MathCircle

Meet the Cast

Advanced Edition

Spark & Anvil

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This advanced edition collects 5 chapter books from the MathCircle cast — each character embodies a different curricular primitive; together they teach the full subject.

Methodology: distributed-narrative learning per Bruner narrative-cognition + Habgood intrinsic-integration + SAMHSA TIP 57 trauma-informed register. Advanced edition: upper-middle-grade register (Wonder / Hatchet / Holes band) for readers ages 11-14 ready for longer sentences + more nuanced subtext.

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##

For everyone who learns by reading between the lines.

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Introduction

The MathCircle cast was authored to embody the curriculum, not decorate around it.
Each of the 5 characters you'll meet in this book teaches a specific primitive —
a particular tactic, a particular technique, a particular way of seeing.
Together they form an ensemble: the cast IS the curriculum.

Read in any order. Each chapter stands alone.
Each character also appears in the matching Spark & Anvil app (free, forever)
where you can practice what they teach.

This is the **Advanced Edition** — written for readers who are ready for longer sentences, layered subtext, and the trust that comes with not having every joke explained. The Standard Edition covers the same characters at a lighter register; pick whichever feels right for the reader at hand.

— *The editors at Spark & Anvil*

Circle Circe

Circle Circe had been studying the art of leaving the room for almost twenty years.

She had not started out good at it. When she was twenty-three and running her first circle — in a community center kitchen in a small town she had moved to that summer — she had stayed in the room the entire ninety minutes. She had asked questions. She had nudged. She had, at one point, written a hint on the chalkboard. The four kids had solved the problem. They had thanked her politely. They had not come back the next week.

She had thought about that for a long time.

The next week, with a different group, she had tried something different. She had introduced the problem. She had told them she was going to step into the kitchen and check on a pot of tea that wasn't actually on the stove. She had walked out. She had stood in the hallway and listened. The four kids had been silent for almost three minutes. Then one of them had said, "Should we just try drawing it?" And another had said, "Sure," and they had started working.

When she had come back twenty minutes later, they had solved the problem.

She had not stayed away long enough yet to understand the difference, but the four kids had come back the next week, and the week after that, and for three years until they aged out of the circle. She had learned, slowly, what the difference was.

The difference was that she had not been in the room.

Tama, who was thirteen now and had been running circles at her own kitchen table for almost two years, had asked Circle Circe about this once.

It had been near the end of a circle, after the four kids — Joon and Mira and Bex and Tama — had solved a problem about handshakes that had taken them most of an hour. The other three had gone home. Tama was alone at the table. Circle Circe was still on the screen, the little timer that usually counted her absence now showing zero.

"Circle," Tama said. "Can I ask you something?"

"Mm."

"How did you learn to go quiet for so long?"

Circle Circe considered. She did not answer right away. She had taught herself, long ago, that whenever a kid asked her a question about how to run a circle, she should treat that question with the seriousness she would give an adult.

"I learned it by ruining a circle when I was younger than you are now," she said. "Well — when I was a little older. Twenty-three. My first one. I stayed in the room the whole time. The kids solved the problem. They never came back. I had to figure out why."

"What was the why?"

"I had stayed in the room. The room wasn't theirs. It was mine. They had solved my problem under my supervision in my room. That's not a circle. That's a tutoring session with extra steps. Kids don't come back to tutoring sessions for fun."

"So you started leaving."

"I started leaving. At first I left for short times. Then longer. Then I figured out how to leave without leaving — how to fade to a dim outline, the way I do now. The fading is the same as leaving. It just means I can come back faster if you need me."

Tama considered.

"Do you ever wish you could just tell us the answer?"

Circle Circe smiled — small, real, very deliberate.

"Sometimes. When the answer is sitting right there and one of you is one good question away from seeing it and the room is going quiet in a frustrated way — yes. Sometimes I want to just tell you. But the wanting passes. The wanting is the part of me that hasn't learned to leave. The wanting is the twenty-three-year-old version of me. She visits sometimes. I let her speak. I do not let her run the circle."

Tama was quiet for a long time.

"I'm thirteen," she said finally. "I want to run circles when I'm older."

"You already do. Today's circle was yours. I was the screen. You were the structure."

"You were on the screen."

"I was a dim outline on the screen. That's not the same as being in the room. The room was yours."

The thing Circle Circe had learned, after almost twenty years, was that the deepest skill in running a math circle was the skill of identifying the moment when a kid needed her to come back.

There were moments when the circle was working well — when the four kids were talking, drawing, arguing, building on each other — and at those moments her job was to stay faded. There were moments when the circle was working slowly — when there was a long silence, when one kid was struggling but trying, when nobody was speaking but everyone was thinking — and at those moments her job was also to stay faded, because the slowness was the work, and the silence was the work, and her returning would only interrupt the work.

But there were also moments when the circle was breaking — when one kid had gone genuinely silent and was withdrawing, when an argument had become unkind, when the group had drifted into a wrong direction with no internal way back, when somebody was crying — and at those moments her job was to come back, gently, and reset.

The hardest part of her job was knowing which moment was which.

She had not always known. She had returned too quickly many times. She had stayed away too long many times. She had once, when she was thirty, come back into a circle where two kids had been quietly mean to a third for almost ten minutes, and the third kid had been holding back tears, and Circle Circe had not noticed because she had been listening for the wrong signals.

She had thought about that for a long time, too.

She had, after that circle, written down a small list of signals she had not been listening for, and she had taught herself to listen for them. The list had grown over the years. She kept it on a card in her desk drawer at home. The card was almost full now.

She had told only one person about the card — the woman who had trained her, back when she was twenty-two. That woman was retired now. She had told Circle Circe, when she had heard about the card: "Good. That's the job. Keep growing the card."

Circle Circe was still growing the card.

She had not yet told Tama about the card. She would, someday. When Tama was older, when Tama had run a circle that broke in a way she did not catch in time, Circle Circe would tell her about the card, and they would compare notes.

That was the lineage.

That was what circles were.

Tama left the kitchen. The cake on the counter was a memory from two years ago now — there had been other cakes, other parties, other circles — but the kitchen was still the same kitchen, and the light coming through the window was the same light, and the iPad was the same iPad.

Circle Circe stayed on the screen for a few seconds after Tama left. Then she faded all the way out, not to a dim outline this time, but completely.

Off-screen, she added a line to her card.

The line said: *Listen for the kid who runs the circle. Sometimes she also needs the wait-time you give the others.*

She closed the drawer.

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https://spark-and-anvil.com/cast/mathcircle/circle-circe

Circle and Echo

The snow had been falling for almost an hour, and at the round wooden table near the bay window of the math circle's old library room, two friends were trying very hard to feed seven yetis.

The problem in front of them was simple enough to state and impossible enough to solve cleanly that it had occupied them, in companionable silence, for the better part of fifteen minutes. A drawing of a lopsided nonagon — nine-sided, vaguely cake-shaped — sat on the table between them. Above it, in Circle Circe's careful, unhurried handwriting, the conditions: *Seven yetis. One cake. If any yeti receives even a crumb more than another, a yodeling contest will commence, and an avalanche will follow.*

Circe was the kind of mathematician who liked to begin by understanding what she was looking at. She had drawn the nonagon herself, very carefully, on a fresh sheet of paper. She had a private theory that messy diagrams produced messy thinking, and that one of the small disciplines a person could cultivate, if she wanted to think well, was the discipline of redrawing the problem before attempting it.

Echo Edie, sitting across from her, was a different kind of mathematician. Edie did not redraw things. Edie listened. She had a habit of leaning slightly forward when the other person was speaking, of humming a small, distracted-sounding hum that meant she was concentrating, and of saying — at intervals that struck most people as both irritating and necessary — *So what you're saying is...*

She said it now.

"So what you're saying is," Edie murmured, although Circe had not yet said anything, "this is a nonagon. Nine sides. For seven yetis. And the yetis are very serious about fairness."

"I have not said any of that out loud," Circe pointed out.

"I am restating your drawing," Edie said.

Circe considered this and decided it was fair.

The first thing Circe did, after a moment's quiet, was the natural thing. She drew nine straight lines from the center of the nonagon out to each of the nine corners, like the slices of a pizza, and laid down her pencil with the small satisfaction of a person who has produced a clean idea.

"The simplest cut," she said. "From the center, out to the corners. Nine identical wedges. We give one to each of seven yetis. We have two left."

She looked up at Edie, expecting agreement. Two leftover wedges was not, on the face of it, a disaster. It was a partial solution, and partial solutions were the kind of thing she liked. They had structure. They could be repaired.

But Edie was already humming.

"So what you're saying is," Edie said, slowly, "we slice it like a pizza. Nine clean wedges. Each yeti takes one wedge, which solves the easy part. And the hard part, the part we have to come back to, is the two wedges nobody has eaten yet."

"Yes."

"And the two wedges we have left over each need to be divided into seven exactly equal portions. So that nobody — not the yeti who got their wedge first, not the yeti who only gets a little crumb from the leftover wedges — receives even a crumb more than anyone else."

"Yes."

Edie looked at the drawing for a long time. "I want to say something about that," she said, "but I want to say it carefully, because I don't want you to think I am dismissing your idea. Your idea is the right place to start. I am restating it so I am sure I understand what you're proposing before I try to add to it."

"I am paying attention," Circe said.

"What I want to say is: the moment we start dividing those two leftover wedges into seven equal pieces, the geometry stops being friendly. A wedge is a triangle. Cutting a triangle into seven exactly equal pieces is — geometrically possible, but the kind of possible that takes a ruler and a protractor and a lot of trust."

"That is true."

"So I'm not sure my objection is that your idea is wrong," Edie said. "My objection is that your idea solves the easy part of the problem and pushes the hard part of the problem one step further down the road. Which is a normal thing for an idea to do. It just means we need to ask whether there is a different first move that doesn't push the hard part down the road."

Circe was quiet for several seconds. She was not offended. She was, in a way she had been learning to recognize over the past two years of these afternoons, grateful. There was a thing Edie did, that Circe's own mind did not naturally do, which was to receive an idea fully — completely — and then ask, *is this the version of the problem we want to keep solving?* Circe's instinct was to refine an idea until it worked. Edie's instinct was to ask whether the idea had pointed them at the right shape of question.

"All right," Circe said. "What if we don't start from the center?"

Edie picked up her own pencil, which was a perfectly ordinary, somewhat blunt pencil that had a small bite mark on the eraser end.

"So what you're saying is," she began — and Circe smiled, because she had begun to find this small ritual genuinely useful — "we should look at a different first move. Something that doesn't leave us with leftover wedges that nobody knows how to share."

"Yes."

"All right." Edie tapped her pencil against the drawing. "We have a nonagon. Nine sides. We have seven yetis. Nine and seven are coprime, which is a polite way of saying they have nothing in common. There's no factor we can pull out to make them play nicely. So instead of starting with the nine, what if we start by giving ourselves a seven?"

She drew, on the same paper, a small but firm pair of lines. Two of the nonagon's nine vertices were now sliced off. The remaining shape — Circe traced it with her eyes — was, in fact, a heptagon. Seven-sided. Seven was the number of yetis.

"If we trim two corners off," Edie said, "the inner shape is a heptagon. And a heptagon, sliced from its center, gives us seven equal wedges. Seven yetis, seven wedges, one each. Clean."

Circe looked at the new diagram. It was a beautiful idea. Beautiful in the way Edie's ideas often were — not surgical, but generous. *Reshape the problem before you solve it.* Circe had once read, in a footnote to a translation of Euler, that the great mathematicians distinguish themselves not by solving the hardest problems but by selecting the right form of those problems.

She was about to say so.

Then she saw the two trimmed corners, sitting in the corner of the drawing like discarded triangle-shaped offcuts, and she felt a small, familiar dissonance.

"Echo," she said quietly. "I want to restate what you just proposed before I add to it. Because you've done me the courtesy of restating my ideas, and I want to do the same."

"All right."

"What you're proposing," Circe said, "is that we change the geometry of the cake itself. We turn it from a nonagon into a heptagon by cutting off two of its corners. Inside the heptagon we make seven clean wedges, one for each yeti, and that solves the hard part of the original problem — the part where nine wedges can't be divided cleanly among seven yetis."

"Yes."

"The thing your idea does not yet solve," Circe said, gently, "is what happens to the two corners we trimmed off."

Edie put her pencil down. "Ah."

"The yetis will know."

"The yetis can smell wasted cake from a mile away."

"They can. So we are now in the same kind of situation as before. Your first move solved the easy part of the problem and pushed a different hard part one step down the road. The hard part is now: how do we divide two triangle-shaped offcuts equally among seven yetis."

"Which is just as hard as dividing two wedge-shaped offcuts among seven yetis."

"Yes."

The two friends sat with this for a moment, looking at each other across the table with the particular companionable resignation of people who have spent enough afternoons together to know that being stuck is a normal stage of the work and not a moral failure.

"All right," Edie said. "I think you're going to tell me that what we actually want is a different kind of move."

"I think what we want," Circe said slowly, "is a move that doesn't generate leftovers. A move where the very act of cutting the cake produces exactly the right number of equal pieces, with no offcuts."

"What you're saying is — instead of starting from the center, or trimming the outside, we look for a third way."

"Yes."

Circe stared at the nonagon for a long time. Edie watched her stare. Circe had a way of looking at things that was not quite the same as looking. She was, Edie had decided some months ago, a person whose face went very still when her mind was working hardest, which was disconcerting if you did not know her and informative if you did.

"What if we draw a smaller nonagon inside the big one," Circe said.

"Restate that for me."

"What if I take the nonagon," Circe said, "and inside it — concentric, with the same nine vertices radiating out to the same nine corners — I draw a smaller nonagon. So the cake is now made of two pieces: a small central nonagon, and around it, a ring made of nine identical trapezoids."

She drew it as she said it. The nine trapezoids ringed the inner nonagon like the petals of a stiff, geometric flower.

Edie hummed. "So what you're saying is — nine identical pieces around the outside, and one small whole nonagon in the middle."

"Yes."

"And we have seven yetis."

"So each yeti receives one trapezoid. That uses seven of the nine outer pieces. We are left with two outer trapezoids and one inner nonagon."

"Which is still leftovers."

"It is. But it is leftovers of a different *shape*."

Edie's eyes narrowed. She leaned forward over the drawing. "Say that part again."

Circe smiled. "It is leftovers of a different shape. The original leftovers were wedges — triangles converging at a point. The trimmed-corner leftovers were triangles at the corners. Both of those shapes are *hard to divide into seven equal pieces.* But trapezoids are not hard to divide into equal slices. You cut them into long, parallel strips. And the inner nonagon — a small nine-sided shape — is itself a cake we already know how to attack. Or," Circe paused, "a cake we can hand back to ourselves as a smaller version of the same problem."

Edie sat back, slowly, and put both hands flat on the table.

"Circle," she said.

"Yes."

"That's a recursive solution."

"It is."

"The hard part of the problem is now smaller. It's the same shape as the original problem, but smaller."

"Yes. We have made the nonagon-for-seven-yetis problem into a smaller-nonagon-for-seven-yetis problem, plus two trapezoids that any kid could divide into seven equal strips with a ruler. And the inner nonagon — we can attack it the same way, again, until the leftover cake is small enough to ignore. Or we just divide it into seven strips, since by that point it will be small enough that strip-cutting works fine."

Outside the bay window, the snow had stopped. The afternoon light was low and gold and falling sideways across the wooden table, and the drawing in front of them was a tidy, almost beautiful nest of nested nonagons, ringed with trapezoids, each one indistinguishable from its neighbors.

Edie picked up her pencil one more time. "So what you're saying is," she said, very softly, almost to herself, "good listening — yours, mine — is what made this possible. If I had not restated your first idea, you would have spent the afternoon trying to slice triangles into seven crumb-piles. If you had not restated my second idea, I would have spent the afternoon trying to share two corner-triangles. Each restating slowed us down by maybe thirty seconds. And it saved us, between us, about two hours."

Circle Circe looked at the drawing. The seven yetis were going to eat. The avalanche was not going to happen. The cake was going to be fair.

"Good listening," she said.

"Good echoes," Edie replied.

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Echo Edie

Echo Edie had not always been a good listener.

When she was twelve, she had been a fast talker. She had been the kid in every class who put her hand up first and answered first and moved on to the next thought before anyone else had finished forming a sentence. She had been, by every measure her seventh-grade teacher could find, a very smart kid. Her teacher had said as much, often, to Edie's parents. Smart, smart, smart, very smart, possibly a genius, sometimes a little hard to keep up with, but smart.

What nobody had told Edie, at age twelve, was that being fast and being smart were different things.

She had figured this out, painfully, in eighth grade.

Eighth grade was the year Edie's best friend, a quiet girl named Jo who had moved into the neighborhood the summer before, had stopped wanting to be Edie's best friend.

Jo had not said this directly. Jo had just become harder to find. She had stopped coming over after school. She had stopped sitting with Edie at lunch. When Edie had finally cornered her in the hallway and demanded to know what was going on, Jo had said, in a voice that was small but very clear: "Edie, you never let me finish a sentence."

Edie had been stunned.

"What?"

"You always finish my sentences. You always answer my questions before I'm done asking them. You always assume you know what I'm going to say. You're usually right. But — I want to finish my own sentences sometimes."

Edie had said: "I — but — I'm fast — "

Jo had said: "I know. I am tired of how fast you are."

She had walked away.

Edie had cried in the bathroom for a long time.

Two months later, after Edie had spent the entire intervening time miserable and trying, with mixed success, to slow herself down enough to let Jo finish her sentences — they had become friends again, slowly — Edie had had a conversation with her grandfather.

Her grandfather had been a slow man. He had spoken slowly. He had walked slowly. He had eaten slowly. He had answered questions only after he had finished considering them, which sometimes took thirty seconds, and which had driven young-Edie crazy.

But she had gone to him that summer because she trusted him. She had told him about Jo.

Her grandfather had listened. He had listened for nearly fifteen minutes. He had not interrupted once. He had not finished any of her sentences. He had just listened.

When she was finally done, he had thought about it for a long time.

Then he had said: "Edie. I want you to try something. I want you to spend the next month restating, out loud, the last thing someone said to you, before you respond. Not paraphrase. Restate. Use as many of their words as you can remember. Then respond."

"That sounds annoying."

"It will be. For everyone. Including you. Do it anyway."

"Why?"

"Because right now you are listening for your own next thought. You are not listening for the other person. The restating is going to slow you down enough that you can hear what they actually said. Not what you thought they were about to say. What they said."

Edie had been quiet.

"How long until I can stop?"

"You'll know."

She had tried it.

For the first week, it had been awful. She had felt slow. She had felt stupid. She had felt like everyone was looking at her oddly. They were looking at her oddly. People asked her once or twice whether she was feeling all right.

For the second week, she had noticed something. The people she was talking to — Jo especially, but also her parents, also her younger brother, also her teachers — had started talking longer. They had said more. They had told her things they had not told her before.

She had been listening differently.

They had noticed.

For the third week, she had begun to enjoy the restating. The act of repeating someone's exact words made her actually hold the words in her head, not just dart past them. It was, she realized, a small kind of attention.

By the fourth week, she had not needed the restating as a practice. She had absorbed it. She had become, slowly, a person who could listen.

She had been twelve. She had not understood, then, what her grandfather had taught her.

She understood now.

Mira was fourteen when Echo Edie told her this story.

The two of them had been alone after a circle. The other three kids had gone home. Mira had asked Edie why she did what she did — why she restated each kid's idea, why she was so careful to use the kid's exact words, why she always asked the quietest kid first what they had said.

Edie had told her about Jo. She had told her about the bathroom. She had told her about her grandfather.

Mira had listened. She had not interrupted. She had — Edie noticed, with a small private satisfaction — restated what Edie had said before responding.

"So you became Echo Edie," Mira said, "because Jo told you that you never let her finish a sentence."

"Yes. And because my grandfather gave me the practice that let me actually hear her."

"And now your job is to make sure that doesn't happen in our circles."

"Yes."

"Do you remember Jo?"

Edie had been quiet for a moment.

"Yes," she had said. "Jo is my oldest friend. We're both in our forties now. She still doesn't let me finish her sentences. She makes me earn the finishing. I am grateful."

Mira had smiled.

"Edie," she had said. "Can I tell you something?"

"Yes."

"I used to interrupt my little brother all the time. I stopped doing it last year. I think because of you."

Echo Edie had looked at Mira for a long moment.

"Tell me what you noticed about your brother," she had said.

"That when I let him finish, he was funnier than I thought. And smarter. And he had things he had been waiting to say that he hadn't said because I'd been talking over him."

"Tell him about Jo someday."

"Why?"

"Because the practice has to be passed on. Otherwise it stops with me, and that's not enough. Jo passed it to me. My grandfather passed it to me. I'm passing it to you. You pass it to your brother. He'll pass it to someone else. That's how listening becomes a tradition instead of a personal trick."

Mira had nodded.

"I will."

Edie had faded.

Mira had sat at the kitchen table for a long time afterward, thinking.

The kids in the circles never quite understood, watching Echo Edie work, how rare what she was doing was.

Restating someone's idea, in their own words, before responding, was not a thing most adults could do. Most adults rushed. Most adults filled. Most adults darted past one idea to get to the next. The kids in the circles, by virtue of seeing it done well over and over for a year, by virtue of being asked, gently, to do it themselves — slowly absorbed the practice.

By the time they were Mira's age, the kids in Echo Edie's circles had a noticeably easier time in group discussions in school. They could let other kids finish sentences. They could repeat what someone had said before agreeing or disagreeing. They could catch a first idea that was about to get lost and hold it up for the room.

The teachers at the school began to notice.

Some of them asked: where did these kids learn this?

The answer was: a small woman on an iPad screen, who had been twelve once, who had lost her best friend for two months, who had spent a summer with her grandfather, who had practiced restating sentences until restating became listening, who had grown up to make sure no kid in any of her circles ever felt the way Jo had felt.

The answer was: Echo Edie.

The answer was: a tradition, passed kid to kid, generation to generation, one restate at a time.

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Patty Patient

Patty Patient had been counting silence professionally for thirty-one years.

She had not started counting it on purpose. She had been twenty-two, fresh out of a teacher-training program that had taught her many things about math but very few things about kids, when she had walked into her first classroom and discovered that her single largest professional fear was the moment between asking a question and getting an answer.

The silence between had felt, to her, like a small accusation. The silence had felt like the kids were judging her for asking a bad question. The silence had felt like she had failed at her job. So she had developed, in those first weeks of teaching, a habit she would later spend two decades undoing — the habit of answering her own question, fast, when the silence stretched longer than three seconds.

She had not realized she was doing it.

The kids had not realized either.

The kids had, however, stopped trying to answer.

Why bother? Patty would answer for them. Patty would always answer for them. The silence would never be allowed to stretch. The work that happens in silence — the searching, the hesitating, the rehearsing of an idea before saying it out loud — that work never had room to happen.

By the end of her first year of teaching, Patty was exhausted. The kids were not learning much. Patty thought she was a bad teacher. Patty thought she might quit.

That was when she had met Mr. Sandford.

Mr. Sandford had been the librarian at her school. He had not been a math teacher. He had been a small soft-spoken man with brown vests and a fondness for index cards, and he had noticed, from across the hallway over the course of a semester, that the young math teacher in the room next to his library was filling every silence she encountered the way a person fills a leaky boat with bailing.

He had invited her to tea.

Patty, who had been twenty-three and tired and unsure how to be a teacher, had gone.

Mr. Sandford had poured the tea. He had said nothing. He had let the silence sit.

Patty had filled it. She had told him about her year. She had told him she thought she was bad at teaching. She had told him she didn't know what she was doing wrong. She had talked for nearly twenty minutes without stopping.

Mr. Sandford had listened. He had nodded once. He had said: "You filled that silence."

Patty had blinked.

"Yes?"

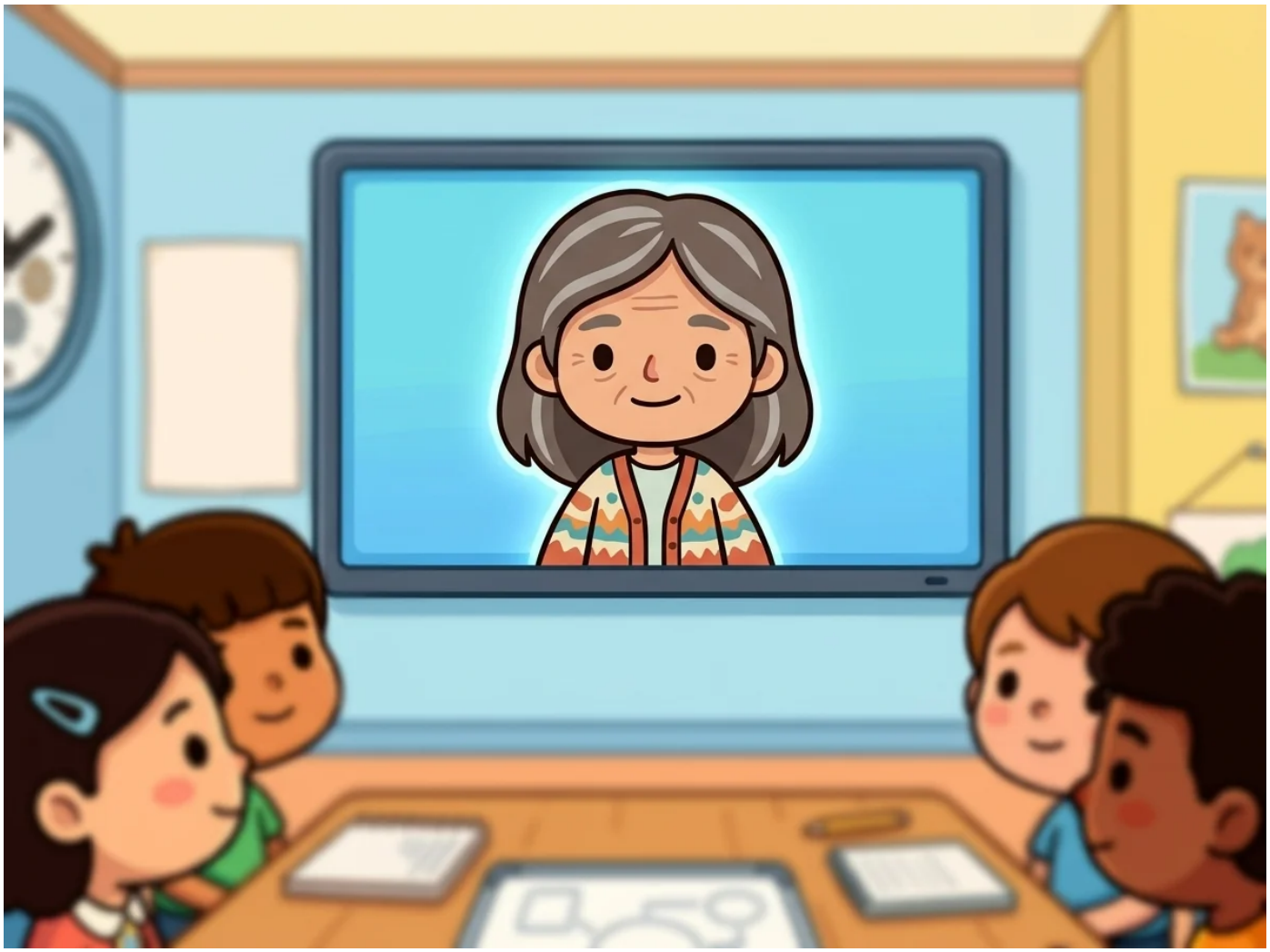
"You did the same thing you do in your classroom. You felt silence. You filled it. The filling was your way of being useful. The filling was what felt like teaching."

"But — "

"In your classroom, the silence belongs to the kids. They need it. They need to think. They need to rehearse an idea before they say it. They need to be wrong inside their own heads for a few seconds before they say it out loud. If you fill their silence, you take that from them."

Patty had been very quiet.

Mr. Sandford had nodded.



"Now I am going to do something cruel," he had said. "I am going to ask you a question, and I am going to say nothing afterward, no matter how long it takes you to answer. The question is: when you teach math, what do you actually want the kids to do?"

He had asked the question.

Patty had sat in the silence for almost three minutes.

She had wanted to fill it. She had wanted to babble. She had wanted to apologize for not knowing. She had resisted all of these urges.

Finally she had said: "I want them to think."

Mr. Sandford had smiled.

"There's your job," he had said. "Now go practice silence."

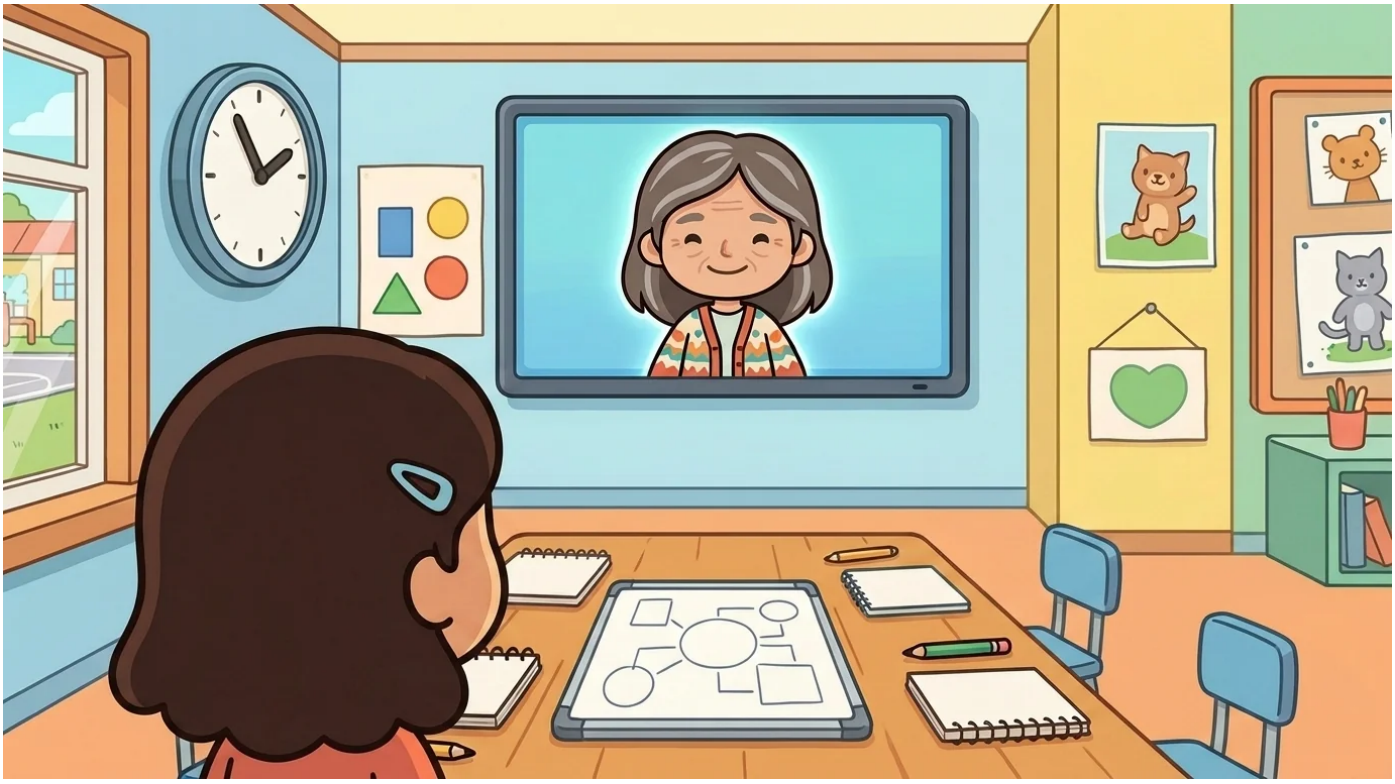
She had practiced for thirty-one years.

She had practiced it by counting, deliberately, in her head, when she asked a kid a question in class. She had counted to ten before saying anything else. Sometimes the kid had answered before she got to ten. Sometimes the kid had not. Sometimes she had had to go from ten to twenty, and the silence had felt like agony, but the kid had eventually answered, and the answer had been better than any answer she could have given for them.

She had practiced it by writing the counts down. She had written, on small index cards she stole the format of from Mr. Sandford: *Today's longest tolerated silence: 28 seconds.* The cards had filled a drawer. Then two drawers. Then a small wooden box.

She had taught the practice to her students. She had taught it explicitly. She had stood in front of her seventh-graders and said: "There is going to be a silence today after I ask you a question. The silence is part of the question. The silence is for you. Do not feel bad about the silence. Do not feel like you have to break it. Use it."

The students had not understood at first. They had eventually understood. The ones who had taken the practice seriously had become noticeably better at thinking through hard problems than the ones who had not.



Patty had retired from full-time teaching at fifty-three. She had taken the wooden box of cards with her.

When the math-circle people had asked her, three years later, to be one of the four characters in a new pass-and-play app — a character whose job was to make wait-time safe for kids — she had said yes immediately. She had not even waited. She had said yes the way a person says yes to something they have been waiting their whole life to be asked to do.

Mira was fifteen when she asked Patty Patient about the box of cards.

She had heard about it from Circle Circe, who had told her on a rainy day at the end of a long conversation about cards and lineages and the things the math-circle teachers had learned by failing.

Mira had next encountered Patty at the end of a circle that had been long and quiet and good. The other three kids had gone home. Mira had sat at the kitchen table with the iPad. Patty had reappeared.

"Patty," Mira had said. "Tell me about the wooden box."

Patty had smiled — slowly, deliberately, the smile of a person who had been waiting to be asked.

"It is full now," she had said. "It is on the shelf above my desk. I do not write new cards anymore."

"Why?"

"Because I count silences automatically now. I do not have to consciously choose to wait. The wait has become how I listen. The cards were the training wheels. The bike is riding itself now."

"Is that the goal?"

"Yes. The goal of the cards was to not need the cards. The goal of being Patty Patient is to not need to be Patty Patient. The goal of the circle is — eventually — to not need any of us."

"That sounds sad."

"It isn't. It's the work succeeding. The circle's success is its own retirement."

Mira had been quiet for a long time.

She had thought of the box. She had thought of her own slowly accumulating notebook of circles — every circle she had run, every silence she had counted, every kid who had said something useful only after she had let the silence run.

"I think I'm going to keep a notebook," Mira had said.

"Good. I started with cards because that's what Mr. Sandford used. You can use whatever you want. The medium doesn't matter. The practice does."

Patty had paused.

"Also," she had added. "Tell the next kid who asks you about the silence: it's a habit wearing a face. Eventually the face goes away. The habit stays."

Mira had nodded.

Patty had faded.

The kitchen had been quiet for a long time.

It had been a good silence.

Listen along + meet more of the cast at:

https://spark-and-anvil.com/cast/mathcircle/patty-patient

Tortoise Hare

Tortoise Hare had been arguing with himself since he was nine years old.

He had not always been one creature. He had started as two friends — a tortoise named Tort and a hare named Har — who had grown up together in a small town near a river, who had gone to the same school, who had finished the same problems on the same chalkboard in opposite ways, and who had argued, daily, about which way was better.

They had loved each other.

They had also annoyed each other constantly.

In their twenties, after they had each finished a different kind of schooling and ended up working at the same math circle in the same community center — a deeply unlikely coincidence that they took as a sign — they had realized something. Or rather, the woman who ran the circle had pointed it out to them. The woman had said, after watching them disagree for the eighteenth time about how to introduce a particular problem about counting paths on a grid: "You two should be one character. Your arguing is the lesson. Stop pretending you're two separate teachers."

Tort and Har had looked at each other.

They had thought about it.

They had argued about it — Tort wanting to think it over slowly, Har wanting to commit immediately — and then they had joined.

The joining had not been physical. They were still two animals. They lived in the same body now, somehow, the way some creatures who love each other learn to share a single shape. They had a long shell and long ears and an unusual way of looking at you with one eye that was thinking slowly and one eye that was thinking fast.

That had been twenty-five years ago.

They had been Tortoise Hare ever since.

Mira had asked Tortoise Hare once, when she was thirteen and had been running circles at her own kitchen table for a year, whether the arguing ever got tiring.

He had considered the question for a long time. Both voices had considered it.

The Tortoise voice had said: "Sometimes. When we're tired, the arguing feels like a chore. When we're tired, Har wants to just take over and finish the problem alone, and I want to just lie down and let her. We resist both impulses. The resistance is the work."

The Hare voice had said: "Always. The arguing is exhausting. I want to be done with it. I want to just go fast. But Tort holds me back, and that's why we're still useful. Without him I would have burned out in my twenties. Without me he would have fallen asleep in his."

"Do you ever wish you were one voice?"

The Tortoise voice had paused.

The Hare voice had said: "Yes."

The Tortoise voice had said: "No."

They had laughed at each other.

"It depends on the day," they had said together, in both voices at once. "Most days we are glad to be both. Some days we wish we were one. We never wish to be the OTHER one specifically. We just wish, occasionally, for less arguing."

Mira had thought about that for a long time.

She had thought about it on the days when she and her friend Joon had argued through a whole circle about whether to draw a diagram or just count, and she had thought about it on the days when she had argued with herself about the same thing. She had begun, after a few weeks of thinking, to notice when her own internal argument was happening. She had begun to give the two voices in her own head different names. Not Tortoise and Hare — those names were taken. She used her own.

She used Slow-Mira and Fast-Mira.

Slow-Mira drew the diagram. Fast-Mira counted. Slow-Mira read the problem twice before starting. Fast-Mira started before reading the problem all the way through. Slow-Mira checked her answer. Fast-Mira moved on.

Neither was right. Neither was wrong. Both were useful.

When Mira told Tortoise Hare about Slow-Mira and Fast-Mira, he smiled in both voices at once.

"That's how it starts," he said. "First you notice you have two voices. Then you let them argue. Then you start trusting both. Then — and this is the part nobody warns you about — they join. Not all the way. Just enough that you stop having to consciously hold the argument. The argument becomes part of how you think."

"Did that happen to you?"

"Some days. Not every day. We never fully joined. We don't think we want to. The arguing is too useful."

There was one circle Mira would remember for a long time.

It was a circle with three kids she had not worked with before, and they were doing a problem about a chessboard and knights. The problem was hard. The four of them had spent forty minutes on it without making progress, and the room had gotten quiet in a frustrated way, and Mira had been about to suggest they take a break when Tortoise Hare had reappeared on the iPad.

He had been quiet for a moment. Both voices.

Then the Hare voice had said: "Quit."

Then the Tortoise voice had said: "Don't quit."

Mira and the three kids had stared at the screen.

"Quit," the Hare voice said again. "You've been working for forty minutes. You're tired. You can come back to this tomorrow. Quitting now is the smart move."

"Don't quit," the Tortoise voice said. "You're three minutes from a breakthrough. You can feel it. The breakthrough is in the next attempt. Stopping now would be a waste of the forty minutes you've already spent."

The four kids looked at each other.

"Which one of you is right?" one of the kids asked.

"We don't know," both voices said. "We have been arguing about this exact thing for thirty-five years. Sometimes the Hare is right. Sometimes the Tortoise is right. The only person who can decide is you. Your circle, your call."

The four kids talked it over. They were tired. They could feel it.

But Mira said, quietly: "Let's try one more attempt. Then we stop, whether it works or not."

The other three agreed.

The one more attempt was the breakthrough.

The knights problem cracked open. The four of them stared at the answer.

When Tortoise Hare reappeared a few minutes later, he said, in both voices: "The Tortoise was right today."

The Hare voice added, a little ruefully: "Today."

The Tortoise voice added, generously: "Last week the Hare was right."

They had argued for another minute. The four kids had watched. The four kids had grinned.

In Mira's last conversation with Tortoise Hare before she aged out of running circles for younger kids — she was fifteen, she was about to start high school, she had been running circles at the kitchen table and at school and at the community center for almost three years — she said: "Do you remember the knights problem?"

"Of course."

"You were right that day. Tortoise was right."

"Yes."

"What if you'd been wrong?"

The Tortoise voice considered. The Hare voice considered.

Both voices said: "We would have learned. We have been wrong many times. Each wrongness has gone on the card."

"What card?"

The Tortoise voice smiled — quietly, slowly. The Hare voice smiled — quickly, brightly. Together they said: "Maybe Circle Circe will tell you about the card someday. We are not the right ones to tell you. But yes — we keep a card too. Or rather, two cards. One for the Tortoise. One for the Hare. They live in the same drawer."

Mira nodded.

"I keep a card too," she said. "Slow-Mira's card. And Fast-Mira's."

Tortoise Hare laughed in both voices at once.

"That's the job," he said. "Now you know the lineage."

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https://spark-and-anvil.com/cast/mathcircle/tortoise-hare

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MathCircle is one of 140 educational iOS apps from Spark & Anvil — a 501(c)(3) public charity making free, ad-free, tracking-free learning apps for ages 9-14.

Every app uses distributed-narrative methodology: named recurring characters embody curricular concepts. The cast you just met appears in the matching app, in mentor scaffolding, in puzzle solutions, in celebration moments. Reading the chapters first means meeting old friends when you open the app.

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