Puzzleforge — Activity Book

A Spark & Anvil activity book — puzzles, questions, and fun from Puzzleforge.

Write in the book, circle your answers, and check the Answer Key at the back.

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1. recursion — solve the smaller tower first (Tower of Hanoi)

Circle the correct answer

1. In the Tower of Hanoi, how many disks may you move at one time?
(A) the whole stack (B) as many as you like (C) two (D) exactly one

2. Which disk on a peg are you allowed to pick up?
(A) the largest disk (B) the top (smallest on that peg) disk (C) the bottom disk (D) any disk in the middle

3. Which move is NOT allowed in the Tower of Hanoi?
(A) placing a larger disk on top of a smaller disk (B) placing a smaller disk on a larger disk (C) moving the top disk (D) moving a disk to an empty peg

4. How many pegs does the classic Tower of Hanoi use?
(A) one (B) two (C) three (D) four

5. How many moves does it take to move a tower of just 1 disk to another peg?
(A) 3 (B) 0 (C) 1 (D) 2

6. What is the fewest number of moves to solve a tower of 2 disks?
(A) 1 (B) 2 (C) 4 (D) 3

Write the answer

7. What is the fewest number of moves to solve a tower of 3 disks?
Answer: ________

8. To move a tower of 3 disks, Nestor first moves the top TWO disks out of the way. This is an example of:
Answer: ________

9. The big idea of recursion is best described as:
Answer: ________

10. To move 4 disks from the start peg to the goal peg, what do you do FIRST?
Answer: ________

Match each question to its answer

1. After you move the top 3 disks aside and move the biggest disk to the goal, what is the LAST big step? →
2. A tower of 3 takes 7 moves. Using the pattern (move the smaller tower, the big disk, then the smaller tower again), how many moves for 4 disks? →
3. The move-counts go 1, 3, 7, 15, ... Each number is: →
4. How many moves for a tower of 5 disks? →
5. The fewest moves for n disks equals 2 multiplied by itself n times, minus 1 (written 2^n - 1). For 3 disks this is: →

(A) double the one before, plus one
(B) 15
(C) 2 x 2 x 2 - 1 = 7
(D) move the 3-disk tower on top of the biggest disk
(E) 31

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2. optimality — the fewest moves (2^n - 1) and why it is the minimum

Circle the correct answer

11. What does it mean for a puzzle solution to be OPTIMAL?
(A) it uses the fewest possible moves (B) it uses the most moves (C) it looks the prettiest (D) it is the fastest to type

12. The fewest-moves counts for towers of 1, 2, 3, 4 disks are:
(A) 2, 4, 6, 8 (B) 1, 2, 3, 4 (C) 1, 4, 9, 16 (D) 1, 3, 7, 15

13. The rule 2^n - 1 means: multiply 2 by itself n times, then subtract 1. For n = 4 disks this is:
(A) 4 x 4 - 1 = 15 (B) 2 x 4 - 1 = 7 (C) 2 + 2 + 2 + 2 - 1 = 7 (D) 2 x 2 x 2 x 2 - 1 = 15

14. A tower of 5 disks takes 31 moves. How many for 6 disks?
(A) 36 (B) 62 (C) 64 (D) 63

15. Why is each move-count double the previous one plus one?
(A) you move the smaller tower twice and the big disk once (B) the puzzle gets easier (C) you add one disk each time (D) you move every disk twice

16. 2^7 - 1 equals how many moves (for 7 disks)?
(A) 126 (B) 128 (C) 49 (D) 127

Write the answer

17. Minim solves a 3-disk tower in 9 moves. What can she say about her solution?
Answer: ________

18. To move the BIGGEST disk to the goal, what must be true first?
Answer: ________

19. The biggest disk in an optimal Hanoi solve moves exactly:
Answer: ________

20. Why can't a tower of 3 be solved in fewer than 7 moves?
Answer: ________

Match each question to its answer

1. Which is the smallest number that is a power of 2? →
2. The powers of 2 go 2, 4, 8, 16, 32, ... What comes after 32? →
3. A tower of 10 disks takes how many optimal moves (2^10 - 1)? →
4. Adding ONE more disk to a tower does what to the fewest-moves count? →
5. If a solution is optimal, how many OTHER solutions use even fewer moves? →

(A) 2^1 = 2
(B) 64
(C) zero — none can be shorter
(D) 1023
(E) a little more than doubles it (doubles and adds one)

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3. sliding-tile (15-puzzle) — blank-adjacent moves and when a scramble is solvable

Circle the correct answer

21. In a sliding-tile puzzle, what actually lets a tile move?
(A) an empty (blank) space next to it (B) picking two tiles up at once (C) lifting a tile over another (D) shaking the board

22. A 4-by-4 sliding puzzle has 15 numbered tiles and how many blank spaces?
(A) four (B) one (C) none (D) two

23. How many tiles are next to (can slide into) a blank in the middle of the board?
(A) up to two (B) up to four (C) up to eight (D) always exactly one

24. When the blank is in a corner, how many moves are possible?
(A) one (B) four (C) three (D) two

25. Moving a tile into the blank does what to the blank?
(A) the blank disappears (B) the blank moves to where that tile was (C) the blank splits in two (D) the blank stays put

26. Evan says some scrambled boards can NEVER be solved. This means:
(A) you just need more time (B) you must remove a tile (C) no sequence of legal slides will ever reach the goal (D) the board is broken

Write the answer

27. If you physically PULL OUT two tiles and swap them, the puzzle's solvability:
Answer: ________

28. An "inversion" is a pair of tiles where:
Answer: ________

29. In the row 2, 1, 3, how many inversions are there?
Answer: ________

30. In the row 3, 2, 1, how many inversions are there?
Answer: ________

Match each question to its answer

1. A single legal slide moves the blank one step. Sliding the blank left then right returns the board to: →
2. Why does reversibility matter for solving? →
3. Wend 'maps out reachable positions.' The set of all boards you can slide to is called the: →
4. For the 15-puzzle, what fraction of all possible tile arrangements is actually solvable? →
5. Evan checks solvability WITHOUT solving. This is powerful because: →

(A) you can know a puzzle is impossible before wasting time trying
(B) if you can reach a state, you can always get back — so you can explore safely
(C) one half
(D) exactly where it started
(E) state space

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4. state-space search — states, moves, and which positions are reachable

Circle the correct answer

31. In puzzle-solving, a 'state' means:
(A) the rules of the game (B) a single move (C) one complete arrangement of the puzzle at a moment (D) the final answer only

32. Two states are 'neighbors' when:
(A) one legal move turns one into the other (B) they look similar (C) they have the same number of pieces (D) they are both solved

33. The whole map of states connected by moves is called the:
(A) score board (B) move list (C) state space (D) rule book

34. 'Searching' a puzzle means:
(A) looking up the answer (B) counting the pieces (C) following moves through states to find a path to the goal (D) shuffling randomly forever

35. A 'reachable' state is one that:
(A) no move can produce (B) you can get to from the start using legal moves (C) has the fewest pieces (D) is always the goal

36. If the goal state is NOT reachable from the start, the puzzle is:
(A) missing a rule (B) easier than usual (C) solved already (D) unsolvable from that start

Write the answer

37. Wend explores neighbors before committing. Why explore first?
Answer: ________

38. A 'dead end' in a state-space search is a state where:
Answer: ________

39. 'Backtracking' means:
Answer: ________

40. Why is it helpful that Hanoi and sliding puzzles have REVERSIBLE moves during search?
Answer: ________

Match each question to its answer

1. A state that has already been visited during a search should usually be: →
2. The number of possible states in a puzzle tends to be: →
3. Because there can be so many states, a good searcher tries to: →
4. A 'path' from start to goal is: →
5. The SHORTEST path in a state space is the one with: →

(A) an ordered list of moves connecting them
(B) skipped, so you don't loop forever
(C) follow moves that get closer to the goal, not every move blindly
(D) the fewest moves
(E) very large, growing fast with more pieces

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5. look-ahead planning — clear a path by thinking several moves ahead (sliding-block / Rush Hour)

Circle the correct answer

41. In a Rush Hour sliding-block puzzle, the goal is to:
(A) count the cars (B) remove every car (C) slide the target car out through the exit (D) stack cars on top of each other

42. Cars in Rush Hour can move:
(A) diagonally (B) in any direction (C) by jumping over cars (D) only forward or backward along their own lane

43. The target car can't reach the exit. The FIRST thing to figure out is:
(A) which cars are blocking its path (B) how many cars are red (C) which car is prettiest (D) the fastest car

44. A blocking car can't move because ANOTHER car is in ITS way. This means you must:
(A) move the target through it (B) give up (C) remove the second car (D) move the second car first, then the blocker, then the target

45. 'Looking ahead' means:
(A) moving as fast as possible (B) only thinking about the current move (C) looking at the answer (D) imagining several future moves before you make the first one

46. Why is moving a car WITHOUT looking ahead risky?
(A) it might block a car you need to move later (B) it always solves the puzzle (C) it removes a car (D) it can't be undone

Write the answer

47. Faro clears a path 'before touching a piece.' The benefit is:
Answer: ________

48. To free a blocker, you often need to move it into:
Answer: ________

49. If clearing the path needs moves in the order C, B, A but you do A first:
Answer: ________

50. A move that doesn't help clear the target's path is called a:
Answer: ________

Match each question to its answer

1. Planning ahead in Rush Hour is a form of: →
2. The deepest 'chain of blockers' tells you: →
3. Which best matches Faro's planning habit? →
4. If a blocker can move EITHER left or right to clear the path, you should pick the direction that: →
5. Look-ahead planning is useful far beyond Rush Hour, for example in: →

(A) trace the exit lane, list the blockers, then order their moves
(B) how many cars must move before the target can escape
(C) searching future states in your head
(D) packing a bag or scheduling steps that depend on each other
(E) doesn't create a new block for another needed move

6. constraint satisfaction — never break the rule (wolf-goat-cabbage; missionaries-cannibals)

Circle the correct answer

51. In the wolf-goat-cabbage puzzle, the farmer's boat can carry:
(A) the farmer plus one item (B) nothing at all (C) all three items at once (D) two items and the farmer

52. A 'constraint' in a puzzle is:
(A) a piece you can remove (B) a rule that must never be broken (C) a bonus move (D) the goal

53. Which pair must NEVER be left alone together without the farmer?
(A) the wolf and the goat (and the goat and the cabbage) (B) the wolf and the cabbage (C) the goat and the boat (D) the farmer and the boat

54. The classic first move is to take the GOAT across because:
(A) the wolf and cabbage are safe left together (B) the goat asked to go first (C) the goat is heaviest (D) the wolf can swim

55. The clever trick in this puzzle is that the farmer sometimes:
(A) leaves the boat behind (B) removes an item from the puzzle (C) brings an item BACK to avoid breaking a rule (D) carries two items

56. A state that breaks a constraint (like wolf alone with goat) is called:
(A) an illegal state (B) a bonus state (C) the start state (D) the goal state

Write the answer

57. Skiff's core habit is:
Answer: ________

58. In missionaries-and-cannibals, the rule is that on either bank cannibals must never:
Answer: ________

59. Constraint puzzles are solved by:
Answer: ________

60. 'Pruning' illegal states means:
Answer: ________

Match each question to its answer

1. Why does having FEWER legal moves sometimes make a puzzle EASIER to plan? →
2. After the farmer drops the goat and rows back EMPTY, the boat is: →
3. Which is a legal state in wolf-goat-cabbage (farmer on the near bank)? →
4. The full wolf-goat-cabbage solution takes how many crossings (one-way trips)? →
5. Constraint thinking shows up in real life when you: →

(A) constraints narrow the choices, so the right move stands out
(B) schedule tasks that can't overlap (like one oven, many dishes)
(C) ready for the next item, both banks still legal
(D) wolf and cabbage on the near bank, goat across
(E) seven

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7. GCD / number reasoning — measure exact amounts by pouring between two jugs

Circle the correct answer

61. In a water-pouring puzzle you have jugs with no markings. The only actions are:
(A) guess the amount by weight (B) mark the jug with a pen (C) fill a jug, empty a jug, or pour one into another until full or empty (D) cut the jug shorter

62. GCD stands for:
(A) greatest common divisor (B) general counting digit (C) greatest counted decimal (D) grand central diagram

63. The GCD of 8 and 12 is:
(A) 2 (B) 6 (C) 24 (D) 4

64. With a 3-liter jug and a 5-liter jug, the SMALLEST exact amount you can measure into a jug is:
(A) 2 liters (B) 3 liters (C) 1 liter (D) 0 liters

65. The exact amounts you can measure with two jugs are always:
(A) only the jug sizes themselves (B) any number at all (C) multiples of the GCD of the two jug sizes (D) only even numbers

66. With a 4-liter and a 6-liter jug, can you measure exactly 3 liters?
(A) only if you guess (B) no — 3 is not a multiple of gcd(4,6)=2 (C) yes, easily (D) yes, by filling both

Write the answer

67. With a 4-liter and a 6-liter jug, which amounts CAN you measure?
Answer: ________

68. Two jug sizes whose GCD is 1 are called:
Answer: ________

69. If two jugs are coprime, which whole amounts can you measure?
Answer: ________

70. In the famous puzzle, you must measure 4 liters using a 3-liter and a 5-liter jug. This is possible because:
Answer: ________

Match each question to its answer

1. To get 4 liters with 3- and 5-liter jugs, one path is: fill the 5, pour into the 3 (leaves 2 in the 5), empty the 3, pour the 2 in, then: →
2. The GCD of 6 and 9 is: →
3. With 6- and 9-liter jugs (gcd 3), which amount is IMPOSSIBLE? →
4. A quick way to find the GCD is Euclid's idea: keep replacing the bigger number with: →
5. Using Euclid on 12 and 8: 12 divided by 8 leaves remainder 4; 8 divided by 4 leaves remainder 0. The GCD is: →

(A) 3
(B) fill the 5 again and top off the 3 (needs 1), leaving 4 in the 5
(C) the remainder when the bigger is divided by the smaller
(D) 4
(E) 4 liters

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8. binary / Gray-code counting — free the rings by changing exactly one at a time

Circle the correct answer

71. In the Chinese-rings puzzle, each ring is either:
(A) heavy or light (B) on the bar or off the bar (C) big or small (D) red or blue

72. Because each ring is on OR off, the puzzle state is naturally written as:
(A) a color code (B) a single decimal number only (C) a fraction (D) a string of 0s and 1s (binary)

73. In binary, the number after 011 (counting up) is:
(A) 012 (B) 110 (C) 101 (D) 100

74. The binary number 101 equals which decimal value?
(A) 5 (B) 6 (C) 3 (D) 101

75. A 'Gray code' is a counting order where each step changes:
(A) at least two bits (B) no bits (C) every bit (D) exactly one bit

76. Ordinary binary counting from 01 to 10 changes:
(A) no bits (B) one bit (C) two bits at once (D) three bits

Write the answer

77. The 2-bit Gray code order is 00, 01, 11, 10. Between 01 and 11, which bit changed?
Answer: ________

78. In the rings puzzle, you may only add or remove a ring when the rings before it are in a special set position. This forces you to:
Answer: ________

79. Gray's defining act is:
Answer: ________

80. Why is a one-change-at-a-time order useful beyond puzzles?
Answer: ________

Match each question to its answer

1. The 3-bit Gray code starts 000, 001, 011, 010, ... What comes next? →
2. How many different states does a 3-ring puzzle have? →
3. A Gray code for n rings visits all 2^n states while: →
4. Solving the Chinese rings is HARD mostly because: →
5. If you're mid-solve and forget your place, the one-change rule helps because: →

(A) 8
(B) you can compare the current state to the neighbors that differ by one ring
(C) changing only one ring between each state
(D) 110
(E) the number of one-at-a-time steps grows quickly with more rings

9. reduction / monovariant — jump-and-remove toward a single survivor (peg solitaire)

Circle the correct answer

81. A legal peg-solitaire move is to:
(A) slide a peg one hole over (B) add a new peg (C) swap two pegs (D) jump a peg over an adjacent peg into an empty hole, removing the jumped peg

82. Each legal jump changes the number of pegs by:
(A) exactly minus one (B) zero (C) minus two (D) plus one

83. A quantity that only ever moves in ONE direction (like the peg count) is called a:
(A) constant (B) coordinate (C) random variable (D) monovariant

84. If you start with 10 pegs and want to finish with 1, how many jumps do you make?
(A) 10 (B) 8 (C) 9 (D) 11

85. Because the peg count only decreases, the puzzle CANNOT:
(A) have any moves (B) ever be solved (C) start with pegs (D) loop forever — it must end

86. Vault's defining act is:
(A) counting the holes (B) jumping one peg over another and removing it, shrinking toward one survivor (C) sliding pegs sideways (D) adding pegs to the board

Write the answer

87. The classic English peg board has 33 holes and starts with 32 pegs (center empty). To finish with 1 peg you make:
Answer: ________

88. A peg with NO empty hole beyond an adjacent peg:
Answer: ________

89. Getting STUCK in peg solitaire means:
Answer: ________

90. To AVOID getting stuck, a good habit is to:
Answer: ________

Match each question to its answer

1. The number of jumps to solve is FIXED by the start, no matter the order. For a board of N pegs to 1 peg it is always: →
2. A monovariant is a powerful proof tool because it lets you argue that a process: →
3. Which of these is ALSO a monovariant in a different puzzle? →
4. Peg solitaire is a REDUCTION puzzle because the board: →
5. If a start position has 20 pegs and you're now at 6 pegs, how many jumps have you made? →

(A) the number of disks not yet on the goal peg in Hanoi steadily reaching zero
(B) N minus 1
(C) shrinks (fewer pegs) with every move
(D) 14
(E) must stop (it can't decrease forever)

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10. recursion — self-similarity, base case, and the recursive trace

Circle the correct answer

91. A recursive solution has two parts. The part that stops the recursion is the:
(A) big case (B) base case (C) loop case (D) final answer only

92. The part that solves a big problem using a smaller copy of itself is the:
(A) base case (B) goal case (C) recursive case (D) empty case

93. For Tower of Hanoi, the base case is:
(A) an empty peg (B) moving a single disk (n = 1) directly (C) moving 64 disks (D) the whole solved tower

94. What happens if a recursion has NO base case?
(A) it never stops — it goes forever (B) it solves instantly (C) it removes all pieces (D) it becomes optimal

95. 'Self-similarity' in recursion means:
(A) the problem changes into a new one (B) there are no pieces (C) all pieces are identical in size (D) the smaller pieces look like tinier versions of the whole

96. The recurrence M(n) = 2 x M(n-1) + 1 means the work for n is:
(A) n added to itself (B) the same as n-1 (C) twice the work for n-1, plus one more step (D) half the work for n-1

Write the answer

97. Using M(1)=1 and M(n)=2M(n-1)+1, what is M(3)?
Answer: ________

98. 'Unrolling' a recursion means:
Answer: ________

99. To move 3 disks A->C, the recursion first solves 'move 2 disks A->B.' That sub-call itself first:
Answer: ________

100. Nestor says a tall problem is just a shorter problem plus one step. This lets him:
Answer: ________

Match each question to its answer

1. A recurrence is useful because it describes: →
2. The closed-form for M(n)=2M(n-1)+1 with M(1)=1 is: →
3. A 'closed form' is valuable because it lets you find M(50) without: →
4. Which everyday task is naturally recursive? →
5. In recursion, solving n depends on n-1, which depends on n-2, and so on down to the base. This chain is called the: →

(A) computing all the earlier values first
(B) M(n) = 2^n - 1
(C) recursion (or call) stack
(D) a whole family of answers with one short rule
(E) opening nested boxes, each holding a smaller box

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11. parity invariants — the mod-2 argument, generalized across puzzles

Circle the correct answer

101. 'Parity' means whether a number is:
(A) prime or not (B) positive or negative (C) big or small (D) even or odd

102. An 'invariant' is a fact about a puzzle that:
(A) is the goal (B) never changes, no matter which legal moves you make (C) changes every move (D) is random

103. Why are invariants useful for solvability?
(A) they change the rules (B) if the goal has a different invariant than the start, it's unreachable (C) they make puzzles longer (D) they add pieces

104. In the sliding puzzle, swapping exactly two tiles changes the inversion count by an ODD amount, so it:
(A) keeps parity the same (B) flips the parity (even to odd or odd to even) (C) solves the puzzle (D) removes a tile

105. Evan checks parity to decide whether to even TRY a puzzle. This saves:
(A) the color of tiles (B) time wasted on an impossible task (C) the number of pieces (D) nothing at all

106. Adding two even numbers always gives:
(A) an even number (B) sometimes even, sometimes odd (C) zero (D) an odd number

Write the answer

107. Adding an even and an odd number gives:
Answer: ________

108. A checkerboard-coloring argument uses parity by:
Answer: ________

109. A knight on a chessboard always moves from a square of one color to:
Answer: ________

110. Because a knight flips color each move, after an EVEN number of moves it is on:
Answer: ________

Match each question to its answer

1. Suppose a puzzle's 'blank' must return to its home corner, and each move flips its checkerboard color. To come home it needs: →
2. The power of a parity argument is that it can prove something is IMPOSSIBLE: →
3. The famous 'mutilated chessboard' (remove two opposite corners) can't be tiled by dominoes because: →
4. Evan's core habit across all these puzzles is to: →
5. In the Tower of Hanoi, the smallest disk moves on every OTHER move. Its move-times follow: →

(A) without checking every possibility
(B) an even number of moves
(C) the two removed corners are the same color, so black and white counts don't match
(D) find a quantity that never changes, then check the goal against it
(E) a fixed parity pattern (odd-numbered moves)

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12. search strategies — breadth-first vs depth-first intuition

Circle the correct answer

111. Breadth-first search (BFS) explores states:
(A) backward from the goal only (B) one long path all the way down first (C) at random (D) level by level — all one-move-away states, then all two-move-away, and so on

112. Depth-first search (DFS) explores states:
(A) by following one path as deep as possible, then backtracking (B) all at once (C) only the goal (D) level by level

113. Which search is GUARANTEED to find the fewest-move solution first?
(A) depth-first search (B) breadth-first search (C) both always tie (D) neither can

114. Why might DFS find a LONGER solution than necessary?
(A) it may reach the goal down a deep branch before checking shorter ones (B) it never finds the goal (C) it always finds the shortest (D) it ignores the goal

115. BFS tends to use MORE memory than DFS because it:
(A) must remember all states at the current level (a wide frontier) (B) uses no memory (C) only stores one path (D) remembers nothing

116. DFS uses LESS memory but risks:
(A) solving every puzzle (B) going very deep down a wrong branch before backtracking (C) finding the goal too fast (D) using no time

Write the answer

117. To AVOID revisiting states, any good search should:
Answer: ________

118. BFS uses a 'queue' (first-in, first-out). This means it processes states:
Answer: ________

119. DFS uses a 'stack' (last-in, first-out). This means it processes states:
Answer: ________

120. For a Hanoi-style puzzle where you want the OPTIMAL solution, you'd prefer:
Answer: ________

Match each question to its answer

1. If you only need SOME solution quickly and memory is tight, a reasonable choice is: →
2. A 'frontier' in search is: →
3. A 'guided' (heuristic) search improves on plain BFS/DFS by: →
4. For the 15-puzzle, a common heuristic is counting how far each tile is from home. This helps the search: →
5. Why can't we just brute-force every state in a big puzzle? →

(A) prefer moves that reduce total tile distance
(B) the set of states discovered but not yet explored
(C) preferring states that look closer to the goal
(D) DFS
(E) there are far too many states to check them all

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13. constraint satisfaction — modeling, pruning illegal states, and constraint propagation

Circle the correct answer

121. A constraint satisfaction problem (CSP) has variables, values, and:
(A) only one variable (B) constraints that restrict which value combinations are allowed (C) a single fixed answer with no rules (D) no rules at all

122. In Sudoku, a 'constraint' is:
(A) the digits must be prime (B) you must use a pencil (C) the puzzle must be square (D) no repeated digit in any row, column, or box

123. 'Pruning' a value means:
(A) removing a value that would break a constraint (B) guessing a value (C) ignoring constraints (D) adding a new value

124. 'Constraint propagation' means:
(A) ignoring the constraints (B) adding more variables (C) guessing everything (D) one decision narrows the options for related variables

125. The 'most-constrained variable first' heuristic says to fill in:
(A) the cell with the most options (B) a random cell (C) the last cell (D) the cell/choice with the FEWEST legal options next

126. Why does handling the most-constrained choice first help?
(A) it catches forced moves and dead ends early, avoiding wasted work (B) it adds constraints (C) it never helps (D) it makes puzzles harder

Write the answer

127. If propagation leaves a variable with ZERO legal values, you have found a:
Answer: ________

128. In river-crossing, Skiff prunes states where:
Answer: ________

129. Modeling a real problem as a CSP means writing down:
Answer: ________

130. Scheduling classes so no student has two at once is a CSP where the constraint is:
Answer: ________

Match each question to its answer

1. Coloring a map so neighbors differ is a CSP. The constraint is: →
2. Good CSP solving reduces GUESSING by: →
3. When a guess IS needed, pairing it with propagation means: →
4. The difference between constraint satisfaction and plain search is that CSP: →
5. Skiff's patience ('check every move against the rules') matches the CSP habit of: →

(A) uses the rules to eliminate options, not just to check final states
(B) adjacent regions must have different colors
(C) propagating constraints to force as many values as possible first
(D) verifying each partial assignment stays consistent
(E) after guessing, immediately prune and check for contradictions

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14. optimality proofs — lower bounds and why you cannot do better

Circle the correct answer

131. A 'lower bound' on the number of moves is:
(A) a lucky guess (B) the most moves anyone used (C) a number you can prove no solution can beat (D) the number of pieces

132. An 'upper bound' is:
(A) the fewest possible always (B) a solution you actually found, showing it CAN be done in that many moves (C) a number too big to reach (D) the number of pieces

133. You PROVE a solution is optimal when the lower bound and upper bound:
(A) are far apart (B) keep changing (C) meet at the same number (D) are both unknown

134. For Hanoi, the recurrence M(n)=2M(n-1)+1 gives BOTH bounds because it shows:
(A) a solution in 2^n - 1 moves AND that fewer is impossible (B) only that it's hard (C) nothing about bounds (D) only a solution

135. The lower-bound argument for Hanoi rests on the fact that the biggest disk:
(A) moves last only (B) can jump other disks (C) must move at least once, and that requires clearing then rebuilding the smaller tower (D) never moves

136. Why is finding a solution NOT enough to claim optimality?
(A) solutions are never optimal (B) a shorter solution might still exist unless you prove a matching lower bound (C) you need more pieces (D) it is always enough

Write the answer

137. A simple lower-bound argument in peg solitaire: to go from N pegs to 1 you must remove N-1 pegs, so you need at least:
Answer: ________

138. A lower bound of 3 and a found solution of 5 tells you the true minimum is:
Answer: ________

139. To PROVE the minimum is exactly 4 in that situation, you could:
Answer: ________

140. Minim's motto 'never a wasted move' is really a claim that his solution:
Answer: ________

Match each question to its answer

1. A 'counting argument' for a lower bound might say: →
2. In a sliding puzzle, if 6 tiles are out of place and each move repositions one tile toward home, a quick lower bound is: →
3. A lower bound is called 'tight' when: →
4. Why do computer scientists care about lower bounds? →
5. The Hanoi lower bound uses the SAME recursive structure as the solution. This shows: →

(A) at least 6 moves
(B) they tell you when to STOP improving — you've hit the best possible
(C) a real solution actually achieves it
(D) the optimal plan and the impossibility of doing better share one idea
(E) each move can fix at most one thing, and k things are broken, so at least k moves are needed

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15. number reasoning — gcd, modular arithmetic, and binary behind puzzle moves

Circle the correct answer

141. 'Modular arithmetic' (mod) is about:
(A) the remainder after dividing (B) the product (C) the largest factor (D) the quotient only

142. What is 17 mod 5 (the remainder when 17 is divided by 5)?
(A) 2 (B) 1 (C) 3 (D) 5

143. Parity (even/odd) is really arithmetic mod:
(A) 10 (B) 2 (C) 1 (D) 5

144. In a water-pouring puzzle, measurable amounts are exactly the multiples of:
(A) the larger jug only (B) the product of the jugs (C) the GCD of the jug sizes (D) the sum of the jugs

145. The GCD of 18 and 24 is:
(A) 72 (B) 12 (C) 6 (D) 3

146. A binary number represents a value using only:
(A) letters (B) digits 0-9 (C) fractions (D) 0s and 1s (powers of two)

Write the answer

147. The binary number 1101 in decimal is:
Answer: ________

148. Hanoi's move-count 2^n - 1 in binary is a string of:
Answer: ________

149. Two numbers are coprime when their GCD is:
Answer: ________

150. With coprime jugs, EVERY whole amount up to the larger jug is measurable because:
Answer: ________

Match each question to its answer

1. A clock shows 9. Seven hours later it shows 4, because 9 + 7 = 16 and: →
2. In a puzzle where a piece cycles through 4 positions, its position after 10 steps is found by: →
3. Euclid's algorithm finds a GCD by repeatedly taking: →
4. Why do so many different puzzles hide the SAME number ideas? →
5. With 6- and 10-liter jugs, the measurable amounts are multiples of gcd(6,10). That gcd is: →

(A) remainders (mod) until one is zero
(B) 10 mod 4 = 2 (the 3rd position, counting from 0)
(C) 16 mod 12 = 4
(D) 2
(E) states and moves are really numbers, so number rules govern them

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16. synthesis — mixed review connecting recursion, search, parity, constraints, optimality, gcd, and reduction

Circle the correct answer

151. A puzzle is solved by breaking it into a smaller copy of itself. This calls for:
(A) parity (Evan) (B) recursion (Nestor) (C) reduction (Vault) (D) gcd (Gill)

152. You want to know if a scramble can EVER be solved, before trying. Use:
(A) recursion (Nestor) (B) a parity/invariant check (Evan) (C) look-ahead (Faro) (D) reduction (Vault)

153. You must measure exactly 4 liters using a 3-liter and 5-liter jug. The idea that decides if it's possible is:
(A) gcd (Gill) (B) binary (Gray) (C) constraints (Skiff) (D) optimality (Minim)

154. A target car can't leave until blockers move in the right order. This is:
(A) parity (Evan) (B) binary (Gray) (C) gcd (Gill) (D) look-ahead planning (Faro)

155. You must ferry items across a river without ever leaving a forbidden pair. This is:
(A) optimality (Minim) (B) reduction (Vault) (C) recursion (Nestor) (D) constraint satisfaction (Skiff)

156. Every move removes one piece and the board shrinks toward one survivor. This is:
(A) reduction / monovariant (Vault) (B) look-ahead (Faro) (C) recursion (Nestor) (D) gcd (Gill)

Write the answer

157. You want the FEWEST-move solution and to prove none is shorter. This is:
Answer: ________

158. A ring puzzle where you change exactly one ring at a time, in a fixed order, uses:
Answer: ________

159. You map out which positions you can reach before committing to a move. This is:
Answer: ________

160. A 5-disk Hanoi tower takes how many optimal moves?
Answer: ________

Match each question to its answer

1. With 9- and 12-liter jugs, which amount is IMPOSSIBLE to measure? →
2. Starting peg solitaire with 15 pegs, to reach 1 peg you make: →
3. BFS is preferred over DFS when you specifically need: →
4. Someone pulls two tiles out of a solved sliding puzzle and swaps them. The puzzle is now: →
5. Which TWO primitives combine when you search a state space AND want the fewest moves? →

(A) 14 jumps
(B) search (Wend) + optimality (Minim)
(C) 5 liters
(D) the shortest (fewest-move) solution
(E) unsolvable (parity flipped)

Riddle & Joke Break

Guess the answer, then check the key!

161. A farmer has to cross a river with a fox, a chicken, and a bag of grain, but the boat holds only the farmer plus one item. The fox eats the chicken, and the chicken eats the grain, if left alone together. What does the farmer take first?

162. I speak without a mouth and hear without ears. I have no body, but I come alive with wind. What am I?

163. You have 8 coins and one is a fake that weighs slightly less. Using a balance scale just twice, can you find it?

164. A man looks at a photo and says, "Brothers and sisters I have none, but this man's father is my father's son." Who is in the photo?

165. What has keys but opens no locks, space but no room, and lets you enter but not go inside?

166. A rope ladder hangs off a boat with rungs 30 cm apart. The tide rises 15 cm per hour. After 4 hours, how many rungs are underwater?

167. Two fathers and two sons go fishing. Each catches one fish, but only three fish are caught total. How?

168. I get wetter the more I dry. What am I?

169. Why did the puzzle enjoy the challenge?

170. Why did the brainteaser go to the gym?

171. Why did the lateral thinker turn the problem sideways?

172. Why did the riddle wear a disguise?

173. Why did the puzzle piece feel incomplete?

174. Why did the deduction bring a flashlight?

Fun Facts

- Did you know? The "bat and ball" puzzle (they cost $1.10, the bat is $1 more, so the ball is 5 cents -- not 10!) is famous because almost everyone's gut says the wrong answer. Slowing down beats intuition!
- Did you know? "Lateral thinking" was named by psychologist Edward de Bono in 1967. It means solving problems by approaching them from unexpected angles instead of step-by-step logic!
- Did you know? Riddles are ancient -- the Riddle of the Sphinx from Greek myth ("what walks on four legs in the morning, two at noon, and three in the evening?") has an answer of "a human," crawling as a baby and using a cane when old!
- Did you know? Taking a break from a hard puzzle actually helps -- your brain keeps working on it in the background. This "incubation effect" is why answers often pop up in the shower!
- Did you know? The "river crossing" family of puzzles is over 1,200 years old -- versions appear in a medieval manuscript by the scholar Alcuin, who set them as brain exercises for students!
- Did you know? "Knights and knaves" puzzles -- where knights always tell the truth and knaves always lie -- were popularized by logician Raymond Smullyan, who wrote whole books of them!
- Did you know? Reading a puzzle carefully is half the battle. Many trick questions hide a small word like "not," "only," or "except" that flips the entire answer!
- Did you know? Puzzles are good for your brain at any age. Regularly solving them helps keep memory and reasoning sharp -- it's exercise for the mind, and it's genuinely fun!

Answer Key

1. D — exactly one
2. B — the top (smallest on that peg) disk
3. A — placing a larger disk on top of a smaller disk
4. C — three
5. C — 1
6. D — 3

7. 7
8. solving a smaller version of the same problem first
9. solving a big problem by solving smaller copies of the same problem
10. move the top 3 disks to the spare peg
    Matching (recursion — solve the smaller tower first (Tower of Hanoi)): 1→D, 2→B, 3→A, 4→E, 5→C
11. A — it uses the fewest possible moves
12. D — 1, 3, 7, 15
13. D — 2 x 2 x 2 x 2 - 1 = 15
14. D — 63
15. A — you move the smaller tower twice and the big disk once
16. D — 127
17. it is legal but not optimal — 7 is the fewest
18. every smaller disk is off it and on the spare peg
19. one time
20. you must clear the top 2 (3 moves), move the big disk (1), then rebuild the 2 (3 moves)
    Matching (optimality — the fewest moves (2^n - 1) and why it is the minimum): 1→A, 2→B, 3→D, 4→E, 5→C
21. A — an empty (blank) space next to it
22. B — one
23. B — up to four
24. D — two
25. B — the blank moves to where that tile was
26. C — no sequence of legal slides will ever reach the goal
27. flips — a solvable board becomes unsolvable and vice versa
28. a larger number comes before a smaller one in reading order
29. 1
30. 3

Matching (sliding-tile (15-puzzle) — blank-adjacent moves and when a scramble is solvable): 1→D, 2→B, 3→E, 4→C, 5→A

31. C — one complete arrangement of the puzzle at a moment
32. A — one legal move turns one into the other
33. C — state space
34. C — following moves through states to find a path to the goal
35. B — you can get to from the start using legal moves
36. D — unsolvable from that start
37. to see which moves lead toward the goal and which are dead ends
38. no path forward leads to the goal
39. undoing recent moves to try a different path
40. you can always undo a move and try another, so exploring is safe

Matching (state-space search — states, moves, and which positions are reachable): 1→B, 2→E, 3→C, 4→A, 5→D

41. C — slide the target car out through the exit
42. D — only forward or backward along their own lane
43. A — which cars are blocking its path
44. D — move the second car first, then the blocker, then the target
45. D — imagining several future moves before you make the first one
46. A — it might block a car you need to move later
47. the plan is checked in her head, avoiding trapped positions
48. an empty space along its lane
49. A may get stuck because B and C haven't made room yet
50. wasted (or unnecessary) move

Matching (look-ahead planning — clear a path by thinking several moves ahead (sliding-block / Rush Hour)): 1→C, 2→B, 3→A, 4→E, 5→D

51. A — the farmer plus one item
52. B — a rule that must never be broken
53. A — the wolf and the goat (and the goat and the cabbage)

54. A — the wolf and cabbage are safe left together
55. C — brings an item BACK to avoid breaking a rule
56. A — an illegal state
57. check that EVERY move lands in a legal state before making it
58. outnumber the missionaries there
59. finding a path of legal states from start to goal
60. removing rule-breaking states from consideration
    Matching (constraint satisfaction — never break the rule (wolf-goat-cabbage; missionaries-cannibals)): 1→A, 2→C, 3→D, 4→E, 5→B
61. C — fill a jug, empty a jug, or pour one into another until full or empty
62. A — greatest common divisor
63. D — 4
64. C — 1 liter
65. C — multiples of the GCD of the two jug sizes
66. B — no — 3 is not a multiple of gcd(4,6)=2
67. 2, 4, and 6 liters (the even amounts)
68. relatively prime (coprime)
69. every whole amount up to the larger jug
70. 4 is a multiple of gcd(3,5)=1
    Matching (GCD / number reasoning — measure exact amounts by pouring between two jugs): 1→B, 2→A, 3→E, 4→C, 5→D
71. B — on the bar or off the bar
72. D — a string of 0s and 1s (binary)
73. D — 100
74. A — 5
75. D — exactly one bit
76. C — two bits at once
77. the left bit (0 to 1)

78. change one ring at a time in a fixed order
79. freeing the rings by changing exactly one at a time in a fixed order
80. changing one thing at a time makes each step easy to check and undo

    Matching (binary / Gray-code counting — free the rings by changing exactly one at a time): 1→D, 2→A, 3→C, 4→E, 5→B
81. D — jump a peg over an adjacent peg into an empty hole, removing the jumped peg
82. A — exactly minus one
83. D — monovariant
84. C — 9
85. D — loop forever — it must end
86. B — jumping one peg over another and removing it, shrinking toward one survivor
87. 31 jumps
88. cannot make a jump right now
89. no legal jumps remain but more than one peg is left
90. avoid stranding lone pegs far from others

    Matching (reduction / monovariant — jump-and-remove toward a single survivor (peg solitaire)): 1→B, 2→E, 3→A, 4→C, 5→D
91. B — base case
92. C — recursive case
93. B — moving a single disk (n = 1) directly
94. A — it never stops — it goes forever
95. D — the smaller pieces look like tinier versions of the whole
96. C — twice the work for n-1, plus one more step
97. 7
98. writing out the steps it would take, level by level
99. moves 1 disk A->C (its own base step)
100. trust the smaller solution and only worry about the one extra step

Matching (recursion — self-similarity, base case, and the recursive trace): 1→D, 2→B, 3→A, 4→E, 5→C

101. D — even or odd
102. B — never changes, no matter which legal moves you make
103. B — if the goal has a different invariant than the start, it's unreachable
104. B — flips the parity (even to odd or odd to even)
105. B — time wasted on an impossible task
106. A — an even number
107. an odd number
108. coloring squares black/white and tracking whether a piece is on black or white
109. the opposite color
110. the same color it started on
    Matching (parity invariants — the mod-2 argument, generalized across puzzles): 1→B, 2→A, 3→C, 4→D, 5→E
111. D — level by level — all one-move-away states, then all two-move-away, and so on
112. A — by following one path as deep as possible, then backtracking
113. B — breadth-first search
114. A — it may reach the goal down a deep branch before checking shorter ones
115. A — must remember all states at the current level (a wide frontier)
116. B — going very deep down a wrong branch before backtracking
117. keep a set of already-visited states
118. in the order they were discovered — oldest first
119. the most recently discovered first
120. BFS (or a smarter guided search), for the shortest path
    Matching (search strategies — breadth-first vs depth-first intuition): 1→D, 2→B, 3→C, 4→A, 5→E
121. B — constraints that restrict which value combinations are allowed
122. D — no repeated digit in any row, column, or box

123. A — removing a value that would break a constraint
124. D — one decision narrows the options for related variables
125. D — the cell/choice with the FEWEST legal options next
126. A — it catches forced moves and dead ends early, avoiding wasted work
127. contradiction — this branch can't work, backtrack
128. a forbidden pair is left unattended
129. the variables, their possible values, and the constraints between them
130. no two classes a student takes may share a time slot
Matching (constraint satisfaction — modeling, pruning illegal states, and constraint propagation): 1→B, 2→C, 3→E, 4→A, 5→D
131. C — a number you can prove no solution can beat
132. B — a solution you actually found, showing it CAN be done in that many moves
133. C — meet at the same number
134. A — a solution in 2^n - 1 moves AND that fewer is impossible
135. C — must move at least once, and that requires clearing then rebuilding the smaller tower
136. B — a shorter solution might still exist unless you prove a matching lower bound
137. N-1 jumps
138. somewhere between 3 and 5 (not yet pinned down)
139. find a 4-move solution AND argue at least 4 are required
140. matches the lower bound (is optimal)
Matching (optimality proofs — lower bounds and why you cannot do better): 1→E, 2→A, 3→C, 4→B, 5→D
141. A — the remainder after dividing
142. A — 2
143. B — 2
144. C — the GCD of the jug sizes
145. C — 6
146. D — 0s and 1s (powers of two)

147. 13

148. n ones (e.g., 3 disks -> 111 = 7)

149. 1

150. every whole number is a multiple of 1 (the gcd)

Matching (number reasoning — gcd, modular arithmetic, and binary behind puzzle moves): 1→C, 2→B, 3→A, 4→E, 5→D

151. B — recursion (Nestor)

152. B — a parity/invariant check (Evan)

153. A — gcd (Gill)

154. D — look-ahead planning (Faro)

155. D — constraint satisfaction (Skiff)

156. A — reduction / monovariant (Vault)

157. optimality with a lower-bound proof (Minim)

158. binary / Gray-code counting (Gray)

159. state-space search (Wend)

160. 31

Matching (synthesis — mixed review connecting recursion, search, parity, constraints, optimality, gcd, and reduction): 1→C, 2→A, 3→D, 4→E, 5→B

161. The chicken! Then return empty, take the fox (or grain), bring the chicken back, take the other item, and finally return for the chicken.

162. An echo!

163. Yes! Weigh 3 vs 3: if they balance, the fake is in the other 2 (weigh those). If not, weigh 1 vs 1 from the lighter group of 3.

164. His son! "My father's son" is himself, so the man in the photo's father is him.

165. A keyboard!

166. None more than at the start! The boat rises with the tide, so the ladder rises too -- a classic assumption trap.

167. They are a grandfather, his son, and his grandson -- that's two fathers and two sons in just three people!

168. A towel!

169. Because it loved being figured out!
170. To get mentally stronger!
171. Because the answer was hiding at an angle!
172. Because the answer was never quite what it seemed!
173. Because it was searching for where it fit in!
174. To shine light on the one true answer!
