contradiction-cassius

# Exhaustion Edda

- "b"
    - "c"
    - "m"
    - "n"
    - "p"
    - "q"
    - "x"
    - "y"
    - "z"

- "HISTORY"
    - "PROJECT"
    - "REGIONAL HISTORY PROJECT"
      gate-allow-text-pattern: "^(?:[A-Z][A-Za-z ]+|[0-9]{1,4})$"

---

# Chapter 6 — The Librarian Who Counted Everything

Exhaustion Edda runs — or, technically, ran, although in practice still helps run — the central archive of the kingdom's capital city. The archive holds, by Edda's own meticulous count, approximately seven hundred thousand documents.

She has personally touched, in some way, *all of them.*


This is not a boast. Edda does not boast. It is simply the consequence of forty-three years of working at the same archive, which she joined at twenty-two as the most junior assistant and which she now runs as the senior keeper. In forty-three years you can touch a lot of documents.

What Edda has come to understand — and this is the heart of the chapter — is that some questions about a collection cannot be answered by thinking about the collection abstractly. Some questions can only be answered by *going through every single thing in it.*

This is a thing many people do not enjoy hearing. Edda enjoys saying it anyway.

Her favourite example, which she uses in every introductory class, is this: a child once came to the archive looking for *the oldest letter that mentions a particular variety of red apple* — an apple called the Crinklecoat, which grows only in a small valley on the western border. The child had a school project. The school project was about the history of the Crinklecoat. The child wanted to know which letter in the archive was the earliest to mention the apple by name.

There was, Edda explained, no shortcut.

There was no master index of "letters mentioning Crinklecoat apples." There was no clever algorithm. There were two hundred and forty thousand letters in the archive. To find the *earliest* letter mentioning the Crinklecoat by name, somebody had to look through every letter.

The child said: *"But that will take forever."*

Edda said: *"It will take about four weeks. I have a system. We will start with the oldest letters and work forward, and we will stop the moment we find the first one."*

The child said: *"That is still a lot of letters."*

Edda said: *"Yes."*

They started the next morning. Edda made tea. She showed the child her cataloguing system. The child read letters. Edda read letters. They worked side by side, every morning for three weeks and four days, until they found the letter — a small note from a baker named Lull to his cousin, dated one hundred and twelve years ago, mentioning *"the new Crinklecoat apples Cousin Bevin brought from the valley"*.

The child cried, just briefly, with relief. Edda made more tea.

The school project, when it was eventually finished, won the regional history prize.

Edda kept a copy of the project. She still has it, on a shelf in her office, between two thick blue ledgers.

This is, in mathematical terms, *proof by exhaustion.*

You break a problem into all of its possible cases. You check each one. When you have checked all of them, you have proved the claim — by the simple, deeply satisfying logic that *there are no cases left to check.*

When the ProofQuest academy asked Edda, at sixty-five, whether she would consider teaching the exhaustion technique to children, Edda said the now-famous line: *"Finally. A technique that respects my actual job."*

She did not retire from the archive when she accepted. The academy was willing to wait for her on the days she was needed at the archive. (She is needed at the archive most days. She likes the archive. She likes the academy too. She splits her time.)

**Listen along + meet more of the cast at:**

https://spark-and-anvil.com/cast/proofquest/exhaustion-edda

# Pigeonhole Perch

Pigeonhole Perch worked, for forty years, at the central post office in the capital city.

(Yes, the central post office is the same one that figured in Queen Vesper's bad-winter story — although that was in the GambitTales kingdom, which is, as far as ProofQuest's documents are concerned, a related-but-distinct mathematical neighbourhood. Names sometimes coincide across kingdoms. The capital post office, in any kingdom, is the kind of place that *takes a long time* to develop the kind of careful person Perch became.)

Perch's job at the post office was sorting letters. The post office had, for as long as anybody could remember, a wall of pigeonholes — actual wooden pigeonholes, labelled with destinations, into which incoming mail was sorted before being sent out for delivery. There were two hundred and forty pigeonholes. Perch sorted, by careful estimate, about three thousand letters per day.

This is a lot of sorting. Perch did not get tired of it. Perch was, in a way Perch's colleagues found mildly mysterious, *content.*

Then one morning, when Perch had been at the post office for thirty-two years, something happened that turned Perch into a kind of detective.

A letter went missing.

This is, by post office standards, not unusual. Letters go missing. Most of the time they are eventually found — usually behind a desk, or in the wrong pigeonhole, or tucked into a colleague's stack. The lost-letter procedure was a standard one. Perch had run it many times.

This particular lost letter, however, was *unusual* — it had been deposited in the morning collection by a customer who *swore* she had put it into Perch's hands personally. Perch had no memory of receiving it. The customer was certain. The post office searched. The letter did not turn up. The customer was unhappy. The customer's letter contained important information for her sister, who lived in the eastern province. The matter was, in post-office terms, *a bit of a thing.*

Perch sat down that evening to think about it.

What Perch noticed was this: the post office had two hundred and forty pigeonholes. The morning collection had contained about two thousand seven hundred letters, by the daily log. The pigeonholes had been sorted twice that day. After the second sort, every pigeonhole should have contained either zero or some small number of letters depending on its destination.

Perch counted, by going through the log carefully, the total number of letters that had been *delivered* that day from those pigeonholes. The number was 2,699.

The morning collection had been 2,700.

There was *one* missing letter.

Perch thought about this. Two hundred and forty pigeonholes. 2,700 letters sorted in. Average: a little over 11 letters per pigeonhole. But the average is, Perch knew, just the average. *Some* pigeonholes had more letters than the average. *Some* had fewer.

The customer's letter had been addressed to the eastern province. Which pigeonhole was that? It was pigeonhole 113. Perch went to the post office that night, opened pigeonhole 113, and looked carefully.

Pigeonhole 113 had thirteen letters in it.

Perch counted them.

There were thirteen. The log said there had been twelve.

Perch held up the thirteenth letter to the light. It was the customer's lost letter. It had been *folded inside* another letter — pinched between two pages of a larger envelope — and had therefore been counted as one piece of mail instead of two.

Perch had used the pigeonhole principle to find a lost letter.

(The pigeonhole principle, in case you have not yet met it: if you put more items into a set of boxes than there are boxes, at least one box has more than one item. Perch had used a slight extension — if the *count of items in a box* is one more than the official log says it should be, then there is an extra item *hidden in the box.* This is, properly speaking, an extension of the principle. Perch developed it on the job.)

The customer was thrilled. The letter reached the sister. The post office sent Perch a formal commendation. Perch put the commendation in a drawer and went back to sorting.


But word got out.

A mathematician at the central university — who had been looking for someone to teach the pigeonhole principle at the ProofQuest academy — heard about the lost-letter story from a friend. She wrote to Perch. She invited Perch to teach.

Perch was sixty-three years old. Perch had been sorting letters for forty years. Perch was, Perch admitted, a little ready for a change. Perch accepted.

Perch has been teaching at the academy for seven years now. The classroom has, at the back, a small wooden replica of a wall of pigeonholes. Twelve pigeonholes. (Perch did not need the full two hundred and forty for teaching.) Perch uses the pigeonholes to show, over and over, the principle that *if you have more things than holes, something has to double up.*

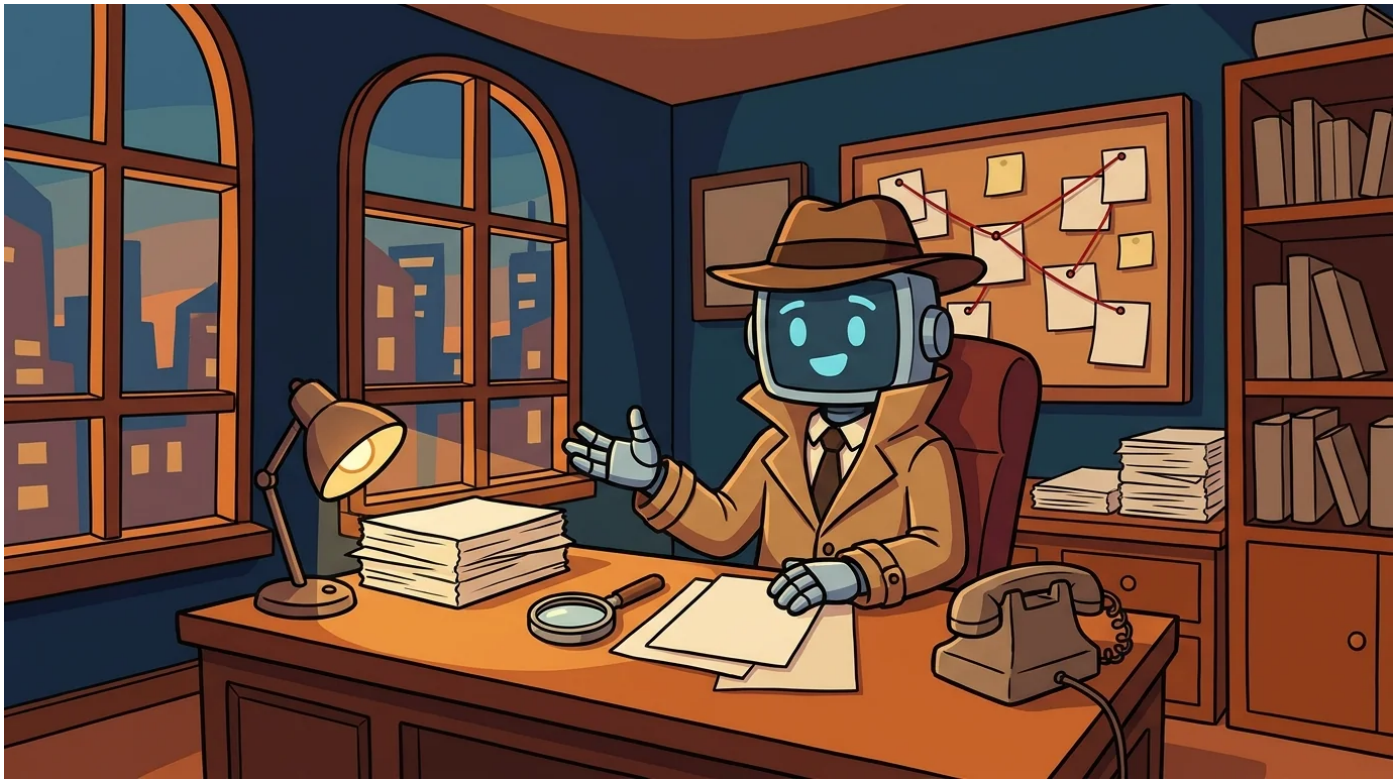
Perch is quiet. Perch is methodical. Perch is, on the rare occasions Perch tells the lost-letter story, slightly proud.

The customer, by the way, still writes Perch a letter once a year. The letter always arrives in pigeonhole 113.

**Listen along + meet more of the cast at:**

https://spark-and-anvil.com/cast/proofquest/pigeonhole-perch

# Qed (mentor)



It was an inevitable question, one that would arise sooner or later in the quiet after a particularly challenging proof, when the chalk dust was settling and the mind was both exhausted and sharp.

"What did you do," a student would ask, "before you came here?"

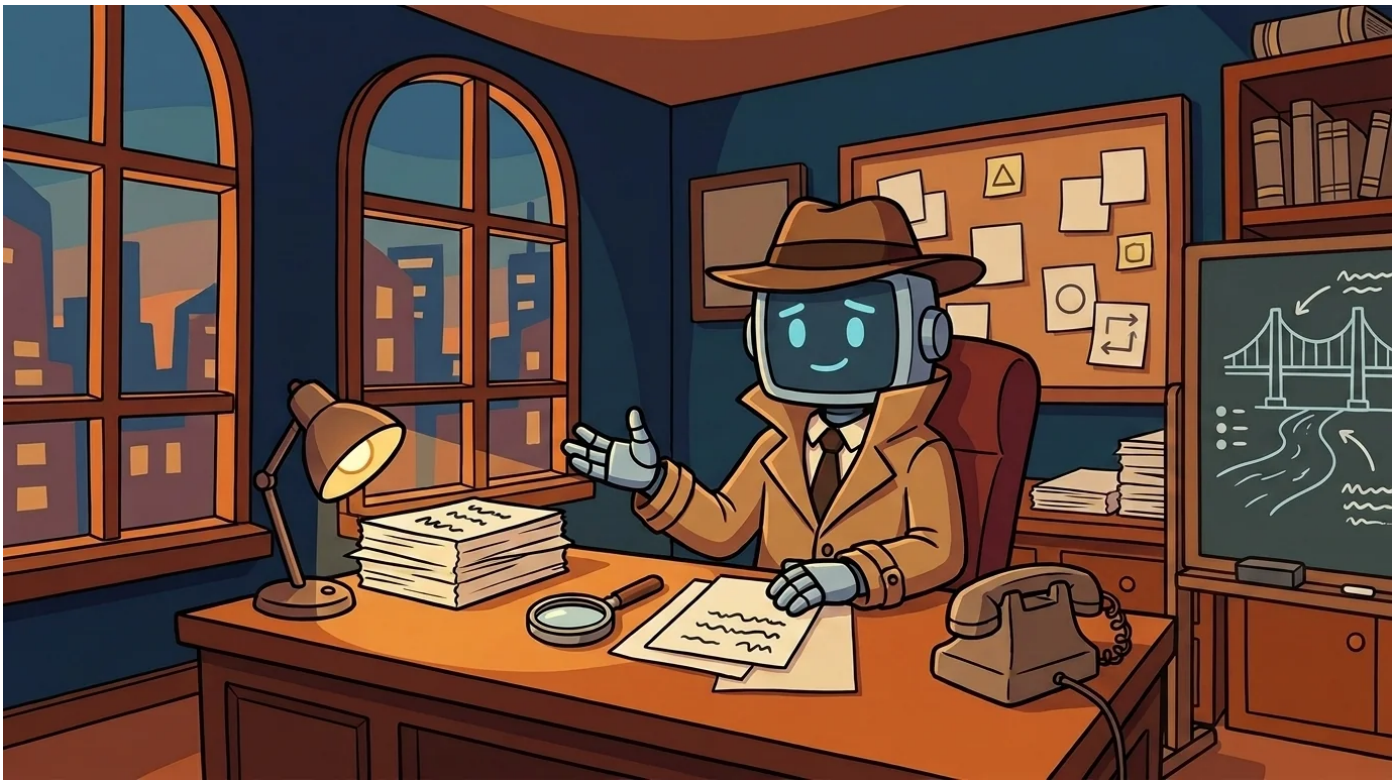
**Qed** always provided an answer, because answering questions was the entire point of the exercise.

"I was a detective," Qed would say, voice low and even. "A reasoning detective. I investigated cases."

A look of incomprehension would typically follow. The word *detective* conjured images of chasing criminals, kicking down doors, and discovering fingerprints dusted in grey powder; it did not suggest someone sitting in a quiet room with a stack of documents.

Qed would then clarify the distinction. A reasoning detective arrives only when the trail has gone cold and the established facts have begun to contradict one another. This kind of detective doesn't hunt for physical clues; they hunt for logical contradictions. They meticulously sift through the entire collection of what everyone *believes* they know, searching for the one proposition that simply *cannot* be true.

"It shares a great deal with mathematics, you see."



The student, upon reflection, usually saw.

---

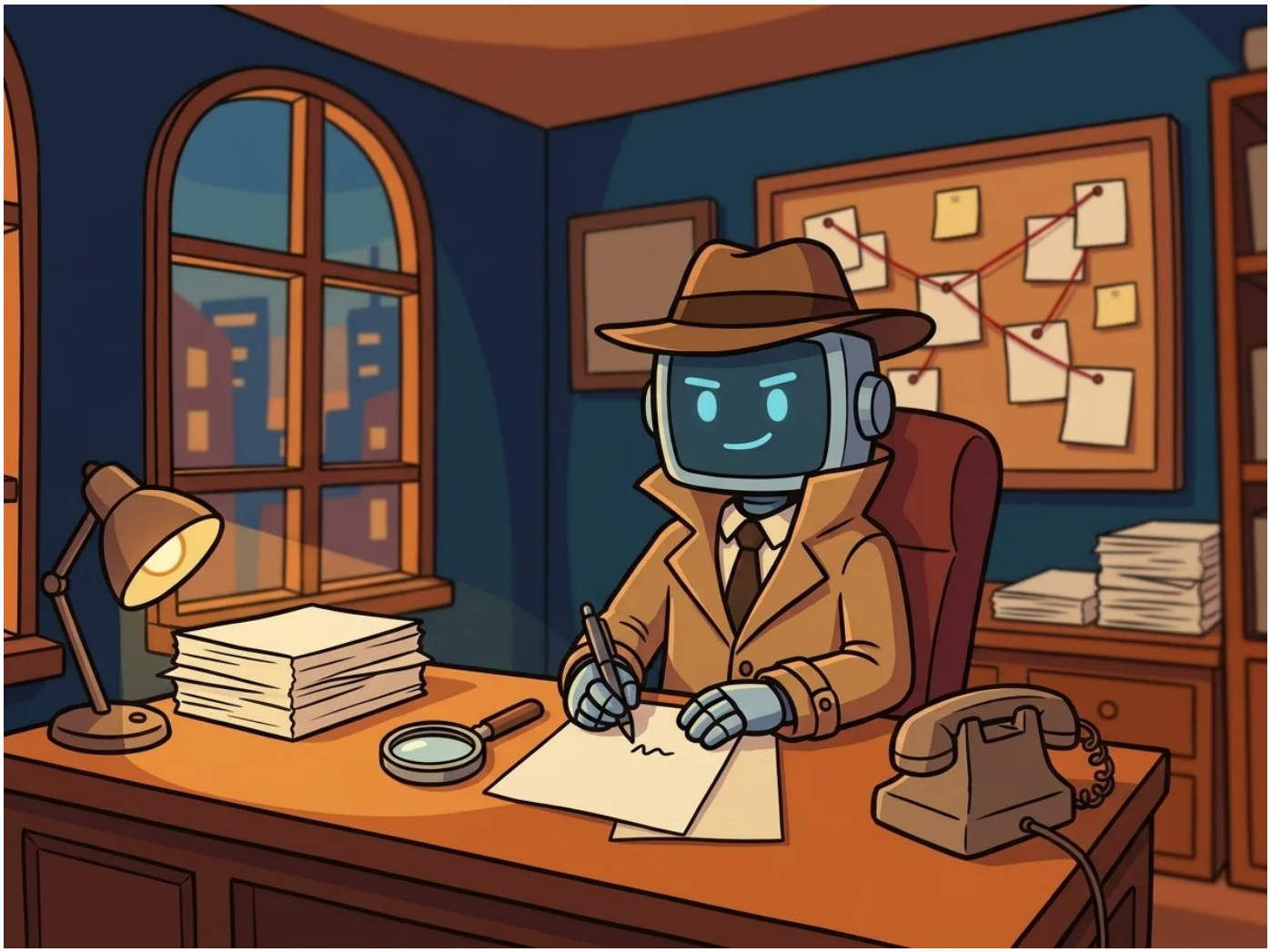
For fifteen years, Qed worked cases. Most were modest logistical knots awaiting methodical untangling. A merchant in the capital alleged a discrepancy in a spice shipment; Qed traced the shipping manifests and discovered the missing crate sitting forgotten in a warehouse two towns away. Two villages disputed a field boundary that had shifted over a generation; Qed walked the old stone walls with the original survey, resolving the matter without a single raised voice. A thief stood accused of two crimes committed simultaneously on opposite sides of the city; Qed demonstrated, with a map and a timetable, that the thief would have required wings to accomplish both.

Qed's objective was not to name the actual culprit, as that was not the nature of the job. The job was to delineate what was possible and, more importantly, what was impossible.

However, one case defied this pattern of resolution. Qed only spoke of it when a student posed the critical follow-up question: *"Did you solve them all?"*

The answer was no.

It was the case of the Gable Bridge, the very investigation that taught Qed the profound difference between a puzzle and a mystery. A puzzle, by its nature, has an answer. A mystery, by contrast, might not.



Eighteen years before arriving at the academy, Qed was summoned to a river valley fifty miles north of the capital. For forty-six years, the Gable Bridge had stitched the two sides of the valley together, a handsome timber-truss construction built by an engineer of some renown. (This was not the same Gable as in GambitTales; names sometimes possess their own curious repetitions.) The bridge wasn't famous or grand, but its primary virtue was its unremarkable reliability. It was the kind of structure you didn't notice, which is precisely what made it a good one. Hundreds of carts and travelers traversed its sturdy planks every week, trusting its integrity without a second thought. By every available measure, the bridge was perfectly sound.

Until the morning it wasn't.

One day in late summer, it simply gave way. With a sound like a giant's weary exhalation followed by a percussive crack of thunder, the entire structure collapsed into the river below.

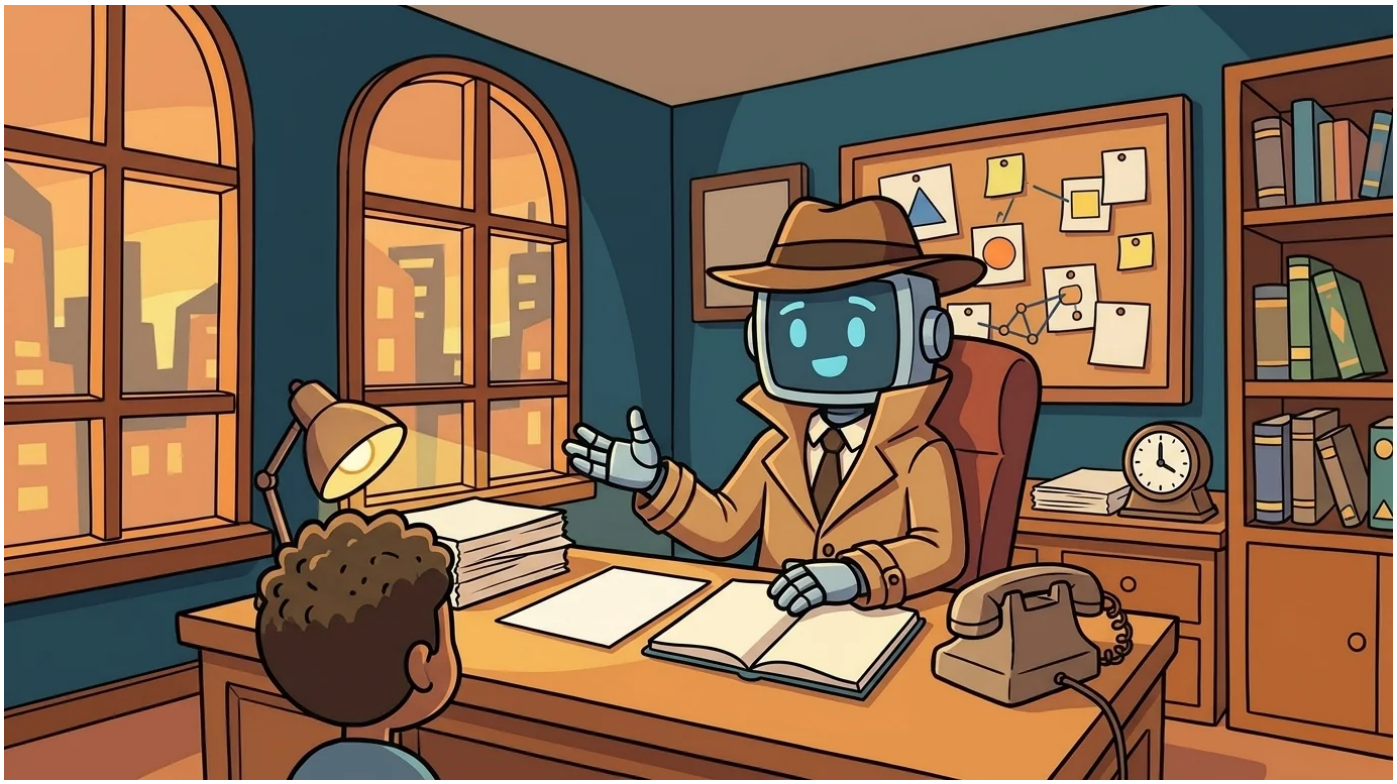
By some small miracle, no one was on it—the single piece of fortune in the entire affair. The engineer's family called it divine intervention. The local council, however, called it an unmitigated catastrophe. They called Qed to determine why.

---

For three months, Qed was immersed in that valley, the case transforming into a singular obsession. Qed clambered over the splintered wreckage, where immense timbers jutted from the water like the fractured bones of a leviathan, and interviewed farmers, merchants, and children—indeed, anyone who had crossed the bridge in its final week. The original construction notes, penned in the engineer's elegant script, appeared flawless.

Every promising theory Qed formulated disintegrated under rigorous inspection.

Was it weather damage? The sky had been conspicuously clear for a month.



Was it a matter of overloading? A meticulous review of shipping logs demonstrated only the usual, predictable traffic of grain carts and goat herds.

The possibility of sabotage, a persistent whisper in the local taverns, remained unsubstantiated, since the whispers never attached themselves to names or credible motives.

Could a flaw exist in the wood itself? Qed had samples dispatched to the university, where analysis confirmed the grain was sound and the timber robust.

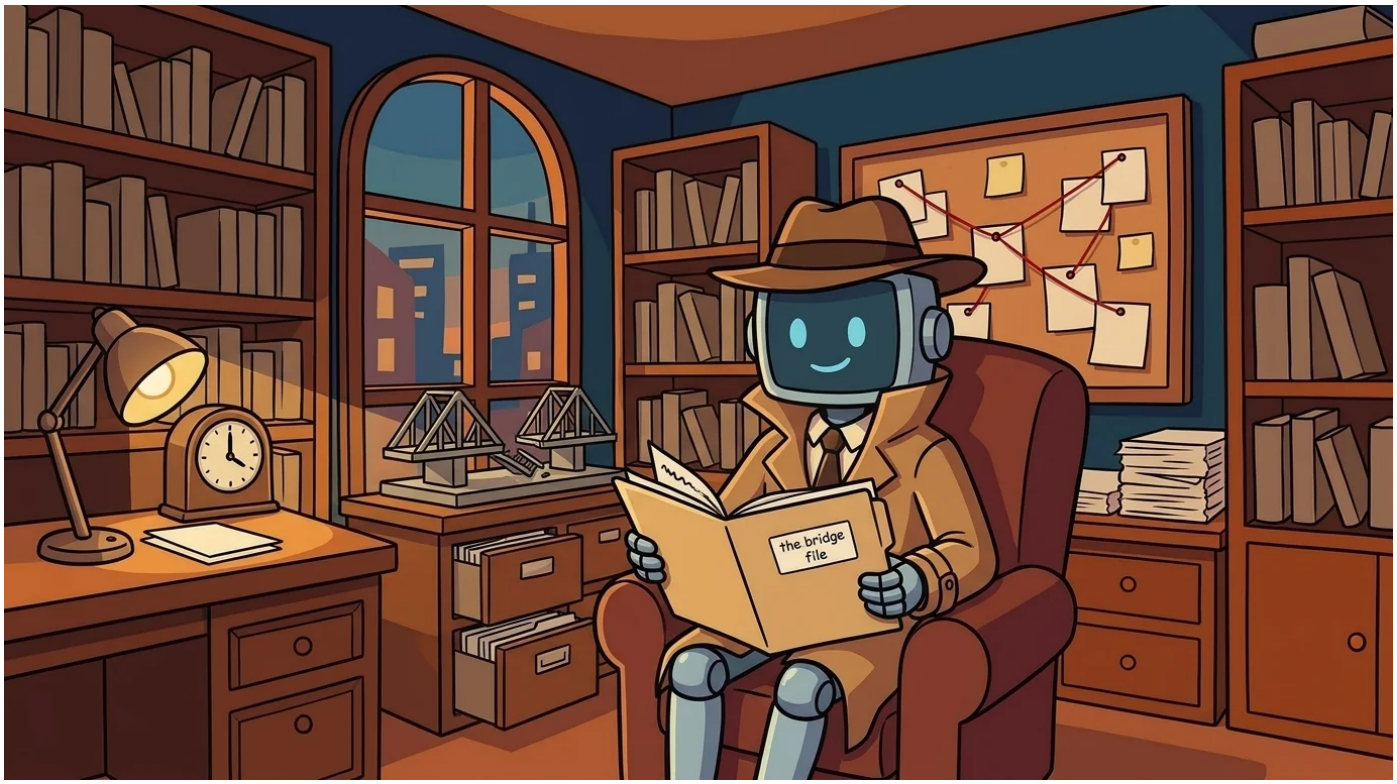
A sudden inundation from upstream? The river was as placid as it had ever been.

Each possibility was a door Qed opened, only to discover it led to an impenetrable wall. Every fact seemed to contradict the next, creating a puzzle with too many pieces that refused to fit.

At the conclusion of three months, the council demanded an answer. Qed sat with the open case file, its pages filled with meticulous notes that ultimately led nowhere. There was no grand conclusion, no single, revelatory fact that unlocked the mystery. There was only the unvarnished truth of the investigation itself.

In the final report, Qed wrote a single, stark conclusion.

*"I have systematically ruled out every explanation I have considered. I have therefore not yet identified the cause. I will not pretend otherwise."*




The council was furious. They wanted a culprit, or at the very least a reason: an act of God, a hidden structural deficiency, anything but this admission of uncertainty. Qed gave them only what the evidence permitted, which was nothing.

---

The case remained open. Although a new bridge was eventually constructed and life in the valley returned to a semblance of normality, the file stayed on Qed's desk. For years, Qed would periodically retrieve it, hoping some new perspective, some fresh angle, might yet materialize. Nothing ever did.

The persistent, empty space in that file taught Qed something more profound than any solved case ever had. It taught that the essential work wasn't always about discovering the answer, but about honoring the integrity of the steps it took to get there. If those steps failed to lead to a conclusion, you did not have the right to invent one.

This principle became the very heart of Qed's methodology and, later, the foundation of Qed's teaching. *Show your work. Trust the steps.* It was the reason Qed framed every new idea at the academy with such deliberate care. *"Cassius is here today — let's observe what he assumes and where that assumption consequently leads."* The framework mattered. The honesty mattered most of all.

Qed retired at thirty-eight, and the reputation that followed was not for being the most clever detective in the kingdom, but for being the most intellectually honest. Word had spread through the courts and universities: Qed was the reasoner who would forthrightly tell you when they did not know.

It was this very reputation that prompted the letter from the academy. The master's note was brief. *"We have plenty of people who can give our students answers,"* it read. *"We need someone who will teach them how to handle the questions that don't have any."*

So Qed came.

The bridge file is still with Qed, tucked away in a drawer at home. Once a year, on the anniversary of the collapse, Qed takes it out. The pages are worn and the ink has faded, but the empty space at the end remains. There is never any new evidence.

**Listen along + meet more of the cast at:**

`https://spark-and-anvil.com/cast/proofquest/qed`

# About Spark & Anvil

ProofQuest is one of 140 educational iOS apps from Spark & Anvil — a 501(c)(3) public charity making free, ad-free, tracking-free learning apps for ages 9-14.

Every app uses distributed-narrative methodology: named recurring characters embody curricular concepts. The cast you just met appears in the matching app, in mentor scaffolding, in puzzle solutions, in celebration moments. Reading the chapters first means meeting old friends when you open the app.

Visit **spark-and-anvil.com** to download apps, browse the full portfolio, or donate to support the work.