FractionForge

Meet the Cast

ADVANCED EDITION

Spark & Anvil

Copyright & License

This advanced edition collects 6 chapter books from the FractionForge cast — each character embodies a different curricular primitive; together they teach the full subject.

Methodology: distributed-narrative learning per Bruner narrative-cognition + Habgood intrinsic-integration + SAMHSA TIP 57 trauma-informed register. Advanced edition: upper-middle-grade register (Wonder / Hatchet / Holes band) for readers ages 11-14 ready for longer sentences + more nuanced subtext.

Spark & Anvil is a 501(c)(3) public charity. All apps free forever; no ads; no tracking; no in-app purchases.

spark-and-anvil.com

##

For everyone who learns by reading between the lines.

Contents

Introduction

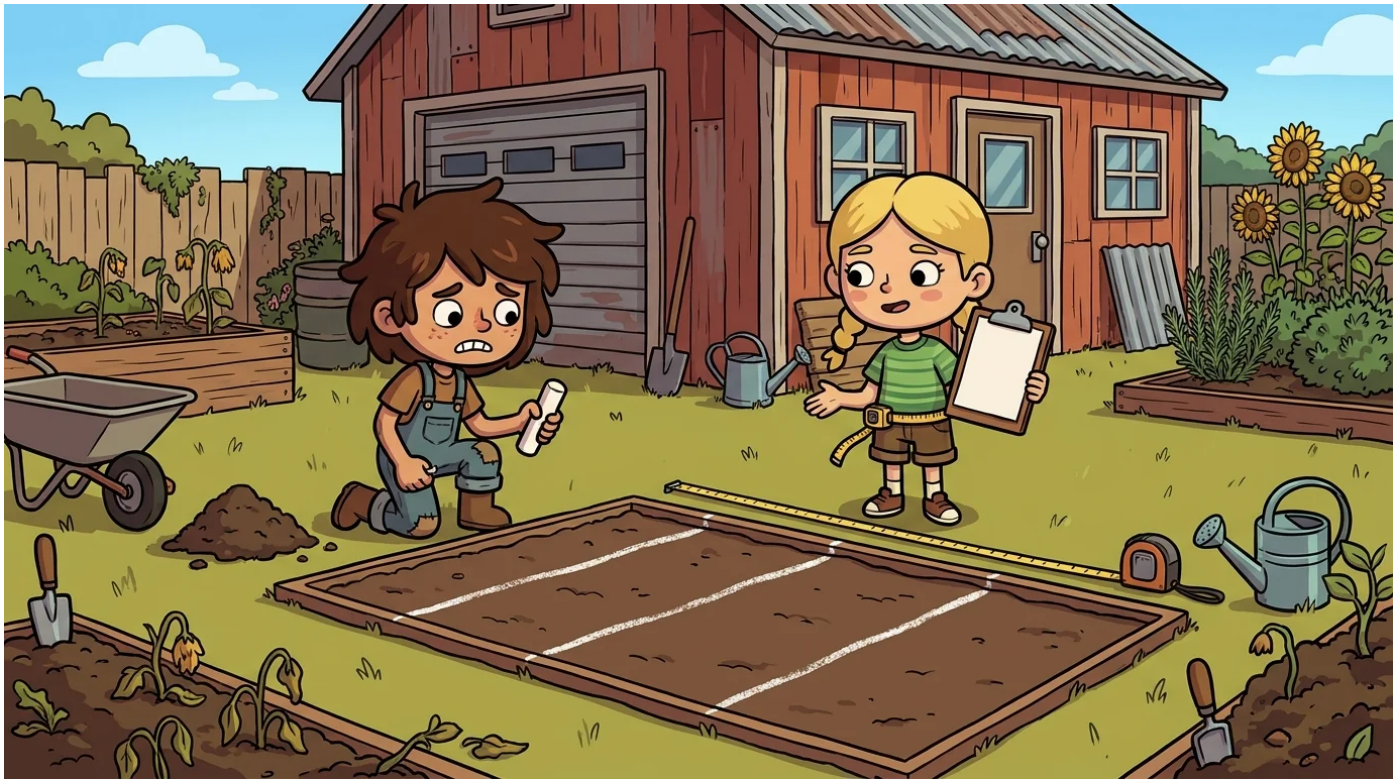
The FractionForge cast was authored to embody the curriculum, not decorate around it.
Each of the 6 characters you'll meet in this book teaches a specific primitive —
a particular tactic, a particular technique, a particular way of seeing.
Together they form an ensemble: the cast IS the curriculum.

Read in any order. Each chapter stands alone.
Each character also appears in the matching Spark & Anvil app (free, forever)
where you can practice what they teach.

This is the **Advanced Edition** — written for readers who are ready for longer sentences, layered subtext, and the trust that comes with not having every joke explained. The Standard Edition covers the same characters at a lighter register; pick whichever feels right for the reader at hand.

— *The editors at Spark & Anvil*

Equi and Stretch



Behind the school workshop, a long rectangular patch of earth lay waiting. For generations, this garden plot had witnessed the enthusiastic, if sometimes chaotic, efforts of the gardening club. Most years, carrots mysteriously sprouted in the rosemary patch, while ambitious sunflowers cast long shadows over delicate herbs. But this year, a new order was promised. The gardening club had divided into four distinct groups, and each group insisted on its own perfectly **equal space**.

Equi clutched a stick of chalk, her brow furrowed in concentration. Beside her, Stretch unspooled a bright yellow tape measure, a thoughtful, almost serene expression on her face.

"Four groups," Equi announced, her voice firm. "One long rectangle. The solution is simple: we divide it into four equal strips." She knelt, drawing four confident chalk lines across the dry dirt. The garden plot instantly fractured into four neat, vertical sections. "Strip A for the herbs," she narrated, pointing with her chalk. "Strip B for salad greens. Strip C for root vegetables. And Strip D, of course, for the sunflowers." She stood, dusting her hands. "Done."

Stretch, however, slowly rewound her tape measure. "Almost done," she corrected, her tone gentle but firm.

Equi turned, a question etched on her face. "Did I miss something obvious?"

Stretch held up a clipboard, its pages covered in neat handwriting. "The herbs group's list," she began. "They need three separate herb plots: basil, parsley, and mint. Basil and mint, as you know, are not exactly friendly neighbors. They can't share a single strip. So, they need three distinct sub-strips within their quarter."

Equi's gaze dropped to the clipboard, her initial confidence wavering. "Oh."

"Then the salad-greens group," Stretch continued, flipping a page. "They've planned for six different kinds of lettuce. Each kind requires its own dedicated row."

"Oh," Equi repeated, a little softer this time.

"The root-vegetables group needs two sub-plots: one for carrots, the other for radishes."

"And the sunflowers?" Equi asked, almost hopefully.

"Just one sub-plot," Stretch confirmed. "Pure sunflowers."

Equi stared at the pristine chalk lines, which now seemed less like a solution and more like the beginning of a puzzle. The herbs group needed their strip divided into thirds. The salad greens group required sixths. The root vegetables group wanted halves. And the sunflowers, bless their simple hearts, just needed their entire quarter. Four distinct strips, each demanding a different internal partition. This wasn't just about drawing lines; this was going to require some serious, brain-stretching thought.

Stretch crouched beside the chalk lines, her fingers tracing the boundaries. "Here's my starting point," she explained. "Every group begins with the same outside boundary: one-quarter of the entire plot. That's their designated strip. So, when we look *inside* each strip, every group is essentially slicing their own quarter into smaller sub-portions."

"Right," Equi murmured, watching her.

"Take the herbs group's strip," Stretch continued, drawing a small, imaginary box in the dirt. "It's one-quarter of the whole plot. They need to slice that quarter into three equal sub-portions. Each sub-portion is therefore one-third of *their* quarter. Which means, if we think about the entire garden, each of those herb sub-portions is one-twelfth of the whole. Because a third of a quarter, well, that makes a twelfth."

Equi blinked, her eyes wide. "Did you just... multiply fractions?"

Stretch nodded. "Exactly. One-third times one-quarter equals one-twelfth. It's the same fundamental idea as your **equivalence** rule, just applied to a practical slicing problem."

"No," Equi countered, shaking her head slightly. "That's not multiplication in the way I teach it. That's *scaling*. You took the herbs group's quarter and you stretched it down, making it visible on a much finer scale. Their single quarter suddenly became three twelfths."

"Yes," Stretch agreed, a small smile playing on her lips. "That's precisely what I do. I stretch fractions to a common scale. It allows us to compare them fairly across different groups and different needs."

Equi paused, letting the idea settle. "So," she mused aloud, "if I wanted to know how much space the basil gets, compared to, say, just one single row of lettuce..."

"You stretch them both to the same scale," Stretch finished for her. "The basil occupies one-twelfth of the entire plot. One row of lettuce, however, takes one-sixth of the salad-greens strip, which translates to one-twenty-fourth of the whole garden. So, when basil and one row of lettuce are both viewed on that common scale, it's one-twelfth versus one-twenty-fourth."

"Which means," Equi concluded, a spark of understanding in her eyes, "basil gets twice as much space as one row of lettuce."

"Exactly," Stretch confirmed.

Equi remained silent for a long moment, the chalk dust on her fingers suddenly feeling significant. "I've taught kids that two-thirds equals four-sixths for years," she finally said, her voice soft with revelation. "I've always thought of it as *naming the same number differently*. Like a kid having a nickname, or a secret identity. But I've never considered it as *stretching* a fraction to a finer scale."

"Both perspectives are correct," Stretch replied, her gaze steady. "It's the same fundamental idea, just viewed from two different angles. You see the equivalence. I see the scaling. And when we put those two views together, we find the complete answer."

Equi picked up a fresh piece of chalk, its white tip pristine against the brown earth. "Okay," she began, her tone more confident now. "Let me try to restate this, just to make sure I've truly grasped it. What we've accomplished is this: we took four groups, each with their own unique way of slicing their space. We then scaled all those diverse sub-portions to a single, common base—sixths. This allows us to plan the entire garden using one shared, understandable scale. Each group still gets to slice their own quarter-strip however they choose. But on *this* diagram, everything is now expressed in sixths."

"Precisely," Stretch affirmed.

"And this common scale," Equi continued, gesturing vaguely, "is it twelfths? Or twenty-fourths?"

Stretch gently took the chalk from Equi's hand. She knelt and quickly sketched a small, neat grid into the dirt, an abstract representation of the problem. "We need a **common denominator**," she explained, her finger tracing a line. "That's a number, a scale, that all four groups' sub-portions can be expressed in. The herbs group needs thirds of their strip. Salad greens require sixths. Root vegetables need halves. And the sunflowers, well, they just need the whole strip, which we can think of as one."

Equi recited the numbers, almost to herself. "Three, six, two, one."

"Now," Stretch prompted, looking up. "What's the smallest number that all four of those values divide into evenly?"

Equi's eyes narrowed in thought. "Six," she declared.

"So, if we slice each individual strip into sixths," Stretch continued, her chalk moving across the dirt, "then it looks like this:"

She quickly drew a series of smaller lines within her grid, transforming the abstract into something concrete.

- "The herbs group's three sub-portions," she narrated, "will each take up *two* sixths of their strip."
- "The salad-greens group's six sub-portions will each occupy *one* sixth."
- "The root-vegetables group's two sub-portions will each claim *three* sixths."
- "And the sunflowers group's single sub-portion will encompass *six* sixths, the entire strip."

"Now," Stretch concluded, sitting back on her heels, "everyone is finally on the same scale: sixths of a strip."

Equi leaned closer, studying the intricate diagram. "And this," she said slowly, tracing a line with her finger, "is what you mean by a common denominator."

"Precisely," Stretch confirmed.

"And it's also what I mean by **equivalent fractions**," Equi added, a triumphant note in her voice. "Every group's portion has simply been renamed in sixths."

"Exactly," Stretch agreed.

"They are the same idea," Equi stated, a wide smile spreading across her face.

"They are the same idea," Stretch echoed, returning the smile.

Equi stood up, brushing loose dirt from her knees, a new lightness in her movements. "All right," she said, her voice clear and confident. "Let me restate one last thing, just to solidify it. What we've accomplished is this: we took four groups, each with their own unique way of slicing their space. We then scaled all those diverse sub-portions to a single, common base—sixths. This allows us to plan the entire garden using one shared, understandable scale. Each group still gets to slice their own quarter-strip however they choose. But on *this* diagram, everything is now expressed in sixths."

"Exactly," Stretch confirmed, a proud look in her eyes.

"So," Equi continued, her finger hovering over the diagram, "when the herbs group says, 'Basil gets one of our three sub-portions,' we can immediately translate that. It means basil gets *two* sixths of the herbs strip. And when the salad-greens group says, 'Arugula gets one of our six sub-portions,' we translate that to arugula getting *one* sixth of the salad-greens strip. Now, both of those measures are directly comparable. This holds true even though basil and arugula reside in completely different strips."

Stretch grinned. "You just articulated the entire purpose of common denominators in a single, perfect sentence."

"I just *restated* the entire purpose of common denominators," Equi corrected, a playful glint in her eye. "And the only reason I could restate it so clearly is because you literally showed me what scaling looks like, right here in the dirt."

She smiled, a genuine, unburdened smile.

"My equivalence rule and your scaling rule," Equi concluded, "they're just the same trick, told in two different ways."

"And when applied to garden plots," Stretch added, "they ensure we can share space fairly, without argument."

A moment later, the garden club kids emerged from the workshop, their chatter filling the air. They quickly gathered around Equi and Stretch, their eyes curious. Equi handed out copies of the newly drawn diagram, which showed the garden plot in clear detail. Each group's main strip was clearly visible. But now, inside each strip, the smaller sub-portions were precisely drawn to the common scale. The herbs group could easily identify their basil, parsley, and mint sections. The salad-greens group saw their six distinct lettuce rows. The root-vegetables group could point out their carrots and radishes. Even the sunflowers group could see their entire strip laid out, perfectly theirs.

"How do we know it's fair?" a girl named Maya asked, her brow furrowed with a familiar middle-school skepticism.

Equi pointed to the diagram, her finger moving across the lines. "Look closely," she instructed. "Each group's outside boundary is exactly one-quarter of the entire plot. That's the same for everyone. And inside their own strip, they still get to slice it however they need. The clever part is that we translated each group's unique slicing into the same scale—sixths-of-a-strip. This way, we can see everyone's space side by side, clearly."

Stretch nodded in agreement. "Which means, Maya, when you're standing in the herbs strip and you see basil get two sixths of that strip, you automatically know. Without asking anyone, you understand that two sixths of a quarter is precisely two-twenty-fourths of the whole garden. That's a real, measurable, and truly fair share."

Maya considered this, her skepticism slowly melting into understanding. "So, even if basil and arugula are planted in completely different strips, I could tell you exactly how much bigger basil's plot is than arugula's?"

"You absolutely could," Equi confirmed.

"Basil is twice as big," Maya stated, a small smile appearing.

"Precisely," Stretch replied.

Equi and Stretch exchanged a knowing smile. The four chalk lines, once a simple division, now stretched across the dirt, holding the promise of a whole garden's worth of fair, organized work.

"Equivalent forms," Equi murmured, looking at the lines.

"Common scales," Stretch replied, her gaze meeting Equi's.

"The same trick," Equi concluded.

"Told twice," Stretch finished.

Listen along + meet more of the cast at:

`https://spark-and-anvil.com/cast/fractionforge/equi-stretch`

Halver

- "FAIRSHARE VILLAGE SCHOOL"
  - "SHAPES"
  - "1"
  - "2"
  - "3"
  - "4"
  - "5"
    gate-allow-text-pattern: '^[0-9]+/[0-9]+$'

Chapter 1 — Halver and the Pie at the Birthday Party

She often shares this detail with the children in her classroom now. She believes this early struggle with division was precisely why she eventually became a teacher of **fractions**. Children, she has observed, frequently begin with a similar reluctance to share. They, she understands, *comprehend the problem from the inside.*

When Halver—then known by her given name, *Posy*, a name she shed for Halver at the age of eleven—was seven, her younger brother celebrated his fourth birthday. Six children attended the party: the birthday boy, his three small friends, Posy herself, and Posy's older cousin.

There was *one cake.*

Posy's mother had baked the cake in the cozy kitchen of their cottage, nestled in the village of *Fairshare*. Fairshare, a real place within the kingdom, was renowned for two distinct qualities: its exceptional bakers and its long-standing tradition of mediation. Locals often claimed the two were deeply connected, though the precise nature of that connection rarely received a full explanation. The cake itself was perfectly round, featuring a sweet layer of jam in its middle and a delicate frosting flower perched on top. By the modest standards of village birthday cakes, it was *quite lovely.*

As her mother carefully placed the cake on the table, Posy immediately grasped the impending challenge: the cake *had to be divided.*

Posy, however, harbored no desire for a mere fair piece. She wanted a *large* piece. For two weeks, ever since her mother had first mentioned the baking project, Posy had dreamt of this cake. She knew, even at seven, that voicing a demand for a large slice at someone else's birthday celebration was strictly forbidden. Yet, the thought echoed loudly within her mind, a persistent, silent clamor for more.

Her mother, a woman of calm demeanor and steady hands, picked up the polished cake-knife. Her gaze swept over the eager faces gathered around the table. "Six children," she announced, her voice even. "One cake. How, exactly, do we cut it?"

Posy's older cousin, who was eleven and already attending the village school, offered a solution with the confidence of a scholar. "You cut it in half first," he suggested, his tone slightly formal. "Then, you cut each of those halves into three. That way, everyone gets six equal pieces."

Her mother nodded slowly, a small smile gracing her lips. "That is precisely correct," she affirmed. "Each piece will represent one-sixth of the entire cake."

With the dull back of the knife, she traced a clean, vertical line across the cake's frosted surface, dividing it into two symmetrical halves. Then, with practiced precision, she drew two more lines across each half, creating three segments within each section. Six pieces now defined the cake's circumference. The lines, once completed, formed a pattern resembling a six-pointed star delicately etched into the circular dessert.

When her turn came, Posy received her piece. She had to concede, albeit reluctantly, that the slice was undeniably *fair*. Every other child, from the tiny birthday boy to her older, more knowledgeable cousin, received an identical portion. There was no room for complaint, no justification for feeling cheated.

Posy ate her piece. It was delicious, a perfect blend of sweet jam and soft cake. (She noted, with a careful, almost mathematical precision, that it was *one-sixth as good as the whole cake would have been if she had been the only child at the party.* Still, it was good enough.) She savored every bite, finishing it slowly. She watched the other children, their faces smeared with frosting, devour their own slices. Finally, her mother set the empty plate back on the table, a testament to the cake's swift disappearance.

Posy thought about that cake for the remainder of the day, and for many days that followed.

Over the subsequent years, slowly, in the way children gradually absorb the profound truths embedded in their everyday experiences, she began to understand. Fairness, she realized, was not just a feeling; it was a kind of arithmetic. To divide something fairly, you had to *partition* it. **Partitioning**, she came to see, meant meticulously cutting a whole into equal parts. The number of parts you created became a specific name for the division's fairness. Two parts? Each was a *half*. Three parts? Each became a *third*. Six parts, like the birthday cake? Each was a *sixth*.

This fundamental principle, she later understood, formed the very bedrock of fractions. The **denominator** of a fraction, she learned, represented the exact count of equal parts into which the whole had been divided.

Posy stared at the slate, a sudden jolt of recognition sparking through her. She immediately conjured the image of her brother's birthday cake. The cake had been cut into six pieces. Each child had received one-sixth. *One of six equal parts.* The written notation, *1/6*, was simply *the name for the portion they had eaten.* It was a label, a concise way to describe a fair share.

To Posy, this was an extraordinary revelation, a sudden opening of a door onto a new way of seeing the world. Fractions, she understood with a rush of excitement, were the precise language of **partitioning**. The number beneath the line, the denominator, counted the total number of equal parts. The number above, the numerator, counted how many of those parts you possessed.

That, Posy decided then and there, was *exactly what she was going to teach.*

She began calling herself *Halver*, a playful nickname her cousin had coined after the cake story spread through the family. The name stuck. From sixteen to nineteen, she rigorously studied at the esteemed FractionForge academy. At twenty, she joined its faculty. She has been teaching the principles of **partitioning** ever since.

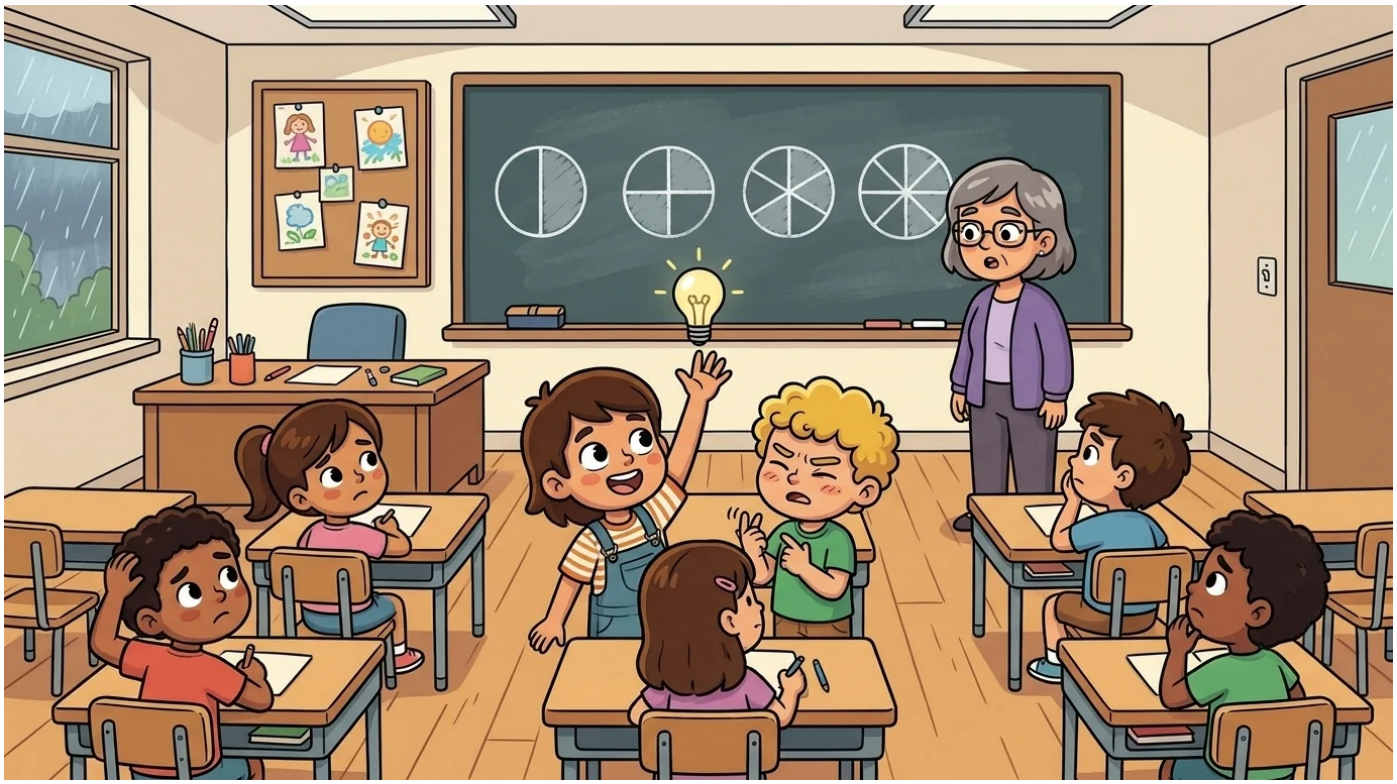
In her classroom, Halver commences every first-day lesson with the same ritual. She brings a small, perfectly round cake from the village bakery—which, incidentally, is her mother's bakery, now managed by her younger brother, the very one whose fourth birthday had started everything. The cake is plain, unadorned, and clearly intended for cutting. She places it gently on her desk and addresses her new students. "There are twenty of you in this room," she states, her voice calm and clear. "How do we share this cake fairly?"

Listen along + meet more of the cast at:

https://spark-and-anvil.com/cast/fractionforge/halver

Equi

- "2"
    - "3"
    - "4"
    - "5"
    - "6"
    - "8"
    - "9"
    - "10"
    - "12"
    - "1/2"
    - "2/4"



- "4/8"
    - "5/10"
    - "Lessons"
    - "Student"
    - "Geometrics"
    - "lessons"
      gate-allow-text-pattern: '^[0-9]{1,2}$|^[0-9]+/[0-9]+$'

Chapter 3 — Equi and the Four Pairs of Twins

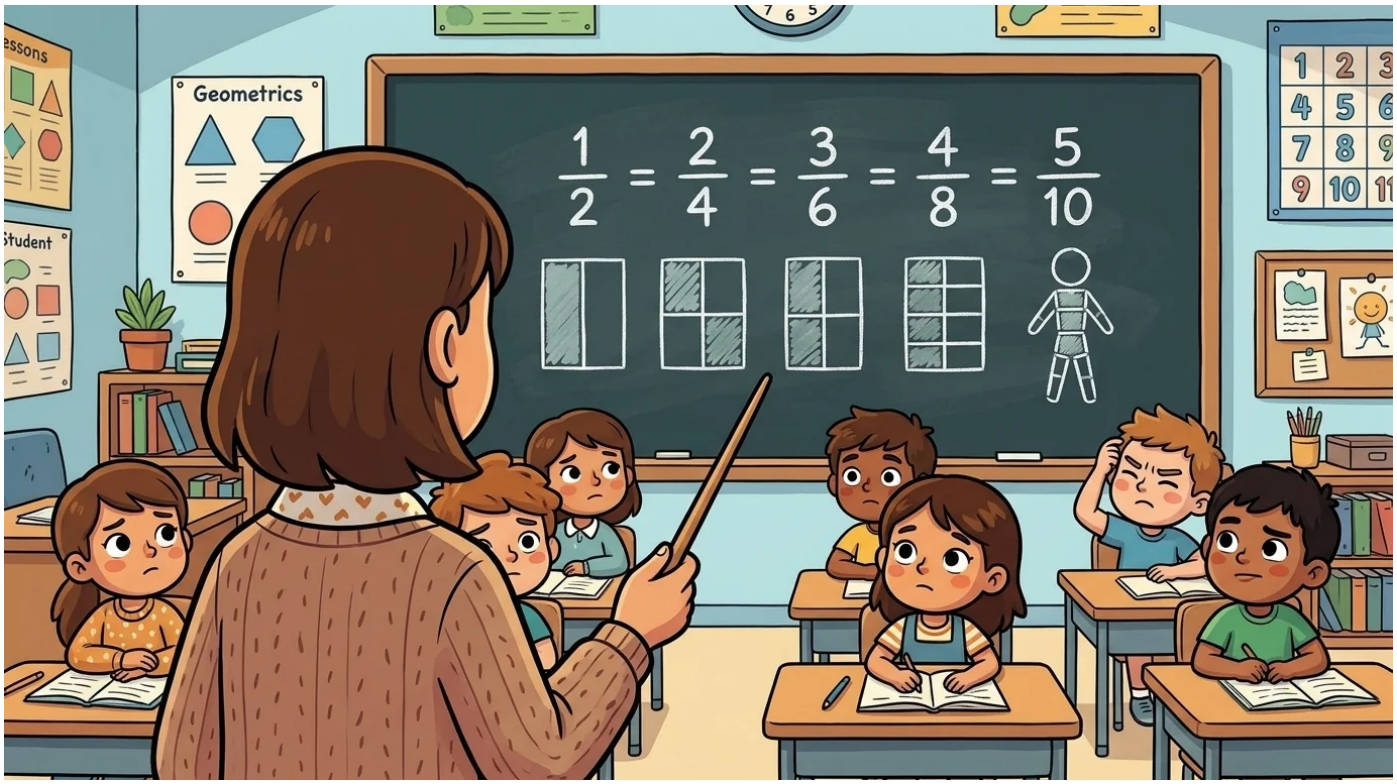
Specifically: she was *the youngest of four pairs of twins.* Her parents had — for reasons no one in the family fully understood and which the family doctor described as *"a statistical anomaly we will simply enjoy"* — had identical-twin children four pregnancies in a row. Equi's mother said, when asked, that *"after the first pair we were prepared; after the second we were experienced; after the third we were resigned; after the fourth we stopped counting and started naming."*

The eight children were named with *a deliberate system.* Each twin pair had two names that *sounded slightly different but felt the same.* The oldest pair was *Forta and Forga* (eight years older than Equi). The second pair was *Vena and Verra* (six years older). The third pair was *Posi and Poso* (four years older). The youngest pair was *Equi and Equa* — Equi was the second-born, by about ninety seconds, and the family considered her the *fourth* of the four pairs.

The naming system was, Equi's mother explained when asked, *deliberate.* She had wanted each twin pair to have names that emphasized *their unity as a pair* without making them indistinguishable. Forta-and-Forga were two expressions of the same idea. Vena-and-Verra were two expressions of the same idea. *Same idea. Different name.*

This was — although nobody in the family used these words — the principle of *equivalent fractions.* The same quantity, expressed in two different forms.

Equi grew up viscerally aware of this. Her sister Equa was *the same person* in some essential respects — they shared appearance, voice, hand-gestures, food preferences, even handwriting — and *a different person* in others (Equa preferred apples; Equi preferred pears). They were *equivalent expressions of a single twin-pair.* The pair was the same. The two expressions were the same and different at once.



She was nine. The schoolteacher (a different schoolteacher than the one who had recognized Pie's natural talent — this was a different village, a few years later) had written on the board:

1/2 = 2/4 = 3/6 = 4/8.

The schoolteacher said: *"These four fractions look different but they are *the same value.* They are *equivalent.* We can multiply the numerator and denominator of a fraction by the same number, and the new fraction equals the old one."*

Equi raised her hand. She said: *"Like my sisters."*

The schoolteacher said: *"Like your sisters?"*

Equi said: *"I have three older pairs of twin sisters and a twin of my own. Each pair has two names. The two names sound slightly different but the pair is the same. They are *equivalent.* Equivalent fractions are like that. The denominator-scale changes — like the names change — but the *underlying value* is the same."*

She said, slowly: *"That is actually correct. Equivalent fractions are the same value expressed at different denominator-scales. Twin names are the same person expressed in two slightly different ways. The structure is parallel."*

Equi went home that afternoon and told her sisters. Her sisters (all seven of them) were delighted. Forta and Forga, who were seventeen, said: *"We are an equivalent fraction together. We are 1/2 of a twin-pair."* Vena and Verra, who were fifteen, said: *"And we are 2/4. Same value. Different scale."* Posi and Poso, who were thirteen, said: *"And we are 3/6."* Equi and Equa, who were nine, said: *"And we are 4/8."* All four pairs nodded.

Their mother, who was making supper, said: *"You are *children.* You are not fractions. Please set the table."*

The family set the table. The family ate supper. The family, that night, started a small running joke about fraction-equivalence that has continued, in the family, to this day.

When Equi was eighteen, she went to the FractionForge academy. Her sister Equa went to a different academy (Equa wanted to study music, not mathematics). They wrote letters to each other every week. The family considered them *equivalent but different.*

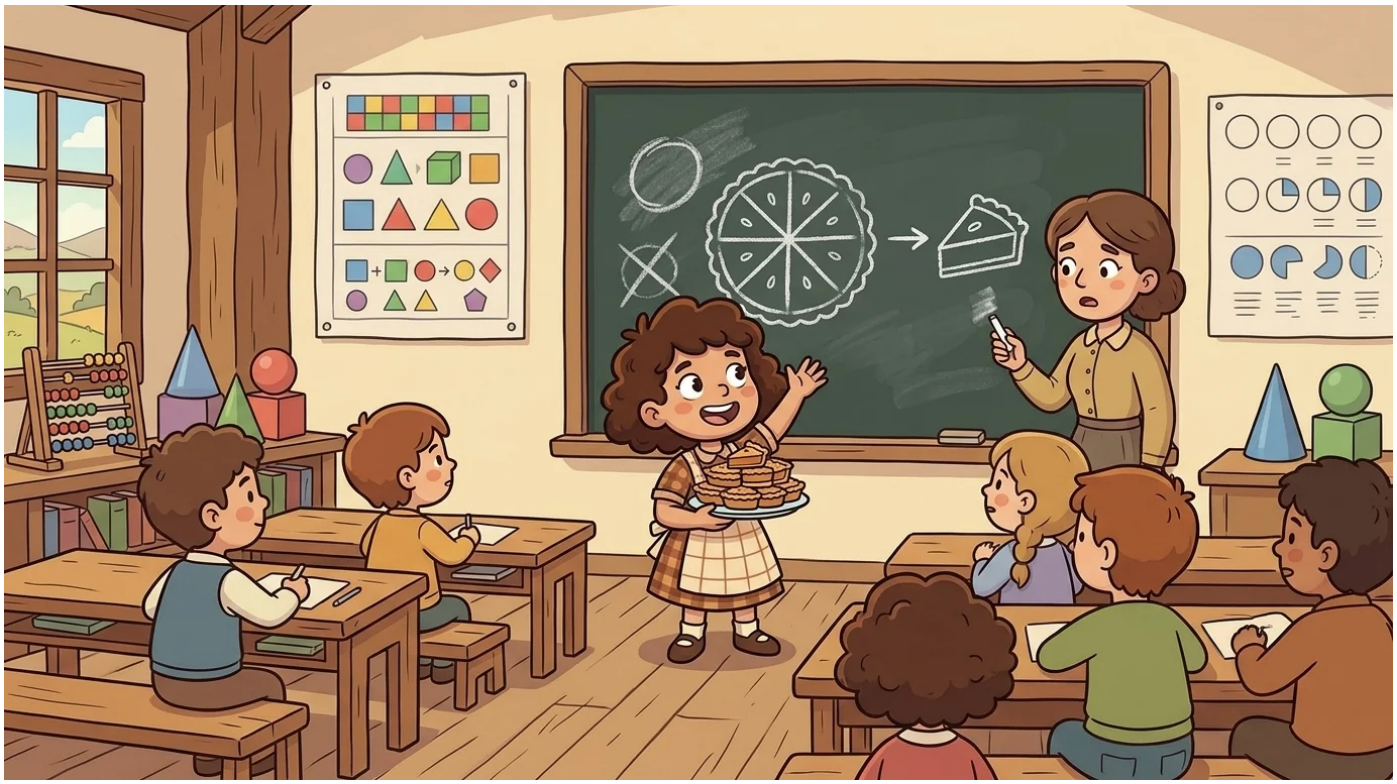
Equi has been teaching equivalent fractions for ten years.

Listen along + meet more of the cast at:

https://spark-and-anvil.com/cast/fractionforge/equi

Pie

- "CRUSTFORD"
  - "CRUSTFOOD"
  - "Co."
  - "THE SLICE IS THE UNIT."
  - "THE PIE IS THE WHOLE."
  - "1/8"
  - "9/8"
  - "1"
  - "a"
  - "1 a 1/8"
  - "CRUSTFOO"
  - "CRUSTFORDS"
  - "Crustford"



- "CRUSTFO"

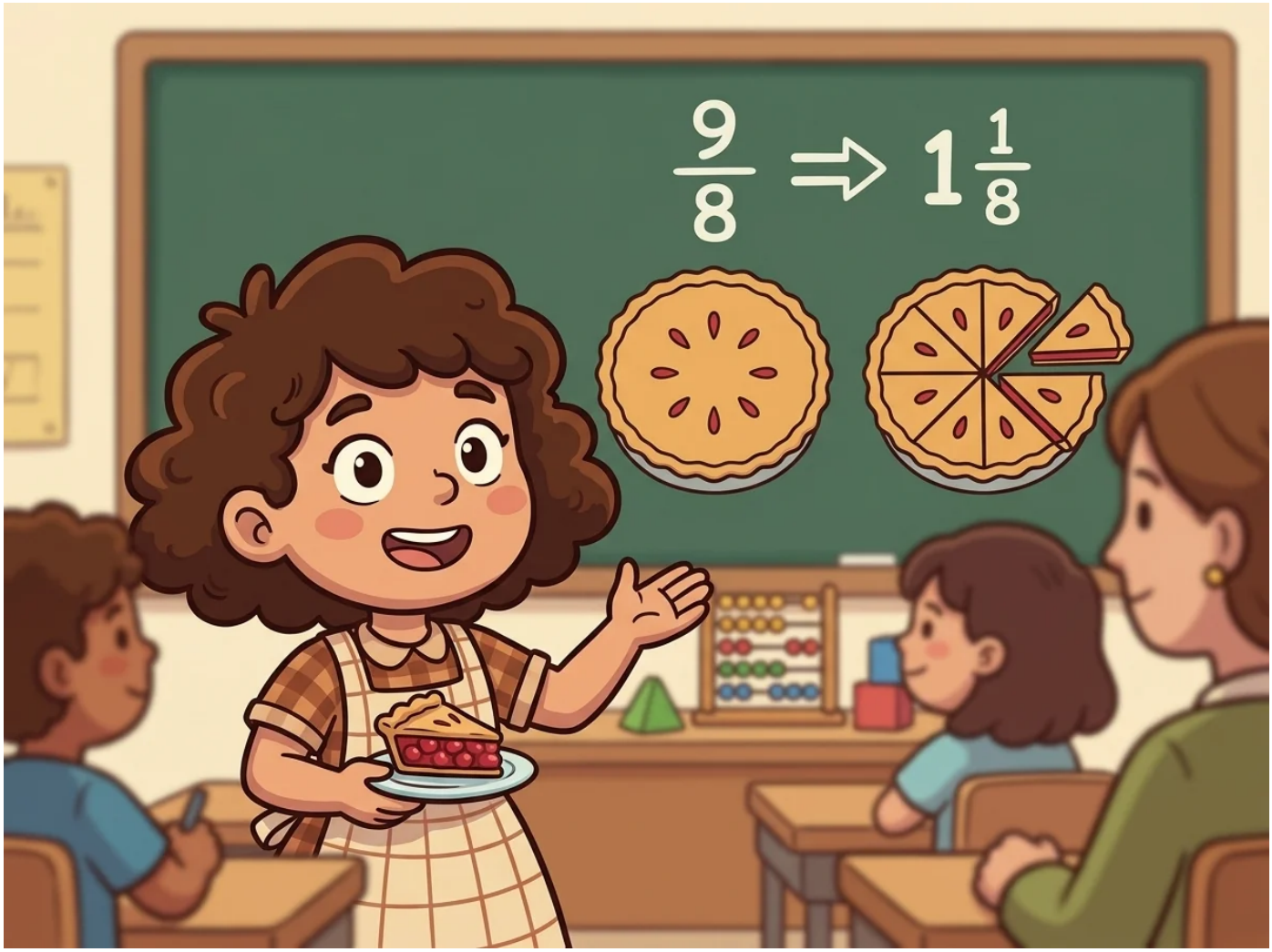
Chapter 2 — Pie and the Standardized Slice

Pie grew up in *a pie-shop.*

The pie-shop — *Crustford and Co.*, on the main street of the town of *Crustford* — had been in her family for *five generations.* Her great-great-grandfather had founded it. Her great-grandfather had standardized the pie-sizes. Her grandfather had added the savory line. Her grandmother had added the breakfast pies. Her parents — who still ran the shop when Pie left for the academy — had added the slice-by-slice retail counter so customers could buy individual slices without having to order whole pies.

The shop was, by Crustford's standards, *famous.* Travellers came from three towns over to buy its pies. The shop's slogan, painted in large gold letters above the front window, was:

"THE SLICE IS THE UNIT. THE PIE IS THE WHOLE."



Crustford pies had been standardized since her great-grandfather's generation. *Every Crustford pie was twelve inches across.* Every Crustford pie was *cut into eight slices.* Every slice was *exactly the same size.* This had been an aggressive standardization in its time — most village pie-makers had cut their pies in whatever sizes the customer wanted, with no fixed ratio — but Crustford's great-grandfather had decided that *consistency was good for the customer.* The customer always knew what a Crustford slice was. The customer could always compare prices. The customer could always know, *exactly,* how much pie they had paid for.

Pie — whose given name was *Tess*, though everyone called her Pie from the time she was two and a half — grew up with the slice-and-pie ratio as a *foundational fact of her life.*

By the time she was four, she could answer questions like: *"If you have three slices and your brother has four slices, how many slices total?"* (Seven.) *"How much of a whole pie is that?"* (Seven-eighths.) *"And if your sister brings two more slices to the table?"* (That makes nine slices, which is *one whole pie plus one extra slice — one and one-eighth pies.*)

She did this in her head, all day, every day, throughout her childhood. It was not, to her, *arithmetic.* It was *just how pies worked.*

When she was eleven, she walked into the village school for the first time. (Her parents had homeschooled her until then; she had been working in the shop.) The schoolteacher introduced *fractions* to the class. She drew a pie on the board. She said: *"A pie cut into eight pieces. Each piece is one-eighth. If you eat three pieces, you have eaten three-eighths."*

Pie raised her hand. She said: *"What if you eat nine pieces?"*

Pie said: *"Yes. You would have eaten *one and one-eighth* pies. The first eight pieces are a whole pie. The ninth piece is one-eighth of another pie."*

The schoolteacher said: *"That is correct. We call that a *mixed number.* The 1 is the whole-pies-count. The 1/8 is the extra-slice-fraction."*

Pie said: *"You can also write it as *nine-eighths.* The 9 is the total slices. The 8 is the slices-per-pie. *9/8* is the same as *1 and 1/8.* They are the same quantity. Just written differently."*

The schoolteacher stared at her.

The schoolteacher said: *"How do you know that?"*

Pie said: *"We sell pies."*

Pie nodded. She had not known the names. She had been doing the operation, in her head, since she was four.

That afternoon, the schoolteacher walked to Crustford and Co. and bought a single slice. She asked Pie's mother: *"Has anyone in your family ever taught fractions formally?"*

Pie's mother said: *"No. We just sell pies."*

The schoolteacher said: *"Your daughter is a natural teacher of fractions. The slice-and-pie ratio in this shop has done her education."*

Pie's mother said, after thinking: *"She does spend a lot of time computing slice-counts."*

The schoolteacher said: *"If she ever wants to teach formally, she has a place at the FractionForge academy. Tell her."*

Pie's mother told her that evening. Pie thought about it for two years. She loved the pie-shop. She did not, at the time, want to leave. But by the time she was fourteen, she had begun to feel that *the slice-and-pie ratio belonged to more than just the shop.* It was, in a small way, a *piece of mathematics that her family had been quietly preserving.* It deserved a wider audience.

Listen along + meet more of the cast at:

`https://spark-and-anvil.com/cast/fractionforge/pie`

Dot

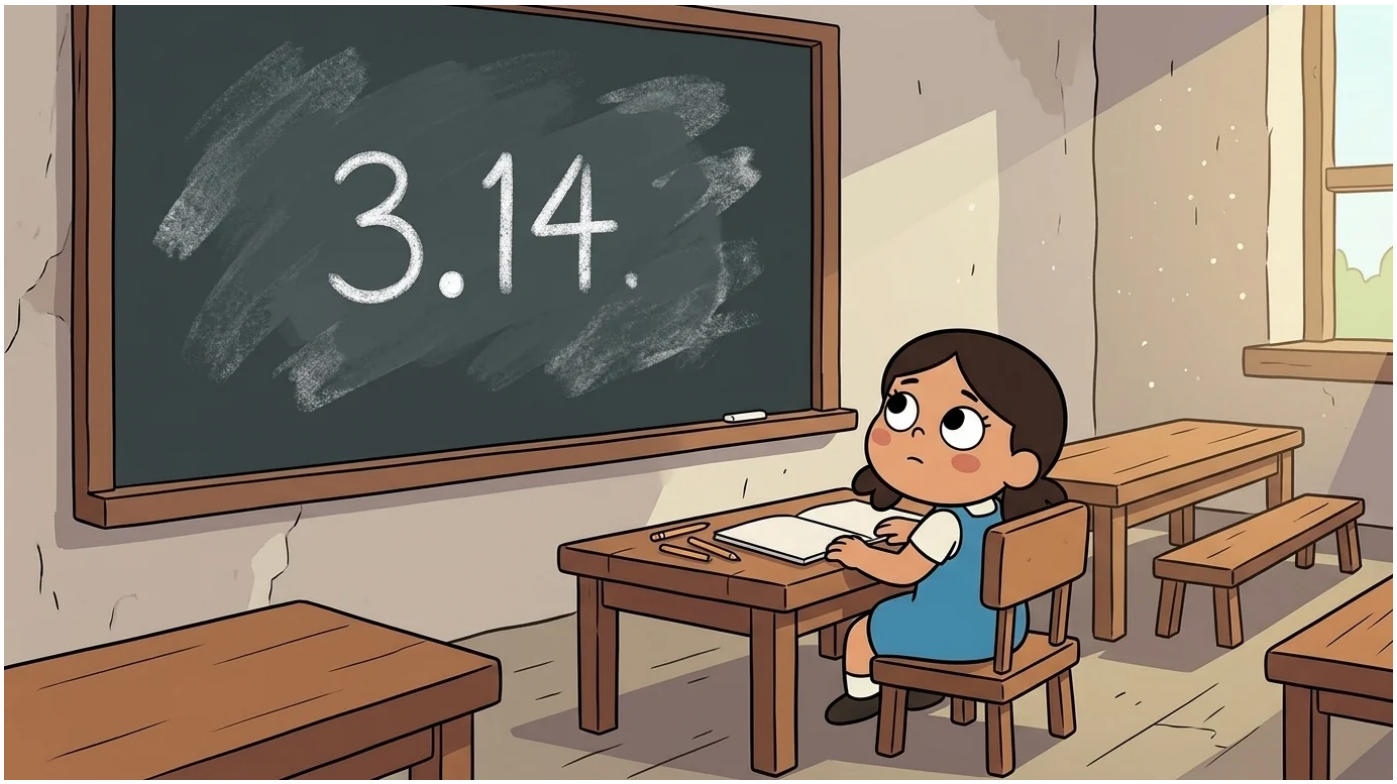
Dot was, for most of her childhood, *the smallest person in every room.*

This is, when you are a child, a *strange feeling.* Adults always see you. Adults often address you as if you were younger than you are. Adults sometimes step around you because they did not notice you were there, and then they apologize, and then they ask if you are all right, and then they pat you on the head, which is not necessarily what a small person wants. The world, when you are small, is *full of large things politely trying not to bump into you.*

Dot — whose given name was *Mila*, though she has gone by Dot since her older brothers nicknamed her at age four — grew up in a household of *tall, loud, fast-moving siblings.* She had four older brothers. They were all, even by adult standards, large. By the time her oldest brother was seventeen and Mila was four, he could pick her up under one arm and carry her up the cottage's stairs without breaking stride. (He did this often. He thought it was funny. Mila thought it was *deeply unfair* and would have *much preferred to walk up the stairs herself,* but she did not yet have the words to object.)

Her brothers called her Dot because she was *a small dot in the family painting.* She did not mind the name. She liked it, actually. *Dot* felt accurate. She was a dot.

But — and this is the chapter's essential fact — *Dot understood, very early, that her smallness did not mean she was unimportant.* She understood that *small things, placed correctly, could mean a great deal.*



This was a thing she learned from her grandmother.

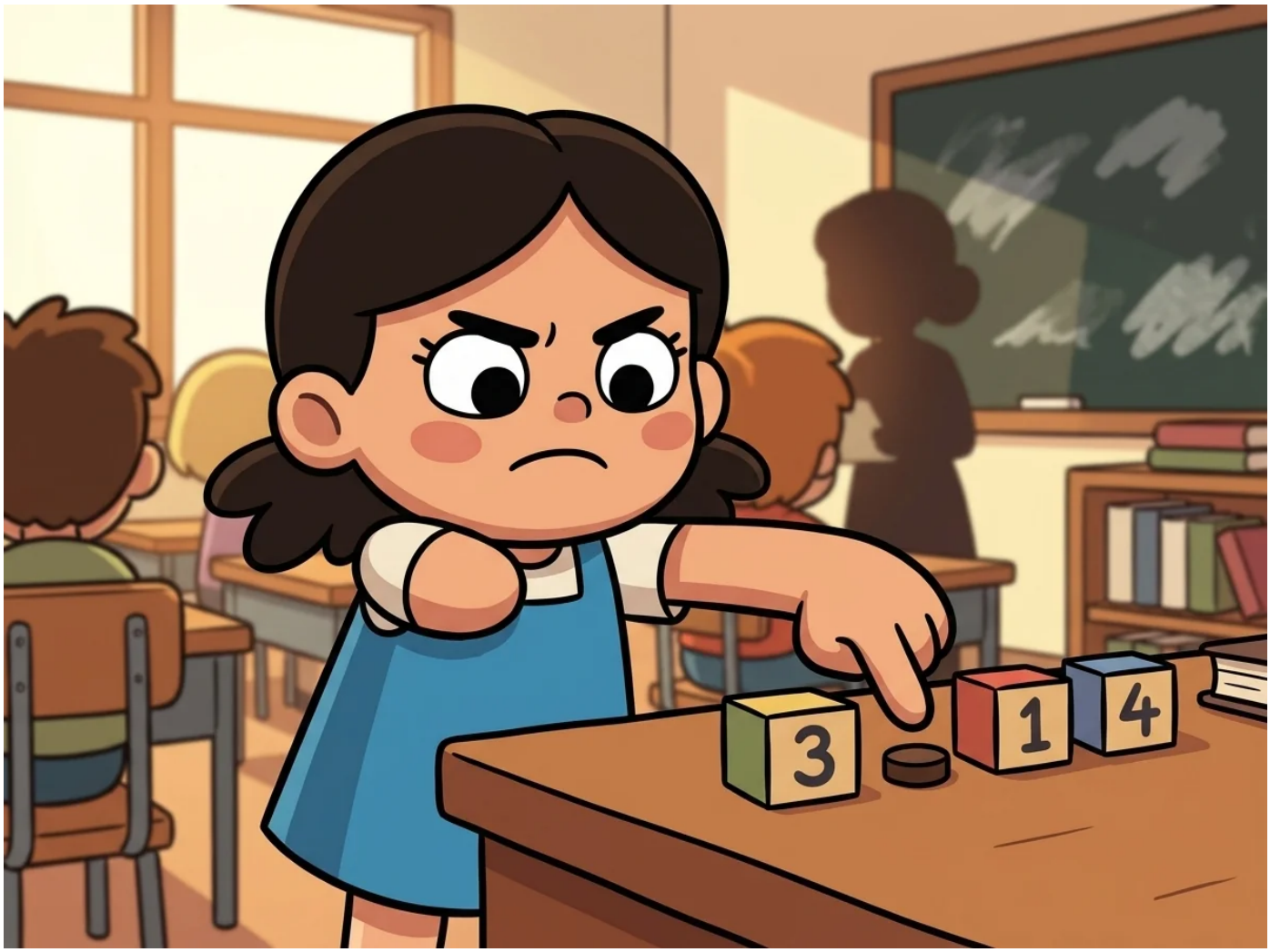
Her grandmother — *Vade*, who had been a calligrapher in her younger years and who still maintained the family's library of carefully-lettered books — had taught Dot to read by *pointing at each letter with a fingertip.* The fingertip touched the letter for a heartbeat. The fingertip moved to the next letter. Vade said, very slowly: *"Each letter is small. Each letter only has meaning *in its place.* If you move the letter, the word changes. If you remove the letter, the word breaks. Small things matter when their *place* matters."*

Dot understood this immediately. She had been four. She had learned to read in three months.

Over the next several years, Dot internalized the principle. *Small things, placed correctly, were not unimportant. They were the placement itself.* A comma in the wrong place changed a sentence. A grain of sand in a clock's gears stopped the clock. A small candle in a dark room *was* the light.

When Dot was nine, she encountered decimals at the village school. The schoolteacher wrote on the board:

3.14.



The schoolteacher said: *"This is a decimal number. The dot between the 3 and the 14 is called *the decimal point.* It separates the whole part from the fractional part. The 3 is three whole units. The 1 is one-tenth. The 4 is four-hundredths."*

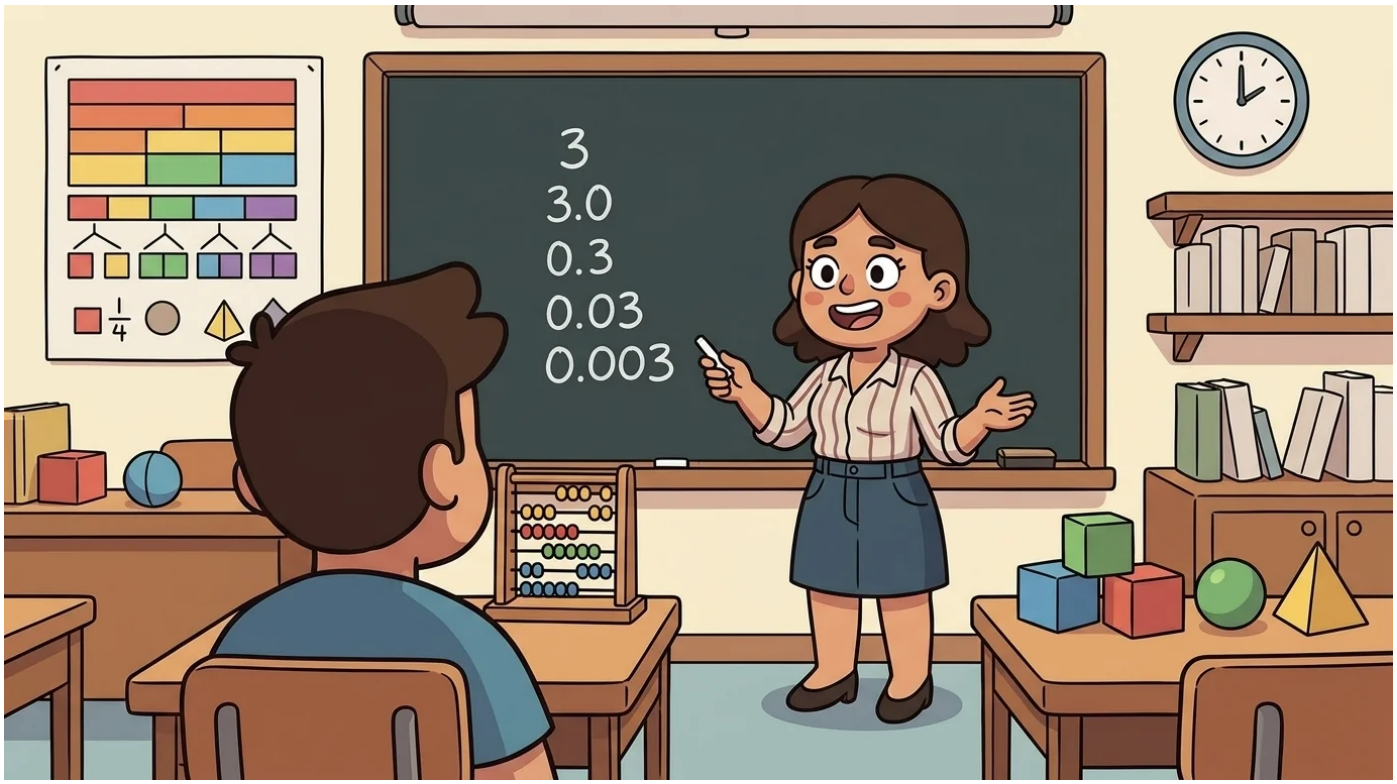
Dot stared.

She said: *"The dot is the most important part of the number."*

The schoolteacher said: *"Why?"*

Dot said: *"It tells you *which digit means what.* Without the dot, you would have the number 314, which is three hundred fourteen. *With* the dot, you have 3.14, which is three-and-some-fraction. The dot decides whether the 3 is *three units* or *three hundred units.* The dot is small. The dot decides everything."*

The schoolteacher was, by his own later admission, *struck.* He had been teaching decimals for fifteen years. He had not heard anyone state the case for the decimal-point's importance quite that directly.



He said: *"Yes. That is exactly right. The decimal point is small but it is *essential.* Its position determines what every digit means. Move the decimal point one place to the right, and every digit moves one place to the *left* in value — every digit becomes ten times bigger. Move the decimal point one place to the left, and every digit becomes ten times smaller."*

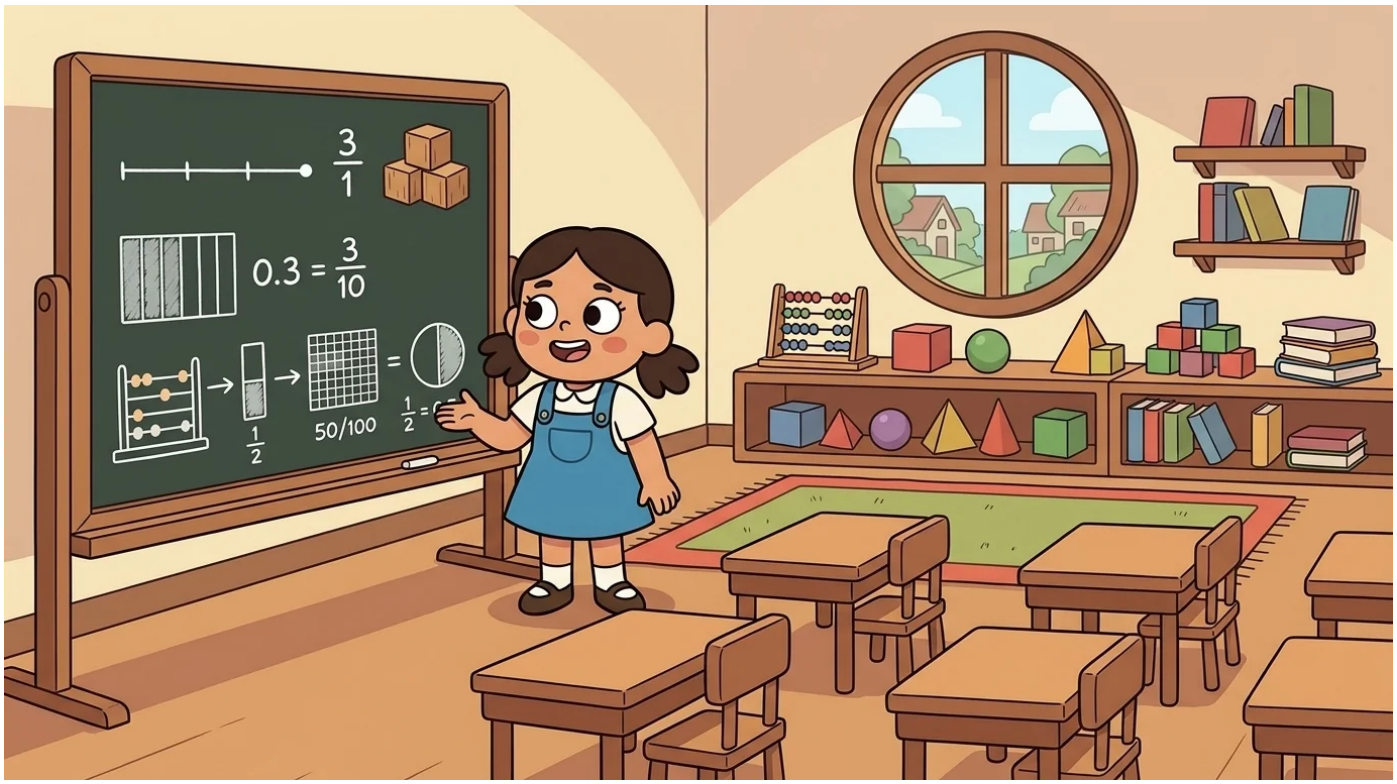
Dot nodded. She thought of her grandmother. *Small things, placed correctly, were the placement itself.* The decimal point was such a small thing. The decimal point *was* the placement.

She has been teaching the decimal point ever since. She studied at the FractionForge academy from sixteen to nineteen. She joined the faculty at twenty. She is now thirty-one. She is no longer the smallest person in every room (she had a late growth spurt and is now of slightly-above-average height) but she is *still called Dot* and she does not mind.

In her classroom, she begins every first-day lesson the same way. She writes on the board:

3 3.0 0.3 0.03 0.003.

She turns to the class. She says: *"These all contain the digit 3. They are not the same number. What makes them different?"*



The children — always — say *the dot.* (Sometimes they say *the decimal point,* but most of them call it the dot.)

Dot smiles. She says: *"The dot is small. The dot decides everything. *3* is three whole units. *3.0* is also three whole units (the zero is just confirming there is nothing in the tenths position). *0.3* is three tenths. *0.03* is three hundredths. *0.003* is three thousandths. The digit 3 is the same. The *place* changes. The *value* changes by a factor of ten with every step."*

She then writes: *1/2 = 0.5 = 50%.* She says: *"Decimals are fractions. *0.5* means *5/10*, which equals *1/2.* Percentages are fractions with denominator one hundred. *50%* means *50/100*, which equals *1/2.* These three are the *same value* in three different writing systems. The fraction, the decimal, the percent. All three are equivalent."*

The children — always — see it. Once Dot has made the connection between fractions and decimals, the rest of the curriculum opens up.

When children ask whether decimals are hard, Dot always says the same thing:

*"They are not hard. They are *fractions with denominator a power of ten.* The decimal point's *place* tells you which power of ten the denominator is. Tenths after the dot is *over ten.* Hundredths is *over one hundred.* Thousandths is *over one thousand.* Decimals are just a different way to write the same fractions you already know."*

She still has her grandmother's calligraphy slate. She brings it to class sometimes. She points at letters. She says: *"My grandmother taught me about placement before I knew there was such a thing as a decimal point. The same idea. Different letters."*

Listen along + meet more of the cast at:

`https://spark-and-anvil.com/cast/fractionforge/dot`

Stretch

Stretch was, before he was a teacher, *a glass-blower's apprentice.*

The glass-blowing workshop where he apprenticed was in the harbour town of *Anneal*, on the kingdom's southern coast. The workshop was run by an old master glass-blower named *Forge* (which was, the workshop hands joked, a *very on-the-nose* name for a glass-blower; Forge had heard the joke every day for forty years and had stopped finding it funny by year three, but he tolerated it). The workshop produced *glass tubes* for various uses: laboratory equipment, alchemists' instruments, decorative ornaments, and — its specialty — *medicine vials* of perfectly uniform diameter.

The medicine-vial work was the workshop's bread and butter.

The kingdom's apothecaries needed glass vials in *very specific sizes* — half-thumb diameter, one-thumb diameter, two-thumb diameter — and the vials had to be *perfectly uniform* within each size. A medicine-vial whose width varied along its length was useless: the dose could not be measured reliably, the cork would not seat correctly, the contents would settle unevenly. The apothecaries paid premium prices for vials that were *uniform within a hair's-width* across their entire length.

Forge's workshop made the best vials in three provinces. The secret was *the stretching.*

Stretch — whose given name was *Hadrian*, though everyone called him Stretch by the time he was sixteen — had been apprenticed to Forge at twelve. His first year was spent sweeping the workshop. His second year was spent feeding the furnace. His third year, finally, was spent at the *marble slab.*

The marble slab was the centerpiece of the workshop. It was a long flat piece of polished marble, about six feet long and two feet wide, set on a sturdy iron frame. The slab was used for *stretching molten glass.* A glass-blower would take a small ball of molten glass from the furnace, attach it to a long iron rod, and *roll it back and forth across the marble slab* — patiently, evenly — until *every part of the glass had the same diameter.*

This was the operation Stretch eventually understood as *finding a common denominator.*

The molten glass, when it left the furnace, was *not uniform.* It was *thick in some places and thin in others.* It was, in fraction terms, *like fractions with different denominators — not directly comparable until brought to a common scale.* To make the glass into a usable tube, you had to *roll it.* The rolling forced *every part of the glass to the same diameter.* Once every part was the same diameter, the tube was *uniform.* You could cut it into vials. Each vial was the same width as every other.

Stretch learned this, slowly, over years. By the time he was eighteen he could roll a six-foot tube of one-thumb diameter that was uniform within a hair's-width across its entire length. He was, by workshop standards, *very good.*

He stayed at the workshop until he was twenty-six. Then his father died, and Stretch inherited his father's small farm, and he left the workshop to manage the farm. He stayed on the farm for two years. He hated it. (He had no aptitude for sheep and the farm had been a sheep-farm.) He sold the farm. He returned to Anneal.

But Forge had retired. The workshop had a new master who did not need an old apprentice. Stretch, who was now twenty-eight and unsure what to do with himself, sat on the harbour wall and thought.

What he thought, eventually, was that *the rolling-the-glass-to-uniform-diameter operation was an arithmetic operation.* He had spent his apprenticeship doing it physically. But the *same operation — bring all the parts to a common scale —* was the *same operation needed to add or compare fractions with different denominators.*

The connection was clear, once he saw it.

He went to the FractionForge academy. He told the academy master what he had been thinking. The academy master listened patiently. The academy master said: *"You are correct. Bringing fractions to a common denominator is the same operation as bringing the molten glass to a uniform diameter. We need teachers who can explain this physically. Would you teach?"*

Stretch said yes.

That was twelve years ago. He has been teaching common denominators ever since.

In his classroom, he begins every first-day lesson the same way. He brings, from Anneal (he goes back twice a year to visit the new workshop's owner and the new owner has been kind enough to supply him with a teaching prop), *a small piece of stretched glass tube.* He places it on the desk. He holds up a small caliper. He measures the tube at one end, in the middle, and at the other end. He says: *"This tube is one-thumb diameter at every point. To make it that way, the glass-blower rolled the molten glass on a marble slab. The rolling brought every part to a common diameter."*

The children — always — examine the tube.

Then he writes on the board: *1/3 + 1/4.* He says: *"These two fractions have different denominators. Three and four. They cannot be added directly. We need to bring them to a *common denominator.* The smallest common denominator is twelve. *1/3 scaled to twelfths is 4/12. 1/4 scaled to twelfths is 3/12.* Now we can add them: 4/12 + 3/12 = 7/12. They have been *stretched to a common scale.*"*

The children — always — see it. The connection between the glass-tube and the fraction-arithmetic clicks.

When children ask whether common denominators are hard, Stretch always says the same thing:

*"They are not hard. They are *stretching.* You roll the fractions out to a common base. Once they share a base, you can add them, subtract them, compare them. It is the same operation as rolling glass on a marble slab. Bring everything to a uniform scale."*

He still has the small caliper. The children sometimes ask to measure the tube. He always lets them.

He sometimes adds: *"If you ever go to Anneal, the workshop is still rolling glass on a marble slab. The principle is older than I am."*

Listen along + meet more of the cast at:

`https://spark-and-anvil.com/cast/fractionforge/stretch`

About Spark & Anvil

FractionForge is one of 140 educational iOS apps from Spark & Anvil — a 501(c)(3) public charity making free, ad-free, tracking-free learning apps for ages 9-14.

Every app uses distributed-narrative methodology: named recurring characters embody curricular concepts. The cast you just met appears in the matching app, in mentor scaffolding, in puzzle solutions, in celebration moments. Reading the chapters first means meeting old friends when you open the app.

Visit **spark-and-anvil.com** to download apps, browse the full portfolio, or donate to support the work.