Reckonquest — Activity Book

A Spark & Anvil activity book — puzzles, questions, and fun from Reckonquest.

Write in the book, circle your answers, and check the Answer Key at the back.

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1. systematic guess-and-check → 2-variable systems (the chickens-and-cows riddle)

Circle the correct answer

1. A chicken has how many legs?
(A) 3 (B) 4 (C) 1 (D) 2

2. A cow has how many legs?
(A) 4 (B) 3 (C) 6 (D) 2

3. Each animal — chicken OR cow — has how many heads?
(A) 0 (B) 2 (C) it depends (D) 1

4. How many legs do 4 chickens have in total?
(A) 16 (B) 6 (C) 4 (D) 8

5. How many legs do 3 cows have in total?
(A) 6 (B) 7 (C) 9 (D) 12

6. A pen holds 5 chickens and 2 cows. How many heads are there?
(A) 9 (B) 14 (C) 7 (D) 10

Write the answer

7. A pen holds 5 chickens and 2 cows. How many legs are there?
Answer: ________

8. "Guess and check" means:
Answer: ________

9. There are 10 heads. If you GUESS they are all chickens, how many legs would that be?
Answer: ________

10. Swapping ONE chicken for ONE cow changes the total leg count by:
Answer: ________

Match each question to its answer

1. You guessed all 10 chickens = 20 legs, but the riddle says 28 legs. You need 8 MORE legs. Each cow-swap adds 2 legs. How many cows are there? →
2. So for 10 heads and 28 legs, how many chickens and cows are there? →
3. CHECK the answer: 6 chickens and 4 cows — how many legs is that? →
4. The riddle: 30 heads and 100 legs. If you GUESS all chickens, how many legs is that? →
5. 30 heads, 100 legs. Your all-chickens guess gave 60 legs. How many MORE legs are needed? →

(A) 60
(B) 4
(C) 6 chickens and 4 cows
(D) 28
(E) 40

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2. name the unknowns and write the relationships (let x = ...)

Circle the correct answer

11. The first step Lettie takes on any word problem is to:
(A) name the unknown with a letter (let x = ...) (B) look for the word 'total' (C) guess the answer (D) add all the numbers

12. "A number increased by 7" is written as:
(A) x + 7 (B) x - 7 (C) 7 - x (D) 7x

13. "5 less than a number" is written as:
(A) 5x (B) x - 5 (C) 5 - x (D) x + 5

14. "Twice a number" is written as:
(A) x + 2 (B) 2x (C) x/2 (D) x - 2

15. "The number of legs is twice the number of heads" becomes:
(A) legs = 2 x heads (B) legs = heads - 2 (C) heads = 2 x legs (D) legs = heads + 2

16. If x is the number of apples and there are 3 more oranges than apples, the number of oranges is:
(A) 3 - x (B) 3x (C) x - 3 (D) x + 3

Write the answer

17. Why give the unknown a LETTER instead of just guessing a number?
Answer: ________

18. "A number and its double add to 18" becomes:
Answer: ________

19. Solve x + 2x = 18. What is x?
Answer: ________

20. "The product of a number and 4" is:
Answer: ________

Match each question to its answer

1. If a rectangle's length is 3 more than its width w, its length is: →
2. "Half of a number is 12" becomes: →
3. Lettie writes 'let c = number of chickens.' The number of chicken LEGS is then: →
4. "7 more than twice a number" is: →
5. The key difference between '5 less than x' and '5 minus x' is: →

(A) w + 3
(B) x/2 = 12
(C) 2x + 7
(D) 2c
(E) the order: x - 5 versus 5 - x

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3. schema-based instruction — recognize the problem type and label every number (anti-keyword)

Circle the correct answer

21. Kindra's first question on a word problem is:
(A) "can I add everything?" (B) "what TYPE of problem is this?" (C) "what is the biggest number?" (D) "which key word is here?"

22. Why are 'key word' tricks (like 'total means add') UNRELIABLE?
(A) the same word can mean different operations in different problems (B) key words are always right (C) they are too slow (D) they use letters

23. "Sara had some marbles. She gave away 4 and has 9 left. How many did she start with?" The word 'left' here means you should:
(A) subtract, because 'left' means minus (B) ignore the 4 (C) multiply (D) ADD (9 + 4) to undo the giving-away

24. Tagg's habit is to write a UNIT next to every number. For '6 chickens' he labels:
(A) 6 total (B) 6 legs (C) 6 chickens (the unit is 'chickens') (D) just 6

25. In the chickens-and-cows problem, the number '20' could be heads OR legs. Labeling units prevents:
(A) mixing up which number means what (B) drawing pictures (C) solving the problem (D) using equations

26. Two problems: (A) 'Tom has 3 bags of 4 apples' and (B) 'Tom has 3 apples and 4 apples.' They differ because:
(A) both use the word 'has' (B) both are subtract (C) they are the same (D) A is a multiply structure (3x4), B is an add structure (3+4)

Write the answer

27. A 'coin problem' schema usually has two facts:
Answer: ________

28. The chickens-and-cows schema has which two relationships?
Answer: ________

29. Recognizing that coins and chickens-and-cows share a schema helps because:
Answer: ________

30. "There are 6 legs." Tagg reminds you 6 is a count of:
Answer: ________

Match each question to its answer

1. A 'rate problem' schema involves: →
2. The BEST reason to ignore the word 'altogether' as an automatic 'add' signal is: →
3. Kindra sorts a new problem by asking what is UNKNOWN and how the knowns RELATE. This is: →

4. "A ribbon is cut into 4 equal pieces of 5 cm." The total length uses which structure? →
5. Labeling units also helps you CHECK an answer because: →

(A) 'altogether' can appear in multiply or subtract problems too
(B) identifying the schema (structure) of the problem
(C) equal groups -> multiply (4 x 5)
(D) an amount per unit of something (like miles per hour)
(E) the answer's unit should match what the question asked for

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4. draw a bar/tape diagram of the relationships

Circle the correct answer

31. A bar (tape) diagram represents a quantity as:
(A) a number line only (B) a circle graph (C) a rectangle whose length stands for the amount (D) a single dot

32. In a 'part-part-whole' diagram, the whole bar equals:
(A) the difference of the parts (B) the two part-bars joined end to end (C) twice a part (D) one part only

33. "Ann has 12 stickers; she has 5 more than Ben." A comparison bar for Ben is:
(A) a bar 5 shorter than Ann's (Ben = 12 - 5 = 7) (B) a bar of length 5 (C) the same length as Ann's (D) a bar 5 longer than Ann's

34. Why draw a diagram BEFORE computing?
(A) it shows the relationships so you pick the right operation (B) it wastes time (C) diagrams are only decoration (D) it replaces the answer

35. Equal groups (like '4 boxes of 6') are drawn as:
(A) 4 equal bars each labeled 6 (B) a bar labeled 24 with no parts (C) one bar labeled 10 (D) two bars of different sizes

36. A bar split into 3 equal parts with the whole labeled 15 shows each part is:
(A) 5 (B) 3 (C) 12 (D) 45

Write the answer

37. For chickens-and-cows, a helpful diagram draws:
Answer: ________

38. "A number and its double" as a bar diagram is:
Answer: ________

39. If a diagram shows the whole as 3 equal parts and TWO of those parts are shaded (30 total), the shaded amount is:
Answer: ________

40. A diagram helps most when a problem has:
Answer: ________

Match each question to its answer

1. "Twice as many apples as pears; 18 fruit total." A bar diagram (pears = 1 unit, apples = 2 units) shows total units: →
2. From q11, the number of APPLES is: →
3. The 'unit bar' method turns a comparison into: →
4. A diagram and an equation are related because both: →
5. "After spending 8 dollars, Mia has half her money left." A bar for her original money split in half shows one half equals: →

(A) 3 units = 18, so 1 unit = 6
(B) 12
(C) 8 dollars (the amount spent), so she started with 16
(D) counting equal units that make up the whole
(E) capture the same relationships in different forms

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5. translate 'in N years / N years ago' into equations

Circle the correct answer

41. If Mia is x years old now, in 5 years she will be:
(A) 5x (B) x - 5 (C) 5 - x (D) x + 5

42. If Sam is x years old now, 3 years ago he was:
(A) x + 3 (B) 3x (C) x - 3 (D) 3 - x

43. The KEY fact about ages over time is that everyone's age changes by:
(A) double for older people (B) nothing (C) the same amount (D) different amounts

44. Ana is 12 and Ben is 8. The difference in their ages in 10 years will be:
(A) 20 (B) 4 (the same as now) (C) 0 (D) 14

45. Lettie sets 'let x = Ben's age now.' If Ann is 5 years older, Ann's age now is:
(A) x + 5 (B) x - 5 (C) 5x (D) 5 - x

46. "In 4 years, Ben (now x) will be..." is written as:
(A) x - 4 (B) x + 4 (C) 4x (D) x + 4 years ago

Write the answer

47. "Ana is 3 times Ben's age. Ben is 6." Ana's age is:
Answer: ________

48. "The sum of Ann's and Ben's ages is 20; Ann is 4 older." Using x for Ben:
Answer: ________

49. Solve x + (x + 4) = 20 for Ben's age:
Answer: ________

50. A common age-problem trap is forgetting that 'in N years':
Answer: ________

Match each question to its answer

1. Now Ben is x. "In 2 years Ben will be 15" means: →
2. "5 years ago, Ann (now a) was 10." This means: →
3. Why is the age GAP a useful anchor in age problems? →
4. "Ann is twice Ben's age now." Using x for Ben, Ann is: →
5. "Ann is twice Ben's age; their ages add to 27." Set up: →

(A) a - 5 = 10, so a = 15
(B) x + 2 = 15, so x = 13
(C) 2x
(D) x + 2x = 27
(E) it stays the same past, present, and future

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6. count + value constraints, unit-labeling (coins, stamps)

Circle the correct answer

51. A coin problem always links two facts:
(A) only the number of coins (B) how many coins there are, and how much they are worth (C) only the coin colors (D) the year on the coin

52. Tagg warns that '5' in a coin problem could mean 5 coins OR 5 cents. The fix is:
(A) always assume cents (B) ignore the 5 (C) label it: 5 coins vs 5 cents (D) always assume coins

53. The VALUE of d dimes (in cents) is:
(A) 10d (B) 10 + d (C) d + 10 (D) d/10

54. The value of q quarters (in cents) is:
(A) q + 25 (B) 25 + q (C) 25q (D) q/25

55. "12 coins, all dimes and quarters." If d is dimes, the number of quarters is:
(A) 12 - d (B) d + 12 (C) d - 12 (D) 12d

56. The value equation for d dimes and (12 - d) quarters totaling 210 cents is:
(A) 10d + 25d = 210 (B) 35(12) = 210 (C) d + (12 - d) = 210 (D) 10d + 25(12 - d) = 210

Write the answer

57. Solve 10d + 25(12 - d) = 210. How many dimes?
Answer: ________

58. Why keep VALUE in cents rather than mixing dollars and cents?
Answer: ________

59. A coin problem has the SAME structure as chickens-and-cows because both have:
Answer: ________

60. "20 stamps worth 5 cents and 10 cents each; total 150 cents." If f = five-cent stamps:
Answer: ________

Match each question to its answer

1. Solve 5f + 10(20 - f) = 150. Number of five-cent stamps: →
2. If a coin answer comes out as a FRACTION of a coin (like 3.5 dimes), it means: →
3. The count equation for coins is usually: →
4. The value equation for coins multiplies: →
5. Using the count equation to write one coin in terms of the other lets you: →

(A) the numbers of each coin type add to the total number of coins
(B) 10
(C) each coin count by that coin's value, then adds
(D) turn two unknowns into one, so a single equation solves it
(E) an error — coin counts must be whole numbers

7. combined rates — work done per unit time (painters, hoses, machines)

Circle the correct answer

61. If a hose fills a tank in 6 hours, its RATE is:
(A) 6 hours per tank only, no rate (B) 1 tank per hour (C) 1/6 of the tank per hour (D) 6 tanks per hour

62. When two workers work TOGETHER, you combine them by:
(A) subtracting their rates (B) adding their rates (C) multiplying their times (D) adding their times

63. Hose A fills 1/6 per hour, hose B fills 1/3 per hour. Together their rate is:
(A) 1/9 per hour (B) 1/3 per hour (C) 1/2 per hour (D) 2/9 per hour

64. If the combined rate is 1/2 tank per hour, filling ONE tank takes:
(A) 1/2 hour (B) 4 hours (C) 1 hour (D) 2 hours

65. The relationship time = job / rate means faster rate gives:
(A) the same time (B) longer time (C) no time (D) shorter time

66. "3 painters paint a room in 4 hours." The TOTAL work is:
(A) 7 painter-hours (B) 4 painter-hours (C) 12 painter-hours (D) 3 painter-hours

Write the answer

67. If the room is 12 painter-hours of work, how long for 6 painters (same speed)?
Answer: ________

68. More workers means LESS time. This is an example of:
Answer: ________

69. The most common WRONG move in a together-problem is:
Answer: ________

70. A sanity check for a together-time is that it must be:
Answer: ________

Match each question to its answer

1. Worker A does 1/4 job/hour, worker B does 1/4 job/hour. Together: →
2. "A machine makes 60 parts per hour." In 3 hours it makes: →
3. To find how long to make 300 parts at 60 parts/hour: →
4. Lettie labels the three quantities in every rate problem as: →
5. One pipe fills in 2 hours, another drains in 4 hours. Net filling rate is: →

(A) 300 / 60 = 5 hours
(B) rate, time, and amount (rate x time = amount)
(C) 1/2 - 1/4 = 1/4 tank per hour
(D) 180 parts
(E) 1/2 job/hour, so 2 hours for the job

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8. weighted averages / concentration mixing

Circle the correct answer

71. A mixture problem combines two things and asks about the:
(A) color of the mixture (B) temperature only (C) the name of the mixer (D) combined amount and its overall price or concentration

72. Mixing 2 lb of nuts at \$3/lb with 3 lb at \$8/lb: the total COST is:
(A) \$40 (B) \$5 (C) \$30 (2x3 + 3x8 = 6 + 24) (D) \$11

73. From q02, the total WEIGHT of the mixture is:
(A) 3 lb (B) 2 lb (C) 6 lb (D) 5 lb

74. The PRICE PER POUND of that mixture is:
(A) \$6/lb (\$30 / 5 lb) (B) \$30/lb (C) \$5.50/lb (D) \$11/lb

75. The blend price \$6/lb is NOT the simple average of \$3 and \$8 (\$5.50) because:
(A) \$6 is the simple average (B) there is more of the \$8 nut, pulling the average up (C) prices don't average (D) the mixer made an error

76. A 'weighted average' weights each value by:
(A) how much of it there is (B) the order it was added (C) nothing (D) its color

Write the answer

77. In a SALT-solution problem, the amount of PURE SALT is:
Answer: ________

78. Mix 2 L of 10% salt with 3 L of 20% salt. Total pure salt is:
Answer: ________

79. From q08, the blend's concentration (0.8 L salt in 5 L) is:
Answer: ________

80. The conserved quantity that makes mixture problems solvable is:
Answer: ________

Match each question to its answer

1. Doodle's diagram for a mixture stacks two bars (each ingredient) whose heights show: →
2. A blend's concentration is ALWAYS: →
3. If a mixture answer gives a concentration ABOVE both ingredients, you know: →
4. "How much 30% solution to add to 4 L of 10% to make 20%?" Kindra recognizes this as: →
5. For q14 with x liters of 30% added: the salt equation is: →

(A) the amount of pure ingredient in each
(B) 0.10(4) + 0.30x = 0.20(4 + x)
(C) there's an error — re-check the set-up
(D) between the two ingredient concentrations
(E) a mixture problem — set up conserved salt

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9. start from the goal and reverse the steps

Circle the correct answer

81. The work-backwards strategy is best when a problem gives you:
(A) the answer already (B) no steps (C) the ENDING and a chain of steps, and asks for the start (D) only the start

82. To UNDO 'add 5', you:
(A) multiply by 5 (B) divide by 5 (C) subtract 5 (D) add 5 again

83. To UNDO 'multiply by 3', you:
(A) subtract 3 (B) divide by 3 (C) add 3 (D) multiply by 3

84. When working backwards, you also reverse the ORDER of steps. If forward was 'add 2 then double', backward is:
(A) subtract 2, then halve (B) halve, then subtract 2 (C) add 2, then halve (D) double, then subtract 2

85. "I think of a number, add 4, get 11." Working backwards, the number is:
(A) 4 (B) 7 (11 - 4) (C) 15 (D) 44

86. "Double a number, then add 3, get 17." The number is:
(A) 40 (B) 7 (C) 8.5 (D) 10

Write the answer

87. Retta's habit is to point herself at:
Answer: ________

88. "After spending half her money, then $3 more, Mia has $5." Working backwards, before the $3 she had:
Answer: ________

89. Continuing q08: $8 was what remained after spending HALF. So she started with:
Answer: ________

90. Working backwards is really the idea of applying:
Answer: ________

Match each question to its answer

1. To UNDO 'subtract 7', you: →
2. "A number divided by 4, then plus 1, equals 6." The number is: →
3. Working backwards is equivalent to solving an equation by: →
4. A great CHECK after working backwards is to: →
5. "A tank was filled, then half drained, then 4 L added, ending at 10 L." Before adding 4 L: →

(A) run your start value FORWARD and see if it gives the ending
(B) add 7
(C) 20
(D) 'undoing' operations one at a time to isolate the unknown
(E) 6 L (10 - 4)

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10. solve a tiny version first to see the structure

Circle the correct answer

91. Tad's strategy when a problem feels too big is to:
(A) guess the answer (B) give up (C) make it even bigger (D) solve a tiny version first

92. Why solve a SMALLER version first?
(A) it wastes time (B) the small case reveals the pattern or method for the big one (C) it changes the answer (D) small cases are the goal

93. "How many handshakes if 10 people all shake hands once?" A smart smaller case is:
(A) count nothing (B) try 2, then 3, then 4 people and look for a pattern (C) try 100 people (D) try 10 immediately

94. Handshakes for 2, 3, 4 people are 1, 3, 6. The pattern is:
(A) add the next-smaller count each time (triangular numbers) (B) double each time (C) no pattern (D) always add 1

95. Handshakes for n people equals n times (n-1) divided by 2. For 10 people:
(A) 90 (B) 45 (C) 55 (D) 100

96. In chickens-and-cows, Tad first solves it with just:
(A) only legs (B) a small number of heads (like 3) to see the method (C) no heads (D) 1000 heads

Write the answer

97. "Sum of 1 + 2 + 3 + ... + 100?" A smaller case to try is:
Answer: ________

98. The small case reveals a pairing: 1+2+3+4 pairs as (1+4)+(2+3) = 5+5. For 1..100 the sum is:
Answer: ________

99. After solving small cases, the KEY step is to:
Answer: ________

100. Trying a smaller case is closely related to Nestor's recursion because both:
Answer: ________

Match each question to its answer

1. "How many small squares on an 8x8 checkerboard?" A smaller case is: →
2. Small cases 2x2=4 and 3x3=9 suggest an nxn grid has: →
3. When is trying a smaller case NOT helpful? →
4. Trying MULTIPLE small cases (2, 3, 4) is better than one because it: →
5. "A knockout tournament with 16 teams — how many games to find a winner?" Small case (2 teams): →

(A) count a 2x2 and 3x3 first (4 and 9)
(B) when the small case is so tiny it hides the pattern
(C) n squared small squares (8x8 = 64)
(D) 1 game; 4 teams -> 3 games; pattern is teams minus 1
(E) lets you see how the answer CHANGES, revealing the rule

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11. constraint + logical elimination in classic brain-teasers

Circle the correct answer

101. In a balance-scale puzzle, one pan going DOWN means that side is:
(A) lighter (B) equal (C) heavier (D) empty

102. If the scale BALANCES, the two sides are:
(A) impossible (B) equal in weight (C) both heavy (D) both empty

103. You have 9 coins; one is heavier. Split into three groups of 3 and weigh two groups. If they balance, the heavy coin is:
(A) in the third (unweighed) group (B) impossible to tell (C) in the right group (D) in the left group

104. With 9 coins (one heavy), how many weighings are needed to find it (best strategy)?
(A) 3 (B) 9 (C) 1 (D) 2

105. Why is splitting into THREE groups (not two) smart for weighing puzzles?
(A) it uses more coins (B) three is a lucky number (C) the balance has three outcomes (left, right, equal), each pointing to a third (D) two groups is illegal

106. Retta approaches a logic riddle by:
(A) adding the numbers (B) tracking what each clue rules OUT until one possibility remains (C) ignoring the clues (D) guessing an answer

Write the answer

107. In a river-crossing, a move is only allowed if AFTER it:
Answer: ________

108. The clever river-crossing insight is that sometimes you must:
Answer: ________

109. In a jug-pouring riddle, the amounts you can measure are limited to:
Answer: ________

110. A grid/table is a great tool for a logic riddle because it:
Answer: ________

Match each question to its answer

1. You have 8 coins, one FAKE (lighter). Weigh 3 vs 3. If the left is lighter, the fake is: →
2. Continuing q11 (fake among 3, lighter): weigh 1 vs 1 of them. If they balance, the fake is: →
3. The heart of these riddles is: →
4. "3 friends, 3 hats (red, blue, green); Ann isn't red, Ben is green." Ann's hat is: →
5. From q14, Carol's hat is: →

(A) among the 3 on the lighter (left) side
(B) using each result to eliminate possibilities systematically
(C) the third coin (the one not weighed)
(D) blue (Ben has green, Ann not red, so Ann is blue)
(E) red

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12. substitute back + reasonableness (the look-back step)

Circle the correct answer

111. The 'look-back' step in problem solving means:
(A) check your answer against the original problem (B) erase your work (C) solve it again from scratch (D) look at the first step only

112. The most reliable check is to:
(A) assume it's right (B) substitute the answer back into the original equation/story (C) compare to a friend (D) check only the last step

113. You solve 3x + 4 = 19 and get x = 5. Substituting back: 3(5) + 4 =
(A) 16 (B) 23 (C) 15 (D) 19 (checks out)

114. A chickens-and-cows answer of 7 chickens and 5 cows should be checked against BOTH:
(A) the head count AND the leg count (B) the colors (C) only the heads (D) only the legs

115. An answer with the WRONG UNIT (legs when the question asked for chickens) is:
(A) fine, numbers are what matter (B) wrong, even if the number is right (C) impossible (D) half right

116. 'Estimation' helps checking by:
(A) giving a ballpark to compare your exact answer against (B) making it harder (C) replacing the exact answer (D) hiding errors

Write the answer

117. "A shirt is 20% off $40." A student answers $60. Vetta's reasonableness check flags it because:
Answer: ________

118. A negative answer for a COUNT of people means:
Answer: ________

119. Vetta's core question is:
Answer: ________

120. A speed of 500 miles per hour for a BICYCLE is:
Answer: ________

Match each question to its answer

1. Checking is MOST valuable because it catches: →
2. For 'x + (x + 4) = 20' you find x = 8. Checking: 8 + 12 = →
3. A mixture concentration answer of 25% from ingredients of 10% and 20% is: →
4. A good habit is to check an answer using a DIFFERENT method than you solved with because: →
5. Estimate 19 x 21 quickly to check an exact answer: →

(A) if both agree, you're much more confident it's right
(B) set-up mistakes that arithmetic alone won't reveal
(C) 20 (correct)
(D) unreasonable — a blend must be BETWEEN 10% and 20%
(E) about 400 (near 20 x 20)

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13. write the rule that works for any numbers

Circle the correct answer

121. To 'generalize' a solution means to:
(A) solve only the given numbers (B) write a rule that works for ANY numbers, not just the ones given (C) make it more specific (D) guess the answer

122. Chickens-and-cows with H heads and L legs: the number of cows is:
(A) L - H (B) (L - 2H) / 2 (C) H - L (D) 2H - L

123. Using cows = (L - 2H)/2 with H = 10 heads, L = 28 legs:
(A) 6 cows (B) 8 cows (C) 4 cows (D) 14 cows

124. A general rule is more powerful than a single answer because it:
(A) only works once (B) is harder to use (C) hides the pattern (D) solves every future problem of that type instantly

125. Genny turns Tad's small-case pattern (1, 3, 6, 10 handshakes) into the rule:
(A) n squared (B) n(n-1)/2 for n people (C) 2n (D) n + 1

126. The general rule for the sum 1 + 2 + ... + n is:
(A) n squared (B) 2n (C) n + 1 (D) n(n+1)/2

Write the answer

127. Using n(n+1)/2, the sum 1 + 2 + ... + 20 is:
Answer: ________

128. After writing a general rule, you should:
Answer: ________

129. "A rectangle's perimeter" generalizes to:
Answer: ________

130. Genny's approach starts AFTER a specific problem is solved because:
Answer: ________

Match each question to its answer

1. "Cost of n tickets at $8 each" is: →
2. "Total cost with a $5 booking fee plus $8 per ticket" generalizes to: →
3. A generalized rule uses VARIABLES so that: →
4. Generalizing connects to Tad because: →
5. "Any even number" can be generalized as: →

(A) 2n (for a whole number n)
(B) the same rule adapts to different numbers
(C) 8n
(D) 8n + 5

(E) Tad finds the pattern in small cases; Genny writes it as a formula

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14. two equations, two unknowns — substitution and elimination

Circle the correct answer

131. A 'system of equations' is:
(A) two or more equations that must ALL be true at once (B) two unrelated equations (C) a single number (D) one equation

132. The chickens-and-cows system is: c + w = heads and 2c + 4w = legs, where:
(A) c = legs, w = heads (B) both are heads (C) c = cows, w = chickens (D) c = chickens, w = cows (count and leg equations)

133. The 'substitution' method solves a system by:
(A) adding the equations (B) solving one equation for a variable and plugging it into the other (C) ignoring one equation (D) guessing both variables

134. For c + w = 10 and 2c + 4w = 28, substitute c = 10 - w into the leg equation to get:
(A) 2(10 - w) + 4w = 28 (B) 10 - w = 28 (C) 2c + 4c = 28 (D) c + w = 28

135. Solve 2(10 - w) + 4w = 28 for w (cows):
(A) 4 (B) 8 (C) 6 (D) 2

136. The 'elimination' method solves a system by:
(A) guessing (B) adding or subtracting the equations to cancel a variable (C) removing an equation (D) multiplying the variables

Write the answer

137. For x + y = 12 and x - y = 4, ADDING the equations gives:
Answer: ________

138. Reck's systematic guess-and-check is the INTUITION behind systems because it:
Answer: ________

139. To use elimination on 2x + 3y = 13 and 2x + y = 7, you can:
Answer: ________

140. From q09, 2y = 6 gives y = 3; then x =
Answer: ________

Match each question to its answer

1. Sometimes elimination needs you to MULTIPLY an equation first so that: →
2. To eliminate x from x + y = 5 and 2x + 3y = 12, multiply the first by 2 to get: →
3. From q12, subtracting 2x + 2y = 10 from 2x + 3y = 12 gives: →
4. The SOLUTION to a two-variable system is: →
5. Substitution is usually easiest when: →

(A) a variable's coefficients match (to cancel)
(B) y = 2
(C) the pair (x, y) that makes both equations true
(D) one variable already has a coefficient of 1
(E) 2x + 2y = 10

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15. which solving route fits which problem

Circle the correct answer

141. A problem gives the ENDING and a chain of steps, asking for the start. Best heuristic:
(A) draw a diagram (Doodle)   (B) generalize (Genny)   (C) work backwards (Retta)   (D) try a smaller case (Tad)

142. A problem has a huge number with a repeating structure. Best first move:
(A) try a smaller case (Tad)   (B) check reasonableness (Vetta)   (C) work backwards (Retta)   (D) set up a system (Reck)

143. A problem with two unknowns linked by two facts. Best heuristic:
(A) set up a system of equations (Lettie + Reck)   (B) work backwards (Retta)   (C) draw one bar (Doodle only)   (D) estimate (Vetta)

144. A problem with confusing relationships between amounts. A great FIRST step:
(A) draw a diagram (Doodle)   (B) guess randomly   (C) generalize (Genny)   (D) work backwards (Retta)

145. You just got an answer. The universal LAST step is:
(A) draw a diagram (Doodle)   (B) check it's reasonable and substitute back (Vetta)   (C) generalize (Genny)   (D) move on immediately

146. You can't tell what a problem is asking. Best FIRST move:
(A) generalize (Genny)   (B) work backwards (Retta)   (C) guess the answer   (D) recognize the type and label the units (Kindra + Tagg)

Write the answer

147. You've solved one instance and expect many more like it. Best move:
Answer: ________

148. The BIG lesson of choosing a heuristic is:
Answer: ________

149. Several heuristics can often be COMBINED. For example:
Answer: ________

150. "I think of a number, triple it, subtract 5, get 16." Which heuristic fits BEST?
Answer: ________

Match each question to its answer

1. "How many diagonals in a 12-sided polygon?" Which heuristic fits BEST? →
2. "12 coins, dimes and quarters, worth $2.10." Which heuristic fits BEST? →
3. Before picking a heuristic, the very first thing to do is: →
4. If your first heuristic isn't working, the right response is to: →
5. "A recipe for 4 needs scaling to 10 people." Which heuristic fits BEST? →

(A) try a smaller case, then generalize (Tad + Genny)
(B) try a different heuristic — it's fine to switch
(C) set up a proportion / ratio (rate thinking)
(D) understand the problem — what's known, unknown, and asked
(E) recognize the coin type, set up count + value (Kindra + Lettie)

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16. mixed-family review across the whole solving scaffold

Circle the correct answer

151. Chickens and cows: 8 heads, 22 legs. How many cows?
(A) 3 cows (B) 4 cows (C) 2 cows (D) 5 cows

152. "Double a number, add 7, get 19." The number is (work backwards):
(A) 6 (B) 13 (C) 5 (D) 12

153. "12 coins, nickels and dimes, worth 95 cents." Number of dimes?
(A) 8 dimes (B) 6 dimes (C) 7 dimes (D) 5 dimes

154. "Ann is 3 times Ben's age; together 24." Ben's age?
(A) 8 (B) 12 (C) 6 (D) 18

155. "Hose fills in 4 h, another in 4 h. Together?"
(A) 2 hours (B) 8 hours (C) 1 hour (D) 4 hours

156. "Mix 2 L of 10% with 2 L of 30%." Blend concentration?
(A) 25% (B) 40% (C) 15% (D) 20%

Write the answer

157. "Sum 1 + 2 + ... + 50" using n(n+1)/2:
Answer: ________

158. A discount answer of $50 for '20% off $40' should be rejected because:
Answer: ________

159. "How many handshakes among 6 people?" using n(n-1)/2:
Answer: ________

160. A problem says 'total' but describes a start you must recover after items were removed. You should:
Answer: ________

Match each question to its answer

1. "x + y = 9 and x - y = 3." Using elimination (add), x = →
2. To find the LAST digit of a huge product, the best heuristic is: →
3. "After giving away half, then 2 more books, Sam has 6." How many did he start with (backwards)? →
4. The cows-formula cows = (L - 2H)/2 came from Genny GENERALIZING which family? →
5. "9 coins, one heavier — fewest weighings to find it?" →

(A) 16
(B) 6
(C) try small cases to find the repeating last-digit pattern (Tad)
(D) 2
(E) chickens-and-cows (count + value)

Riddle & Joke Break

Guess the answer, then check the key!

161. Compute 7 x 8 in your head. What is it?

162. What's a fast way to find 25 x 16?

163. Compute 99 x 6 quickly. What do you get?

164. What is 48 + 37 using "make a ten"?

165. Evaluate 3 + 4 x 2 using order of operations. What is it?

166. What is 12 x 15?

167. Halve 68 in your head. What do you get?

168. What is 6 x 9 using the nines trick?

169. Why did the mental math wizard skip the calculator?

170. Why did 5 love multiplying by even numbers?

171. Why did the times table go to school early?

172. Why did the number get split in two?

173. Why did 9 always feel a little off?

174. Why did order of operations bring a to-do list?

Fun Facts

- Did you know? Any multiple of 9 has digits that add up to 9 (or a multiple of 9). Check 63: 6 + 3 = 9. Check 81: 8 + 1 = 9. This is a real divisibility test for 9!
- Did you know? To multiply any number by 5, you can halve it and multiply by 10. So 48 x 5 = (48 / 2) x 10 = 24 x 10 = 240. Much easier than the long way!
- Did you know? "PEMDAS" (Parentheses, Exponents, Multiplication/Division, Addition/Subtraction) tells you the order of operations -- but multiplication and division are equal rank, done left to right, and so are addition and subtraction!
- Did you know? Mental-math champions don't have magic brains -- they memorize a few key facts (like squares up to 25) and use clever shortcuts. Anyone can learn the tricks!
- Did you know? To multiply a two-digit number by 11, add its two digits and place the sum in the middle. 24 x 11: put 2 and 4 apart with 2+4=6 between them, giving 264!
- Did you know? The "lattice method" of multiplication, using a grid of diagonals, was used in India and the Middle East over 700 years ago -- and it still works perfectly today!

- Did you know? Subtraction by "counting up" is how store cashiers used to make change. Instead of 10.00 - 6.35, they'd count 6.35 up to 10.00 and hand you the difference as they went!
- Did you know? To square a number ending in 5, multiply the front part by one more than itself, then tack on 25. For 35^2: 3 x 4 = 12, then add 25 to get 1,225!

Answer Key

1. D — 2
2. A — 4
3. D — 1
4. D — 8
5. D — 12
6. C — 7
7. 18
8. make a smart guess, check it against the riddle, then adjust
9. 20
10. +2 legs (a cow has 2 more legs than a chicken)
    Matching (systematic guess-and-check → 2-variable systems (the chickens-and-cows riddle)): 1→B, 2→C, 3→D, 4→A, 5→E
11. A — name the unknown with a letter (let x = ...)
12. A — x + 7
13. B — x - 5
14. B — 2x
15. A — legs = 2 x heads
16. D — x + 3
17. a letter lets you write the relationships exactly, then solve
18. x + 2x = 18
19. 6

20. 4x

    Matching (name the unknowns and write the relationships (let x = ...)): 1→A, 2→B, 3→D, 4→C, 5→E
21. B — "what TYPE of problem is this?"
22. A — the same word can mean different operations in different problems
23. D — ADD (9 + 4) to undo the giving-away
24. C — 6 chickens (the unit is 'chickens')
25. A — mixing up which number means what
26. D — A is a multiply structure (3x4), B is an add structure (3+4)
27. a count of coins AND their total value
28. total heads (count) and total legs (value)
29. the same solving method works for both
30. legs, not animals

    Matching (schema-based instruction — recognize the problem type and label every number (anti-key-word)): 1→D, 2→A, 3→B, 4→C, 5→E
31. C — a rectangle whose length stands for the amount
32. B — the two part-bars joined end to end
33. A — a bar 5 shorter than Ann's (Ben = 12 - 5 = 7)
34. A — it shows the relationships so you pick the right operation
35. A — 4 equal bars each labeled 6
36. A — 5
37. a bar for total heads and a bar showing legs stacked by animal type
38. a short bar and a bar exactly twice as long
39. 20
40. relationships between quantities that are hard to hold in your head

    Matching (draw a bar/tape diagram of the relationships): 1→A, 2→B, 3→D, 4→E, 5→C
41. D — x + 5
42. C — x - 3

43. C — the same amount
44. B — 4 (the same as now)
45. A — x + 5
46. B — x + 4
47. 18
48. x + (x + 4) = 20
49. 8
50. adds N to BOTH people, not just one
    Matching (translate 'in N years / N years ago' into equations): 1→B, 2→A, 3→E, 4→C, 5→D
51. B — how many coins there are, and how much they are worth
52. C — label it: 5 coins vs 5 cents
53. A — 10d
54. C — 25q
55. A — 12 - d
56. D — 10d + 25(12 - d) = 210
57. 6
58. one consistent unit avoids arithmetic mistakes
59. a count equation and a value/leg equation
60. 5f + 10(20 - f) = 150
    Matching (count + value constraints, unit-labeling (coins, stamps)): 1→B, 2→E, 3→A, 4→C, 5→D
61. C — 1/6 of the tank per hour
62. B — adding their rates
63. C — 1/2 per hour
64. D — 2 hours
65. D — shorter time
66. C — 12 painter-hours

67. 2 hours
68. inverse variation
69. adding the two times (6 + 3 = 9) instead of the rates
70. less than the faster worker's solo time

    Matching (combined rates — work done per unit time (painters, hoses, machines)): 1→E, 2→D, 3→A, 4→B, 5→C
71. D — combined amount and its overall price or concentration
72. C — $30 (2x3 + 3x8 = 6 + 24)
73. D — 5 lb
74. A — $6/lb ($30 / 5 lb)
75. B — there is more of the $8 nut, pulling the average up
76. A — how much of it there is
77. the concentration times the volume
78. 0.8 L (0.2 + 0.6)
79. 16% (0.8 / 5)
80. the total amount of the pure ingredient (salt, or dollars of value)

    Matching (weighted averages / concentration mixing): 1→A, 2→D, 3→C, 4→E, 5→B
81. C — the ENDING and a chain of steps, and asks for the start
82. C — subtract 5
83. B — divide by 3
84. B — halve, then subtract 2
85. B — 7 (11 - 4)
86. B — 7
87. the goal, then walk the steps in reverse
88. $8 (5 + 3)
89. $16 (8 is half, so double it)
90. inverse operations in reverse order

    Matching (start from the goal and reverse the steps): 1→B, 2→C, 3→D, 4→A, 5→E

91. D — solve a tiny version first
92. B — the small case reveals the pattern or method for the big one
93. B — try 2, then 3, then 4 people and look for a pattern
94. A — add the next-smaller count each time (triangular numbers)
95. B — 45
96. B — a small number of heads (like 3) to see the method
97. 1 + 2 + 3 + 4 (= 10) to find a shortcut
98. 5050 (100 x 101 / 2)
99. find the pattern/rule and check it predicts the next small case
100. use a smaller version of the same problem
    Matching (solve a tiny version first to see the structure): 1→A, 2→C, 3→B, 4→E, 5→D
101. C — heavier
102. B — equal in weight
103. A — in the third (unweighed) group
104. D — 2
105. C — the balance has three outcomes (left, right, equal), each pointing to a third
106. B — tracking what each clue rules OUT until one possibility remains
107. no forbidden pair is left unattended
108. bring something back to keep every state legal
109. multiples of the GCD of the jug sizes
110. lets you mark what's ruled in or out for each possibility
    Matching (constraint + logical elimination in classic brain-teasers): 1→A, 2→C, 3→B, 4→D, 5→E
111. A — check your answer against the original problem
112. B — substitute the answer back into the original equation/story
113. D — 19 (checks out)
114. A — the head count AND the leg count

115. B — wrong, even if the number is right
116. A — giving a ballpark to compare your exact answer against
117. a discount should make it CHEAPER, not more expensive
118. an error — counts of people can't be negative
119. 'does this answer make sense in the real situation?'
120. unreasonable — bikes don't go that fast, re-check
     Matching (substitute back + reasonableness (the look-back step)): 1→B, 2→C, 3→D, 4→A, 5→E
121. B — write a rule that works for ANY numbers, not just the ones given
122. B — (L - 2H) / 2
123. C — 4 cows
124. D — solves every future problem of that type instantly
125. B — n(n-1)/2 for n people
126. D — n(n+1)/2
127. 210
128. test it on a known case to confirm it's correct
129. P = 2(length + width)
130. the worked example shows the structure to generalize
     Matching (write the rule that works for any numbers): 1→C, 2→D, 3→B, 4→E, 5→A
131. A — two or more equations that must ALL be true at once
132. D — c = chickens, w = cows (count and leg equations)
133. B — solving one equation for a variable and plugging it into the other
134. A — 2(10 - w) + 4w = 28
135. A — 4
136. B — adding or subtracting the equations to cancel a variable
137. 2x = 16, so x = 8
138. narrows guesses using both conditions until they're satisfied
139. subtract the second from the first to cancel 2x

140. 2 (from 2x + 3 = 7)
Matching (two equations, two unknowns — substitution and elimination): 1→A, 2→E, 3→B, 4→C, 5→D
141. C — work backwards (Retta)
142. A — try a smaller case (Tad)
143. A — set up a system of equations (Lettie + Reck)
144. A — draw a diagram (Doodle)
145. B — check it's reasonable and substitute back (Vetta)
146. D — recognize the type and label the units (Kindra + Tagg)
147. generalize into a formula (Genny)
148. match the strategy to the problem's shape, don't force one method on all
149. draw a diagram, set up an equation from it, then check reasonableness
150. work backwards (Retta)
Matching (which solving route fits which problem): 1→A, 2→E, 3→D, 4→B, 5→C
151. A — 3 cows
152. A — 6
153. C — 7 dimes
154. C — 6
155. A — 2 hours
156. D — 20%
157. 1275
158. a discount can't raise the price above $40
159. 15
160. read the structure — this likely means ADD back, not the word 'total'
Matching (mixed-family review across the whole solving scaffold): 1→B, 2→C, 3→A, 4→E, 5→D
161. 56! A handy memory aid: 5, 6, 7, 8 -- "56 = 7 x 8."
162. 400! Because 25 x 4 = 100, and 100 x 4 = 400 (since 16 = 4 x 4).
163. 594! Because 100 x 6 = 600, then subtract 6 to get 594.

164. 85! Add 48 + 2 = 50, then add the remaining 35 to get 85.
165. 11! Multiply first: 4 x 2 = 8, then add 3.
166. 180! Because 12 x 15 = 12 x 10 + 12 x 5 = 120 + 60 = 180.
167. 34! Halve 60 to get 30, halve 8 to get 4, then add.
168. 54! Because 6 x 10 = 60, then subtract 6 to get 54.
169. Because they already had all the numbers up their sleeve!
170. Because the answer always ends in a nice round 0!
171. To learn its multiples by heart!
172. Because breaking it apart made it way easier to multiply!
173. Because it's just 10 minus 1, and it never lets you forget it!
174. Because it always does things in the right order!
