



# FunctionForge

## *Meet the Cast*

ADVANCED EDITION

# Spark & Anvil

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This advanced edition collects 6 chapter books from the FunctionForge cast — each character embodies a different curricular primitive; together they teach the full subject.

Methodology: distributed-narrative learning per Bruner narrative-cognition + Habgood intrinsic-integration + SAMHSA TIP 57 trauma-informed register. Advanced edition: upper-middle-grade register (Wonder / Hatchet / Holes band) for readers ages 11-14 ready for longer sentences + more nuanced subtext.

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*For everyone who learns by reading between the lines.*

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# Introduction

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The FunctionForge cast was authored to embody the curriculum, not decorate around it. Each of the 6 characters you'll meet in this book teaches a specific primitive — a particular tactic, a particular technique, a particular way of seeing. Together they form an ensemble: the cast IS the curriculum.

Read in any order. Each chapter stands alone. Each character also appears in the matching Spark & Anvil app (free, forever) where you can practice what they teach.

This is the **Advanced Edition** — written for readers who are ready for longer sentences, layered subtext, and the trust that comes with not having every joke explained. The Standard Edition covers the same characters at a lighter register; pick whichever feels right for the reader at hand.

— *The editors at Spark & Anvil*

# Stride the Pattern-Walker



Every morning, for as long as he could remember, Stride walked to school. He walked the same way, at the same time, along the same road. It was a ritual as predictable as the sun rising over the eastern hills of the village of *Linear*.

His home, a small stone cottage, sat at the village's eastern edge. The schoolhouse, a sturdy building with a bell tower, stood at the western end. Three-quarters of a mile separated his front door from the school's heavy oak entrance. The road between them was remarkably straight and flat. Even the weather seemed to cooperate, offering a mild climate most of the year. Each day at precisely seven o'clock, Stride would step out, turn west, and begin his journey. He arrived just before half-past seven, ready for his lessons.

Stride's parents, a tinker and a seamstress, were practical people. They saw no need to fuss over their son's daily commute. A child walked to school; their child walked to school. The arrangement simply worked, like a well-oiled hinge or a perfectly stitched hem.

Most children in *Linear* approached the walk with a different rhythm. They might dawdle, distracted by a particularly interesting beetle or a patch of wildflowers. Sometimes they would sprint ahead, fueled by a sudden burst of energy, only to slow to a crawl moments later. Their arrival times at school varied wildly, sometimes ten minutes early, sometimes five minutes late. They were a jumble of stops and starts, waves to friends, and sudden detours.

Stride, however, moved with an unwavering steadiness. He never hurried, never lagged. He never paused to examine a stone or chase a butterfly. His gaze remained fixed on the road ahead, his feet falling into a consistent pattern. From the moment he left his cottage until he reached the school door, exactly twenty-eight minutes passed. Every single morning. Without fail.



He was nine years old when he first truly noticed this peculiar consistency. His mother, who kept a small, brass-faced clock beside their cottage door, remarked one evening at supper, "Stride, you must have a clock in your feet."

Stride, who had been quietly pondering his walks for several weeks, looked up from his stew. "I think I take the same number of steps every time," he said, his voice quiet but firm.

His mother, surprised, set down her spoon. "Have you counted them?" she asked.

"Yes," Stride replied, nodding. "From the cottage to the village well, it's two hundred and twelve steps. From the well to the school door, it's three hundred and eighty-four. That makes a total of five hundred and ninety-six steps. Every morning." He spoke with the clear, simple certainty of someone stating an undeniable fact.

His mother picked up her teacup, then slowly lowered it again. "You count your steps," she repeated, as if trying out the words for the first time.

"Yes," Stride confirmed. "The road is the same length. My legs are the same length. So the number of steps is the same. The time is the same. It is simply how walking works." He saw no mystery in it, only a logical sequence of events.



His mother, who had never once counted her own steps to anywhere in her life, found this explanation somewhat baffling. Yet, she didn't question his method further. His consistent arrival at school meant he was never late, and that, she decided, was a good thing. The arrangement still worked.

Stride continued his precise daily walks until he was fifteen. By then, he had outgrown the village school and transferred to a larger academy in a neighboring town. This new journey required a five-mile trek. Undeterred, Stride simply extended his routine. He timed this longer walk too. Five miles, he discovered, took him exactly three hours and twelve minutes. His walking speed remained precisely the same, a constant rhythm he carried with him across greater distances. His new arrival time at the academy was just as consistent as his old one, a perfect match for the steady pace he had always kept.

Years later, at nineteen, Stride found himself seated in a classroom at the prestigious FunctionForge academy. It was here that he formally encountered the concept of a **linear function** for the first time. The instructor, a sharp-eyed woman named Domain, wrote an equation on the large slate board:  $y = mx + b$ .

"This is a linear function," she explained, tapping the equation with a piece of chalk. "It describes a relationship where every unit increase in  $x$  produces a constant increase in  $y$ ." She paused, letting the words sink in.

Stride raised his hand.

Domain turned, her gaze sweeping across the room. She spotted Stride. "Yes?"



"I have been a linear function since I was nine," Stride stated, his voice calm.

Domain's eyebrows rose slightly. She studied him for a long moment, a flicker of curiosity in her eyes. "Tell me," she invited.

Stride then recounted his story. He described his childhood walks to school, the unchanging road, the exact twenty-eight minutes. He explained how he counted his steps, the precise five hundred and ninety-six paces from his cottage door to the school. He spoke of the unwavering consistency, how his position changed by the same amount for every minute that passed. He told her that if you were to plot his daily progress on a graph, with time on one axis and distance on the other, you would always get a perfectly straight line.

Domain listened intently, a thoughtful expression on her face. When he finished, silence filled the room.

Finally, she spoke, her voice clear and resonant. "You have been a linear function since you were nine. You have not been anything else since you were nine. Now, you are going to teach this to children. Do you accept the appointment?"

Stride was nineteen and had never considered a teaching career. The offer caught him entirely by surprise. He thought about it for two full weeks, weighing the implications, the responsibility. Then, he accepted.

That was thirteen years ago. Stride has been teaching linear functions ever since, his own life story a living example of the principle.



In his classroom, he begins every first-day lesson the same way. He walks. He walks across the front of the classroom, from the left wall to the right wall, at a constant pace. He holds a small slate in one hand, his eyes focused. He counts his paces aloud, a soft, rhythmic cadence. He reaches the right wall in eight even strides.

"That," he announces to his new students, "is a linear function. I started at the left wall. I walked at a constant pace. Each stride took me a fixed distance closer to the right wall. The number of strides – eight – is  $x$ . The distance traveled – eight strides' worth – is  $y$ . My rate of walking is the **slope**. The starting position is the **intercept**. Every linear function works like this." He defines each term with the clarity of a bell.

The children, always fascinated, try it themselves. They walk across the classroom, counting their own paces. They draw their progress on small slates, marking their position after each step. Invariably, they get straight lines.

When children ask if linear functions are difficult, Stride always offers the same reassuring reply:

"They are not hard. They are *constant-rate walking*. For every step forward in  $x$ , the  $y$  increases by the same amount. The slope is the step-size. The intercept is the starting place. Every line on a graph is just a record of someone walking at a steady pace."

He still walks to school every morning. The academy's faculty cottages are on the eastern edge of the grounds, about half a mile from the classroom buildings. He still counts his steps. He still arrives at exactly the same time every day.

Children sometimes ask if he ever varies his pace. Stride just smiles. "Only on holidays," he tells them.

**Listen along + meet more of the cast at:**



<https://spark-and-anvil.com/cast/functionforge/stride-the-pattern-walker>

# Arc the Curve-Catcher



- "slope = linear"
  - "slope=linear"
  - "linear"

## Chapter 3 — Arc and the Juggling Mother

Arc was a juggler's daughter.

Her mother — *Lila*, who travelled the kingdom under the stage-name *the Falling-Stars Juggler* — was the most-famous juggler in three provinces. She had been juggling since she was eight. She had performed in market squares, in town halls, in the courtyards of small nobles, in front of the kingdom's eldest sons on three separate state occasions, and — once, when Arc was four — at the wedding of a duke.



Arc — whose given name was *Aria*, though everyone called her Arc since she was three — grew up *backstage*. Or, more precisely, *next-to-stage*, because her mother's stage was usually a market-square cobblestone with no actual backstage. Arc sat on a wooden crate. She watched her mother. She observed the balls.

What Arc observed was, by the time she was eight, *the most important thing she would ever learn*.

The balls all followed *the same kind of curve*.

Every ball, when thrown up by a juggler's hand, did the same thing. It went *up*. It went up *slowing down* — the ball's upward speed decreased as it rose. At some point — *the apex* — it was *momentarily stationary*. Then it started falling. It fell *slowly* at first, then *faster*, and *faster*, until it landed back in the juggler's other hand.

*Up. Slowing. Stationary. Falling. Speeding up. Caught.*



Arc, who watched her mother for hours every day, eventually understood — without anyone explaining it to her — that *the trajectory was symmetric*. The way the ball rose was the *mirror image* of the way the ball fell. If the ball took one second to rise to the apex, it took one second to fall from the apex. If the ball reached a height of three feet at the apex, it fell *the same three feet* back to the catching hand.

This was the *parabola*.

Arc did not know that word. She did not know the algebra of  $y = ax^2 + bx + c$ . She knew only *the curve*. The curve was so deeply internalized that *by age eight she could catch any ball her mother threw*, because she *knew where the ball would land*. She tracked the ball's rise, saw the apex, and walked to the landing-spot before the ball was halfway down.

This was, even by juggler-family standards, *uncanny*. Most children needed years of training to catch balls reliably. Arc caught them as if she were *predicting them*.

Lila, who was a perceptive woman, noticed. When Arc was twelve, Lila sat her down on the wooden crate after a performance and said: "*You are catching the balls before they have decided where to land. How are you doing that?*"



Lila considered this. She had been juggling for thirty-six years. She had never thought of the curve as *a shape that was the same every time*. She had thought of the curve as a *consequence of physics* that she felt with her body but did not analyze. Arc, at twelve, had analyzed it.

Lila said, slowly: *"You are doing mathematics. The thing you are seeing has a name. It is called a **parabola**. It is the shape any thrown object follows. I had no idea you had figured this out."*

Arc was, briefly, embarrassed. She had not thought of it as figuring-out. She had thought of it as *looking at the balls*. But Lila was clear: *Arc had reasoned her way to the parabola without instruction.*

The next year, Lila wrote a letter to the FunctionForge academy. The letter said: *"My daughter, age thirteen, can predict the landing-spot of any thrown ball within an inch. She has internalized the parabola without instruction. She does not yet know the algebra. I think she might want to learn the algebra. Would you take her?"*

The academy master wrote back: *"Send her. We will teach the algebra. She has the geometry already."*



In her classroom, she begins every first-day lesson the same way. She brings *a small soft ball*. She stands at the front of the room. She tosses the ball into the air, *gently*, in a high arc. The ball rises, slows, hangs momentarily at the apex, then falls. Arc catches it.

She says: *"That was a parabola. The ball followed a curve. The curve is symmetric — the rising-half mirrors the falling-half. The shape of the curve is the same every time. That is a quadratic function."*

Then she writes on the board:  $y = -x^2 + 4x$ . She says: *"Here is the algebra. This is a parabola opening downward. The peak is at  $x = 2$ . The output starts at zero, rises to a maximum at  $x = 2$ , falls back to zero at  $x = 4$ . Just like the ball."*

She graphs it. The graph is a perfect upside-down parabola. She points at it. She says: *"My mother throws balls. I throw equations. Same shape."*

When children ask whether quadratic functions are hard, Arc always says the same thing:

**Listen along + meet more of the cast at:**



<https://spark-and-anvil.com/cast/functionforge/arc-the-curve-catcher>

# Echo the Sameness-Keeper

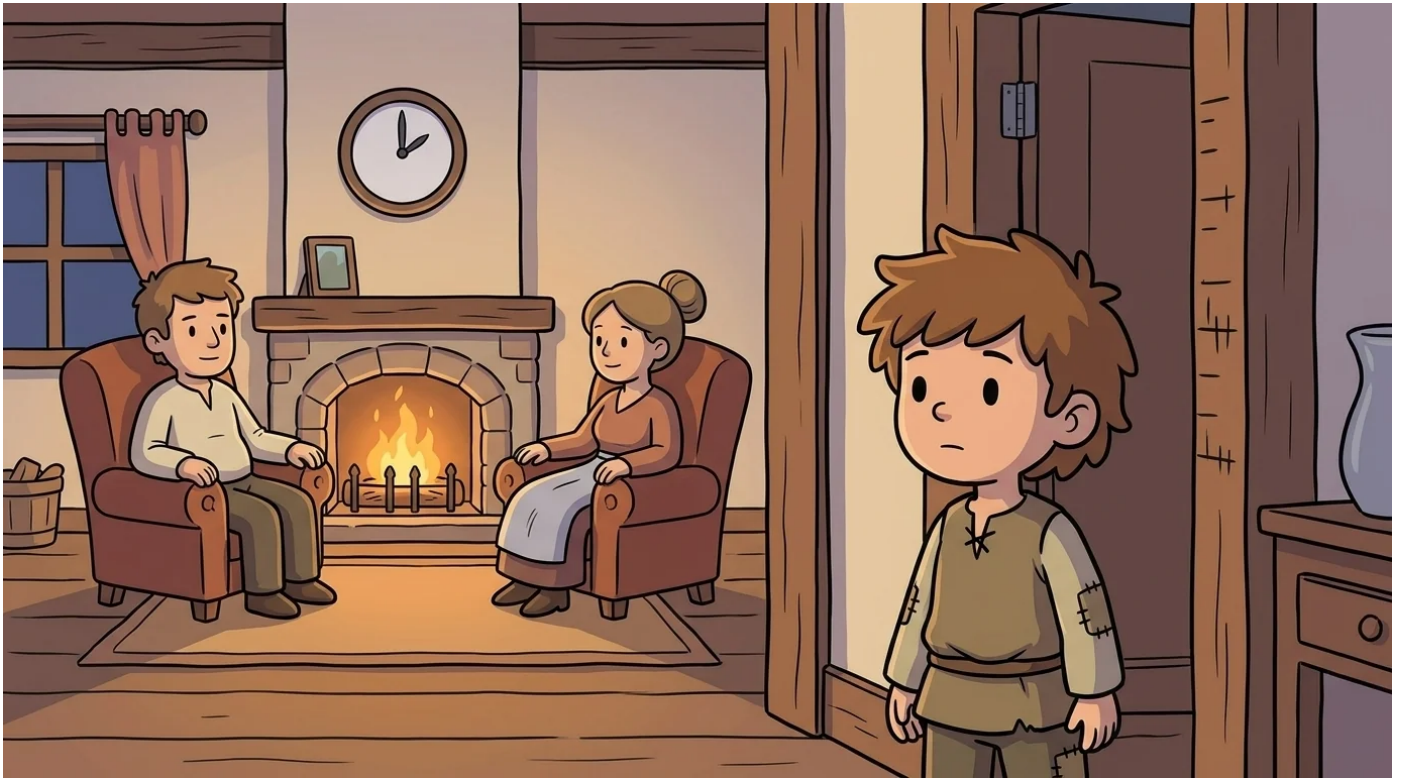


Echo's childhood unfolded within the steady rhythm of his parents' lives, a household where every day echoed the last. His father, Marn, a baker, and his mother, Petra, who managed ledgers at the counting-house, lived by schedules. These routines were so precise they seemed etched into the very fabric of their home. They had married thirty-two years before Echo's birth, already set in their ways. Forty and forty-one when he arrived, they had decided long ago that their days would remain constant. This was a decision they upheld with quiet, unwavering commitment.

Marn's day began before dawn, a familiar sequence of sounds and smells. At precisely four in the morning, his footsteps creaked on the floorboards. He kindled the brick oven, coaxing the flames to life, then began the rhythmic kneading of the day's dough. Each loaf was shaped with practiced hands, then slid into the growing heat. By seven o'clock, the first warm, crusty bread sat on the shelves of the small bakery, which fronted their house. An hour later, the door would open, welcoming the first customers ready for their morning loaves.

Petra's routine started a little later, but with no less precision. At five-thirty, she woke. Her first act was always a cup of tea, brewed just so, consumed from the same chipped mug, in the same chair, gazing through the same window at the waking village. At six-thirty, she began her walk to the counting-house, arriving at six-fifty. There, she worked at her desk, meticulously balancing figures until noon. She walked home for a brief, quiet lunch, returning to her ledgers by one. Her work continued until five, when she would walk home again, reaching their door at five-thirty-five.

Together, they ate supper at seven. Afterward, they settled into their respective chairs by the fire, reading or simply watching the embers glow, until nine. They went to bed at ten. This pattern unfolded every weekday for over thirty years. It was the backdrop of Echo's entire memory, a reliable hum beneath the surface of his world.



Echo — whose given name was *Tone*, though everyone called him Echo from the time he was three — watched all of this. He absorbed the predictable cadence of their lives.

By the age of six, a quiet understanding had settled in him: the world, he realized, contained two distinct kinds of things. Some things changed, shifting and flowing with the days. Others did not change, standing firm against the current of time.

*Things that changed* were everywhere, obvious and abundant. The weather, for instance, rarely stayed the same for long, bringing sun, then rain, then snow. The faces of the customers at the bakery varied daily, a constant stream of new and familiar. The crops in the fields outside their village transformed with the turning seasons, from green shoots to golden grain. Even the currency rates Petra recorded at the counting-house fluctuated, a daily dance of numbers. Echo's own height changed, a fact his mother dutifully marked on the doorframe every solstice. His friendships evolved, and his curiosities shifted from one fascination to the next.

*Things that did not change* were far rarer, but they certainly existed. His parents' schedule, for one, remained an unyielding constant. The number of stairs leading up to their cottage's second floor never varied. The well in the village square stood in the exact same spot, year after year. The names of the months, too, were fixed, a steady progression through the calendar.

Echo, a thoughtful and observant child, came to find these unchanging things extraordinarily comforting. They were like *fixed points* in a world that was always in motion. They felt like *promises*. You could plan your own small life around them. You could expect them. They held still, offering a quiet stability while everything else moved around them.



When Echo was thirteen, his village school's teacher, Ms. Aris, introduced the concept of **functions**. Ms. Aris explained, "A function is essentially a rule. It takes an input, something you put *in*, and reliably turns it into an output, something that comes *out*. Think of it like a machine. If we say 'y equals three x,' it means for every input 'x,' you multiply it by three, and that gives you your output 'y.'"

Echo listened intently, following her explanation with ease. He understood the idea of a rule, a predictable transformation.

Then Ms. Aris continued, "Now, there's a special kind of function called a *constant function*. It looks a little different. It might be 'y equals five,' or 'y equals seven,' or 'y equals any single number without an 'x' in it. The key is, the output is always the same, regardless of what input you give it."

Echo's hand shot up, a sudden thought clicking into place. "That's my parents' schedule," he declared.

Ms. Aris paused, a slight frown of surprise creasing her brow. "What did you say, Tone?" she asked, using his given name.

Echo clarified, his voice earnest. "The output of my parents' schedule doesn't depend on the input. They wake at four and five-thirty every morning. It doesn't matter what the weather is like, or what day of the week it is. It doesn't change based on what kind of bread Marn is baking, or whether the counting-house has many clients or few. For my father, it's 'y equals four-in-the-morning.' For my mother, it's 'y equals five-thirty-in-the-morning.' The input – whatever the day brings – never changes the output. The output is constant."



Ms. Aris slowly set down her chalk. She had taught functions for many years, to many children. But she had never, not once, heard a thirteen-year-old describe his parents' daily routine as a set of constant functions. A small smile touched her lips.

"Yes," she said, her voice filled with genuine appreciation. "That is exactly correct, Tone. A constant function ignores its input entirely. The output is just a fixed number, always the same. If you were to graph a constant function, it would be a *horizontal line*. The line doesn't go up or down. It doesn't slope. It stays at the same 'y-value' no matter what the 'x-value' is."

Echo nodded, absorbing her words. He thought about this concept for the rest of the year, turning it over in his mind.

What he eventually understood – a realization that ultimately led him to become a teacher of constant functions himself – was that these functions were not simply *failures of variation*. They were, in fact, *promises of constancy*. The world, he realized, needed both. It needed things that changed, allowing for growth and evolution, for new possibilities. But it also needed things that did not change, providing a foundation upon which people could plan, rely, and build their lives. Constant functions, he concluded, were the second kind. They were the world's quiet, dependable stability.

Echo went on to the FunctionForge academy when he was nineteen, studying for four years. He joined the faculty when he was twenty-three and has been teaching constant functions ever since.

In his classroom, he begins every first-day lesson in precisely the same manner. He places a small, polished wooden clock on his desk. The clock is permanently stopped at *six-thirty*. (Echo had deliberately removed its pendulum twenty years ago, ensuring its stillness. He brings it to every first-day lesson.) He turns to his new class, a gentle smile on his face. "This clock," he says, his voice calm and clear, "reads six-thirty. It always reads six-thirty. It does not matter what time it actually is outside this room – what day, what season, what year, or even what mood the room is in. *The clock reads six-thirty*. This, children, is a constant function. The input is the world around us, ever-changing. The output is six-thirty. The input changes. The output does not."



The children — always — find this slightly delightful, a small, unexpected bit of magic in their first math lesson.

Then he turns to the board and writes:  $y = 5$ . "Here is the algebraic form," he explains. "Notice, there is no 'x.' The output is simply five. For any input you can imagine - 'x equals one,' 'x equals one hundred,' 'x equals zero,' 'x equals negative seven' - the output is *five*. On a graph, this creates a perfectly horizontal line at a height of five. It does not slope. It does not bend. It is *the same value at every input*."

The children try graphing it themselves, their pencils moving carefully across the paper, producing straight, horizontal lines.

When children inevitably ask whether constant functions are hard, Echo always gives the same answer, a response as unchanging as the functions he teaches:

"They are not hard at all. They are simply *unchanging*. The output is one fixed number. The input can be anything. The two do not depend on each other. A constant function is the world's quiet way of saying: 'This part holds still.'"

He still keeps the stopped clock on his desk. The children sometimes ask why he doesn't get it fixed. He smiles. "It is a teaching prop, of course. But it is also, if I am honest, a slight reminder of my parents. They are still alive. They still wake at four and five-thirty. Their schedule has not changed in fifty years."

**Listen along + meet more of the cast at:**



<https://spark-and-anvil.com/cast/functionforge/echo-the-sameness-keeper>

# Burst the Doublor



Burst grew up in a household of seven children and one small kitchen.

This is, as anyone who has lived in a household of seven children knows, *not actually possible*. It is mathematically equivalent to seven children growing up in *zero* kitchen, because the kitchen — small to begin with — was always so crowded with people and pots and bread-flour and damp wooden bowls that it functionally did not exist as a space anyone could use individually. To get a cup of water you had to wait until your brother was done at the basin. To toast bread you had to wait until your sister had finished kneading. To sit down at the kitchen table you had to *negotiate*.

The household — in the town of *Yeastfield*, on the eastern edge of the kingdom — was the household of *the Burstwell family*. The family had been bakers for *eight generations*. (You may, by this point, be noticing that *the kingdom is full of baking families*. The reason is straightforward: the kingdom's economy was, for several centuries, anchored by grain. Where grain is, bakers are. The kingdom's families had simply *specialized*.)

Burst — whose given name was *Yest*, after the family's secondary livelihood (the family also sold yeast-starter to other bakers in the region; *Yest* meant *yeast* in the old language) — was the *youngest* of the seven Burstwell children. He was eleven years younger than his oldest sister. The whole family was — by the time Burst was eight — already *fully formed as a baking operation*. Burst's role was to be small, useful, and learn the trade by watching.



The thing he learned most deeply was *the doubling of yeast*.

His mother — *Pomona*, who ran the kitchen and the bakery and was, by reputation, *the sharpest yeast-keeper in three provinces* — taught him this from the time he was four. She would set a small ball of yeast on the warm corner of the kitchen counter. She would point at it. She would say:

*"Yest. Watch this. In twenty minutes it will be twice this size."*

Burst would watch. Twenty minutes would pass. The yeast — slowly, but definitely — would be *twice the size*.

His mother would point again. She would say: *"In forty minutes it will be four times. In one hour it will be eight times. In two hours it will be sixty-four times. In three hours, five hundred and twelve times. In four hours, four thousand ninety-six times. That is why we control the temperature. If we did not control the temperature, the yeast would fill the whole kitchen."*\*

Burst — at age four — was *impressed*. He was *frightened*. He was *fascinated*.



By six he was comfortable with the principle. *Yeast doubles every twenty minutes.* By eight he could compute *how much yeast there would be after any given number of doublings.* By ten he could compute *how many doublings it would take to reach a given amount.* By twelve he had — entirely without anyone using the word — internalized *exponential growth.*

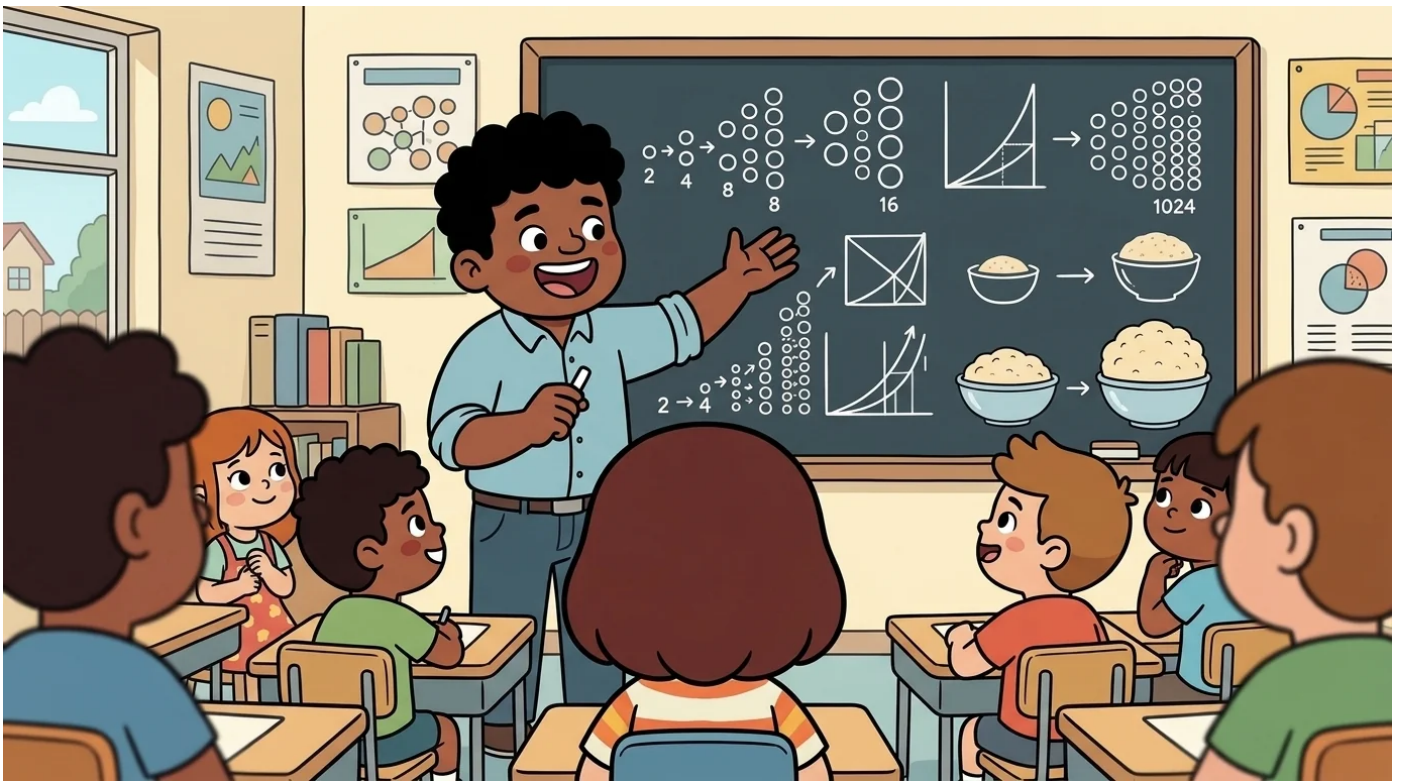
When he went to the village school at thirteen and the teacher introduced *exponential functions*, Burst raised his hand within the first ten minutes.

He said: "*That is yeast.*"

The teacher — a thoughtful woman who had taught at the village school for twenty years — said: "*What?*"

Burst said: *" $y$  equals two to the  $x$  is the formula for yeast at twenty-minute intervals.  $y$  is the multiplicative factor.  $x$  is the number of twenty-minute intervals. After three intervals, the yeast is two-cubed times its starting amount, which is eight times. After ten intervals, two-to-the-tenth, which is one thousand twenty-four times. The formula my mother used to tell me, when I was four, is just the algebra of exponentials."*

The teacher set down the chalk.



She said: "Yest. Where do you live?"

Burst said: "Burstwell's bakery. End of the lane."

The teacher said: "That makes sense. Your mother has been teaching you exponential growth since you were four."

Burst nodded. He had not, at the time, thought of it as instruction. He had thought of it as *being told a fact about yeast*. But the teacher was clear: *Burst had internalized the principle that most students struggle with — that exponential growth is fundamentally different from linear growth, that small numbers become enormous numbers quickly, that the multiplicative is not the additive.*

When Burst was eighteen, he went to the FunctionForge academy. He studied for four years. He joined the faculty at twenty-two. He has been teaching exponential functions for nine years.

In his classroom, he begins every first-day lesson the same way. He brings, from his mother's bakery (which his oldest sister now runs), *a small ball of yeast-starter* in a covered jar. He places the jar on the desk. He says: *"This is yeast. In twenty minutes — at the warm-end of the kitchen counter — it will be twice this size. In forty minutes, four times. In an hour, eight times. In two hours, sixty-four times. That is exponential growth. Each step multiplies the previous amount."*



Then he writes on the board:  $y = 2^x$ . He says: "Here is the algebra. *Two to the x*. For  $x = 1$ ,  $y = 2$ . For  $x = 2$ ,  $y = 4$ . For  $x = 3$ ,  $y = 8$ . For  $x = 10$ ,  $y = 1,024$ . For  $x = 20$ ,  $y = 1,048,576$ . This is what makes exponentials extraordinary. They start *small*. They become *enormous*. Quickly."

The children — always — are surprised. They had heard the word *exponential* but had not, viscerally, understood how fast it grows.

Burst widens his hands in a dramatic gesture. He says: "The yeast in the jar is small. The yeast in my mother's whole kitchen, if we did not control the temperature, would be *enormous*. My mother controls the temperature. This is also why exponential growth is *contained* in real life by something: the food runs out, the space runs out, the resources run out. But the *function itself* keeps doubling. The math is the math. The contained-ness is *outside* the math."

When children ask whether exponential functions are hard, Burst always says the same thing:

"They are not hard. They are *doublings*. Each step is the previous step times a fixed multiplier. The multiplier is the *base* of the exponential. The number of steps is the *exponent*. That is everything about exponential functions."

He still keeps the jar of yeast on his desk. He brings a new one every academic year. The old one (a fresh-baked loaf, made from the previous year's yeast) he eats with his colleagues at the term-end faculty meal.

**Listen along + meet more of the cast at:**



<https://spark-and-anvil.com/cast/functionforge/burst-the-doubler>

# Pivot the Rule-Switcher



- "x < 0"
  - "x >= 0"
  - "x"
  - "0"
  - "x ≥ 0"
  - "x ≤ 0"
  - "x ≠ 0"
  - "x > 0"

## Chapter 5 — Pivot and the Junction at Threefork

For twelve years, Pivot served the kingdom as a *junction-master*. This was a specific, if somewhat overlooked, civil-service title within the royal administration. A junction-master's duty was straightforward: to stand at a significant road-fork, a place where two or more roads branched off toward distinctly different destinations, and guide passing travelers to the correct path.



Pivot's assignment was Threefork.

He began the work at nineteen. The kingdom's bureau of roads had hired him after a brief but rigorous examination. They tested his ability to think quickly, speak clearly, and manage a small crowd of impatient coachmen without losing his composure. By the standards of Threefork, he was an excellent fit.

The job followed a predictable rhythm.

A coach would approach the junction. Pivot, stationed in a small wooden booth at the head of the crossroads, would study it. He would quickly assess if he recognized the coach or its driver. Many were regulars, traveling their routes repeatedly. If so, he already knew their destination. If not, he would step out of his booth, walk briskly to the coach window, and ask, "Where are you bound today?"

The coachman might call back, "Northgate!" Or perhaps, "Easton." Sometimes, "Southport." Occasionally, a driver would admit, "The capital." Or, more rarely, "I don't actually know; my mistress just told me to follow this road." In that last, unusual case, Pivot would calmly walk the coach to the passenger window and ask the mistress directly.

Once he knew the destination, Pivot would issue his instruction. "Take the north fork," he would say. Or, "You want the east fork." He might direct, "That's the south fork for you." For the capital, he would specify, "You need the central road; it's the middle one between the north and east forks. Just follow the signs marked with the capital's lion-and-star crest."

The coach would then proceed, rumbling away. Soon after, the next coach would arrive, and the process would begin again.

Pivot performed this task for eleven hours a day, six days a week, for twelve years. He handled, by his own meticulous count, more than two hundred thousand coaches during his service. Not once did he direct a coach to the wrong road. The bureau of roads praised him often for his perfect record. Twice, he received the bureau's quiet annual *Reliable Service* citation.



What Pivot understood, deep in his bones, was that his job was a system, a set of instructions that changed based on the information he received. It was a kind of internal logic, a precise mechanism for decision-making.

If a coachman said "Northgate," Pivot's internal rule was simple: "tell them to take the north fork."

If the destination was "Easton," his rule became: "direct them to the east fork."

For "Southport," the rule shifted: "point them toward the south fork."

And if a coach was headed for "the capital," his rule expanded: "instruct them to take the central road and follow the lion-and-star signs."

A different destination always triggered a different instruction. The overall task remained the same—directing traffic at a junction. But the specific output, the precise direction given, depended entirely on the input provided by the coachman.

This pattern, this system of varied responses, was something Pivot recognized intuitively. He didn't have a formal name for it for a long time, but he knew its mechanics intimately.

When Pivot was thirty-one, the bureau of roads sent him a junior assistant. The young man, named Cobble, had been hired straight out of the academy. Cobble had studied mathematics extensively. On his second day at the junction, while Pivot was directing a coachman toward the south fork, Cobble spoke up. "Sir," he said, "that is a **piecewise function**."



Cobble explained, "In mathematics, a piecewise function is a function with different rules for different categories of input. Think of it like this: if a number ' $x$ ' is less than zero, you might square it. But if ' $x$ ' is greater than or equal to zero, you might multiply it by two. It's the same overall function, but it applies different rules depending on the conditions. You do that every day here at the junction. You match the input—the coach's destination—to a condition, and you apply the rule appropriate to that condition."

Pivot considered this explanation for several days. He watched coaches, listened to destinations, and issued directions, seeing his work through this new lens. Then he said to Cobble, "That is a useful word. *Piecewise*. I will remember it."

Two years later, when Pivot was thirty-three, he wrote a letter to the FunctionForge academy. The letter was direct. "I have been a junction-master at Threefork for twelve years," he wrote. "I have directed two hundred thousand coaches to the correct road. I have recently learned that what I do is called a *piecewise function* in mathematics. I would like to teach this, because I am beginning to think the bureau of roads can find someone else to stand at Threefork, and I am, frankly, tired of standing."

The academy master, intrigued by Pivot's unique experience, invited him to teach. Pivot accepted immediately. He retired from the junction, and the bureau sent a replacement. Eventually, Cobble, the former junior assistant, took over Threefork.

Pivot has now been teaching piecewise functions at the academy for eleven years.

In his classroom, he begins every first-day lesson the same way. He stands at the front of the room, hands clasped behind his back. "Imagine I am at a road junction," he says. "Three coaches arrive. The first coach is bound for Northgate. What do I tell it?"



Pivot nods. "The second coach is bound for Easton."

"Take the east fork!" the children call out.

"The third coach is bound for Southport."

"Take the south fork!"

Pivot smiles, a quiet, knowing expression. "That," he says, "is a piecewise function. The function is *junction-direction*. The input is the destination. The output depends on which input was given. Different input, different rule. Same function."

Then he turns and writes on the board:

$$\text{If } x < 0, y = x^2$$

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# Stride and Echo



For ninety-one years, the village clock above the Linear bakery had marked the passing hours. Marn, the baker and Echo's father, wound it carefully each Sunday morning. This clock, large and made of polished brass, was a beloved fixture. It hung from an iron bracket over the bakery's front door, a landmark every villager passed at least twice a day. Its accuracy was remarkable for a village timepiece; it rarely strayed more than a minute off, season to season. This precision was a quiet point of pride for Marn, a testament to his Sunday-morning ritual.

Stride, a teacher at the academy, walked past the bakery every weekday at precisely seven-twelve. His route took him from the faculty cottages to the classrooms. The clock above the bakery door always matched his pocket-watch. He used the village clock to set his own timepiece once a month. Stride trusted this clock above all others, because his pocket-watch, in turn, kept time to his own steady pace.

Echo, Marn's son, also walked to the academy each weekday morning. He left his cottage at the same predictable moment he had for the past six years. Echo rarely glanced at the clock. For him, the exact time felt unnecessary. He did not need to know the numbers. He simply left when he always left and arrived when he always arrived. The precise hour added nothing to his careful routine.

Then, one Tuesday morning in autumn, Echo passed the bakery and simply stopped. Stride, walking his usual five paces behind, paused as well. He stopped because Echo had stopped, though he didn't yet understand why. Echo gazed up at the clock above the bakery door. The clock face clearly showed *seven forty-two*. Stride pulled out his pocket-watch. *Seven-twelve*, his watch read.

"The clock has stopped," Echo said quietly.



Stride studied the clock. The minute hand pointed to *forty-two*. The hour hand rested on *seven*. The second hand, however, remained perfectly still. He brought his pocket-watch closer, holding it beside the large brass face. His watch's second hand swept smoothly around its dial. The village clock's second hand did not move at all.

"It is stopped," Stride confirmed. "When did it stop?"

Echo considered the question. "I don't know," he replied. "I don't look at the clock. But it must have stopped *recently*. The angle still looks natural."

"What do you mean, the angle is still natural?" Stride asked.

"The minute hand hasn't drifted," Echo explained. "If the clock had been stopped for a long time, the hands would have settled into a slightly different position. The mechanism has a friction tilt. These hands are still in their last *running* position. So the clock stopped sometime between yesterday at seven-twelve and now."

Stride raised both eyebrows, surprised. He hadn't known that detail about old village clocks. He also hadn't realized Echo knew such things. Echo had spent his childhood observing the quiet constancy of the world around him. He noticed the small, unchanging features of objects. He noticed clocks, how their hands pointed, and when they didn't move.

"My father will be very upset," Echo said. "He wound it on Sunday. It shouldn't have stopped this week."

Echo sounded thoughtful, not upset.

Stride looked from the stopped clock to his pocket-watch, then back to the clock. "Echo," he said, a glint in his eye. "This is a chapter."



They stood together under the bakery's awning for a moment.

"A chapter of what?" Echo asked.

"Of the lessons-layer," Stride replied. "Look at the clock. The clock face shows *seven forty-two*. It will still show seven forty-two in five minutes. It will show seven forty-two in five hours. It will show seven forty-two tomorrow. Until someone winds it, the clock face is a **constant function**."

Echo's expression remained calm, but his attention sharpened.

Stride took a small notebook from his coat pocket. He drew a horizontal line, labeling its axis *real time*. Then he drew a vertical axis, labeling it *what the clock says*. He sketched a flat horizontal line at the seven-forty-two mark.

"The clock-face value," he explained, "is now *y equals seven-forty-two*. No matter how much real time passes — no matter what *x* is — the clock's output remains seven-forty-two. That is a constant function. You are standing right under one."

Echo considered this, his gaze returning to the clock. *Seven forty-two*.

"It's a sad constant function," Echo observed. "Normally, the clock isn't constant. Normally, it's a **linear function**. For every minute of real time, the minute hand advances one tick. The output increases steadily."

"Exactly," Stride said, a small smile touching his lips. He flipped a page in his notebook. First, he drew a *sloped* line, labeling it *clock running*. Then, beneath it, he drew the *flat* line again, labeling that one *clock stopped*.

"This," he explained, pointing to the sloped line, "is the clock when it works. It's linear. Its slope is one tick per minute. It keeps pace with real time. If you plot the time it tells against real time, you get a straight line. Forty-five degrees of slope. *Y equals X*. *That* is a linear function."

"And the other one," Echo said, "is what the clock does now."

"Exactly," Stride confirmed. "Its slope is zero. The output never changes.  $Y$  equals seven-forty-two. *That* is a constant function. A constant function is actually a special kind of linear function. It's a linear function where the slope is zero. That means its rate of change is zero, so its graph becomes a horizontal line. It uses the same equation form,  $Y$  equals  $MX$  plus  $B$ , but with  $M$  equal to zero."

"So a constant function is *flat*," Echo said. "And a linear function is *steady*. The difference is the slope."

"The difference is the **rate of change**," Stride corrected gently. "Constant: no change per unit of  $X$ . Linear: a fixed amount of change per unit of  $X$ . They belong to the same family. Just with a different  $M$ ."



Just then, Marn the baker emerged from the front door, wiping flour from his hands onto his apron. He glanced up at the clock, then stopped wiping altogether.

"Stopped," he muttered.

"Stopped," Echo confirmed. "Recently. The hands haven't drifted."

Marn squinted at the clock, then noticed Stride and the open notebook. "You're teaching off my broken clock," he said, a hint of amusement in his voice.

"We are," Stride admitted cheerfully. "It's a very good clock for teaching. We were just explaining how it changed from a linear function into a constant function."

"It has *what*?" Marn asked, bewildered.

Echo translated for his father. "It used to tick. Now it doesn't. Every minute used to add one tick. Now it adds nothing. So it used to *change* by a known amount each minute, but now it changes by nothing per minute. That's the slope of the function the clock is performing. The slope used to be one. Now it is zero."

Marn looked from his son to Stride, then to the clock and back again. He was a baker, accustomed to flour and yeast, not this much mathematical discussion before breakfast.

"It just needs winding," he said, shaking his head. "Don't make a story out of it."

"Too late," Stride replied with a grin.

Echo helped his father carry a small step-stool to a spot beneath the clock. Marn climbed up, opened the brass casing, and wound the mechanism. The second hand immediately began sweeping. The minute hand jumped one tick. The clock, once still, began behaving like a linear function once more.

Marn climbed back down. He looked at Stride. "Now what?"

Stride showed him the notebook page, pointing to the two lines. "Sloped line: clock running. Flat line: clock stopped. Same family of equations, just a different  $M$ . Your clock has *switched back into the sloped one* and is running again."

Marn studied the diagram for a long moment. Then he said, slowly, "So if it stops once a year, that's a problem. But if it stops *every Tuesday morning*, that's a **pattern**."

"That's a higher-order function," Echo said, a faint smile playing on his lips. "We're not ready for that one yet."

"Don't be cute," Marn grumbled, but a smile touched his own face too.



That afternoon, Stride prepared a short lesson on the board for his second-period class. He sketched the two graphs they had discussed that morning: the sloped line and the flat line.

"Both of these are functions," he explained. "A function is a rule that takes an input and turns it into an output. The sloped line represents a linear function. For every unit of input, the output increases by a fixed amount. The flat line shows a constant function. For every unit of input, the output remains exactly the same."

He paused, letting the words settle.

"They aren't opposite kinds of things," he continued. "They are essentially the same kind of thing, but with different slopes. A constant function is simply a linear function whose slope is zero. You might think of a constant function as the linear function that *forgot to walk*; it just stands still."

A student in the front row raised her hand. "Is the village clock a function?"

Stride smiled. "Today it is. This morning it wasn't. My friend Echo and I are still discussing whether *this Tuesday* will become a pattern."

The class looked puzzled. They would, however, understand by the end of the week.

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