Proofquest — Activity Book

A Spark & Anvil activity book — puzzles, questions, and fun from Proofquest.

Write in the book, circle your answers, and check the Answer Key at the back.

---

1. Logic

Circle the correct answer

1. Which of these is a mathematical statement (has a definite truth value)?
(A) x + 2 (B) 5 + 3 = 8 (C) Solve for x (D) Is math fun?

2. Is the statement 'All squares are rectangles' true or false?
(A) True (B) False (C) Cannot be determined (D) Sometimes true

3. Classify: 'The number 7 is even.' Is this true or false?
(A) False (B) Neither (C) It depends on the context (D) True

4. Which statement has the OPPOSITE truth value of 'All triangles have 4 sides'?
(A) Triangles do not exist (B) Some triangles have 4 sides (C) All triangles have 3 sides (D) No shapes have 4 sides

5. Why is 'This sentence is false' NOT a valid logical statement? What goes wrong?
(A) It is just a normal false statement, as this is what research and standard practice suggests (B) It is always true, and this principle applies across similar situations (C) It creates a paradox: if it is true, then it must be false; if it is false, then it must be true — so it has no definite truth value (D) It is not written in math notation, making this the most widely accepted explanation

6. In the statement 'If it rains, then the ground gets wet,' what is the hypothesis?
(A) The ground gets wet (B) If it rains and the ground gets wet (C) Rain causes wetness (D) It rains

Write the answer

7. 'If a number is divisible by 10, then it is divisible by 5.' Is this true?
Answer: ________

8. 'If an animal is a dog, then it has four legs.' A cat has four legs. Can you conclude it is a dog?
Answer: ________

9. What is the contrapositive of 'If it is snowing, then it is cold'?
Answer: ________

10. Explain why 'If it is a cat, then it is an animal' is true, but 'If it is an animal, then it is a cat' is false.
Answer: ________

Match each question to its answer

1. Is the compound statement '3 > 2 AND 5 > 10' true or false? →
2. 'A square has 4 sides AND 4 right angles.' Why is this true? →
3. How many of these AND statements are true? (a) 2+2=4 AND 3+3=6, (b) 10>5 AND 10<3, (c) Dogs bark AND fish swim →
4. If P is true and Q is false, determine: (i) P AND Q, (ii) Q AND P, (iii) P AND P →
5. Create a mathematical AND statement that is true for the number 12 but false for the number 15. →

(A) Two: (a) and (c)
(B) The number is even AND the number is less than 14
(C) Both individual statements are true, and AND requires both to be true
(D) False
(E) (i) False, (ii) False, (iii) True

---

2. Patterns

Circle the correct answer

11. What is the next number in the pattern: 2, 6, 18, 54, ...?
(A) 108 (B) 72 (C) 162 (D) 216

12. Find the pattern and determine the 10th term: 3, 7, 11, 15, 19, ...
(A) 40 (B) 43 (C) 39 (D) 41

13. The sequence 1, 1, 2, 3, 5, 8, 13... follows what pattern?
(A) Each number is the sum of the two before it (B) The differences alternate between 1 and 2 (C) Each number adds 1 more than the previous addition (D) Each number doubles

14. The differences between consecutive terms of a sequence are 2, 4, 6, 8, 10. If the first term is 1, what is the 6th term?
(A) 31 (B) 36 (C) 21 (D) 30

15. Look at this pattern of equations: 1=1, 1+3=4, 1+3+5=9, 1+3+5+7=16. What conjecture can you make about the sum of the first n odd numbers?
(A) The sum is always a prime number (B) The sum equals $n \times (n+1)$ (C) The sum of the first n odd numbers equals n^2 (D) The sum always equals 2n

16. A pattern of dots forms triangular numbers: 1, 3, 6, 10, 15. What is the next triangular number?
(A) 21 (B) 18 (C) 20 (D) 25

Write the answer

17. How many small squares are in the 5th figure if the pattern is 1, 4, 9, 16, ...?
Answer: ________

18. A growing staircase pattern uses 1 block, then 3 blocks, then 6 blocks, then 10 blocks. How many blocks for a 7-step staircase?
Answer: ________

19. A pattern shows 4, 8, 14, 22, 32. Examine the first and second differences to find the next term.
Answer: ________

20. Examine the number of handshakes when each person in a group shakes hands with everyone else once: 2 people=1, 3 people=3, 4 people=6. Predict for 6 people and explain the pattern.
Answer: ________

Match each question to its answer

1. Test this conjecture: 'The sum of any two odd numbers is even.' Try 3+5, 7+9, and 11+13. →
2. Conjecture: 'Every prime number is odd.' Test with small primes: 2, 3, 5, 7, 11. →
3. Make a conjecture about the product of two consecutive numbers (like 3×4, 7×8, 5×6). Test three examples. →
4. Conjecture: '$n^2 + n + 41$ is always prime.' Test for n = 0, 1, 2, 3. Then test n = 40. →
5. Why is testing examples not the same as proving something? Use the conjecture 'all swans are white' as an analogy. →

(A) It works for 0-3 (giving 41, 43, 47, 53 — all prime) but fails at n=40: $40^2 + 40 + 41 = 1681 = 41^2$ — not prime!
(B) False — 2 is prime but even, so the conjecture is disproven by one counterexample
(C) The conjecture appears true: 3+5=8, 7+9=16, 11+13=24 — all even
(D) The product of two consecutive numbers is always even. 3×4=12, 7×8=56, 5×6=30

— all even.
(E) Testing examples can only support a conjecture, never prove it — one counterexample (like a black swan) disproves it. You would need to check ALL cases for certainty.

---

3. Deductive Reasoning

Circle the correct answer

21. All mammals breathe air. A dog is a mammal. What can you conclude?
(A) A dog breathes air (B) All dogs are the same (C) All animals that breathe air are mammals (D) Mammals are dogs

22. All rectangles have four sides. A square is a rectangle. Therefore:
(A) A square has four sides (B) A square has five sides (C) Rectangles are squares (D) All four-sided shapes are squares

23. 'All prime numbers greater than 2 are odd. 7 is a prime number greater than 2.' What follows?
(A) 7 is the largest prime (B) 7 is odd (C) All odd numbers are prime (D) 2 is odd

24. 'Some students like math. Maria is a student.' Can you conclude that Maria likes math?
(A) Only if Maria is in the 'some' group (B) No, Maria definitely does not like math (C) No — 'some' does not mean 'all,' so Maria might or might not like math (D) Yes, Maria must like math

25. Construct a valid syllogism using these premises: 'All equilateral triangles have three equal sides' and 'Triangle ABC is equilateral.' State the conclusion.
(A) Triangle ABC has three equal sides — this follows necessarily from the two premises (B) Equilateral means different side lengths, and this principle applies across similar situations (C) Triangle ABC has four sides, and this has been consistently demonstrated in practice (D) All triangles are equilateral, and this principle applies across similar situations

26. Which argument form is valid? (A) If P then Q; P is true; therefore Q. (B) If P then Q; Q is true; therefore P.
(A) Neither is valid (B) Both are valid (C) Only A (Modus Ponens) (D) Only B

Write the answer

27. 'If a shape is a square, it has equal sides. This shape has equal sides. Therefore it is a square.' Is this valid?
Answer: ________

28. 'If n is divisible by 6, then n is divisible by 3. n is NOT divisible by 3.' What can you conclude?
Answer: ________

29. Identify the logical structure: 'All cats are animals. All animals are living things. Therefore, all cats are living things.'
Answer: ________

30. Evaluate: 'Most birds can fly. Penguins are birds. Therefore, penguins can probably fly.' Is this reasoning valid even though the conclusion is wrong?
Answer: ________

Match each question to its answer

1. Given: (1) All multiples of 10 end in 0. (2) 350 is a multiple of 10. What must be true? →
2. Given: If today is Saturday, the library is closed. The library is open. What can you conclude about today? →
3. Given: No reptiles have fur. A gecko is a reptile. What follows? →
4. From a chain of deductions, determine: All A are B. All B are C. All C are D. What is the relationship between A and D? →
5. Given three clues, determine which fruit Sam likes. Clue 1: Sam does not like yellow fruits. Clue 2: Sam likes a fruit that grows on trees. Clue 3: The choices are banana (yellow, tree), grape (purple, vine), apple (red, tree), strawberry (red, bush). →

(A) All A are D
(B) 350 ends in 0
(C) Today is NOT Saturday
(D) Apple — it is not yellow (satisfies clue 1) and grows on a tree (satisfies clue 2)
(E) A gecko does not have fur

---

4. Inductive Reasoning

Circle the correct answer

31. After testing 1+3=4, 2+4=6, 5+7=12, and 3+9=12, what generalization can you make about the sum of an even and odd number?
(A) The sum is always greater than 10 (B) The sum is always even (C) No pattern exists (D) The sum of an even and an odd number is always odd

32. You observe: $3^2=9$, $5^2=25$, $7^2=49$, $9^2=81$. All results are odd. Is 'the square of any odd number is odd' a valid generalization?
(A) Only for single-digit odd numbers (B) Yes — $(2n+1)^2 = 4n^2+4n+1$, which is always odd (C) No, some odd squares might be even (D) Only for prime numbers

33. From the data: 6÷2=3, 6÷3=2, 12÷4=3, 12÷6=2 — a student generalizes 'division always produces a smaller number.' Find a counterexample.
(A) There is no counterexample, which is why this approach is recommended by experts (B) 12÷1=12 is the same, and this is the standard approach used in most cases (C) 6÷0.5=12, which is larger than 6. Dividing by a number between 0 and 1 makes the result larger. (D) 6÷6=1 is the counterexample, as this is what research and standard practice suggests

34. Examine: 2 is prime, 3 is prime, 5 is prime, 7 is prime. A student says 'numbers that do not end in 0, 4, 6, or 8 are prime.' Test this.
(A) True for all such numbers, and this has been consistently demonstrated in practice (B) True for numbers under 100, and this is the standard approach used in most cases (C) False. 9 does not end in those digits but 9 = 3×3, so it is not prime. Also 15 = 3×5, 21 = 3×7, etc. (D) Only false for very large numbers, which is why this approach is recommended by experts

35. Explain the fundamental limitation of inductive reasoning compared to deductive reasoning.
(A) Inductive reasoning is always wrong, which is the most common explanation for this phenomenon (B) Inductive reasoning can never prove a claim with certainty — it only provides supporting evidence. Deductive reasoning from true premises guarantees the conclusion. (C) Deductive reasoning requires more examples, which is why this approach is recommended by experts (D) There is no real difference, and this principle applies across similar situations

36. Which provides stronger inductive evidence for 'all multiples of 5 end in 0 or 5': testing 5 examples or 500 examples?
(A) 5 examples is sufficient, which is the most common explanation for this phenomenon (B) Neither provides any evidence, which is the most common explanation for this phenomenon (C) 500 examples — more evidence is stronger, though neither proves it with certainty (D) Both are equally strong, and this has been consistently demonstrated in practice

Write the answer

37. A conjecture passes testing with numbers 1-1000. Then it fails for 1001. What does this teach us?

Answer: ________

38. Rate these as weak, moderate, or strong inductive evidence: (A) 'I tested 3 cases.' (B) 'I tested 100 diverse cases.' (C) 'I tested all cases up to 1 million.'

Answer: ________

39. Compare inductive evidence: (A) Testing 'all primes > 2 are odd' with 2,3,5,7,11. (B) Testing 'all numbers > 1 are prime' with 2,3,5,7,11. Why is A's evidence trustworthy but B's is misleading?

Answer: ________

40. Design a testing strategy to evaluate the conjecture 'the sum of three consecutive even numbers is always divisible by 6.' What cases would you choose?

Answer: ________

Match each question to its answer

1. The polynomial $n^2 - n + 41$ gives prime numbers for $n = 1$ through $n = 40$. What happens at $n = 41$? →
2. How can a generalization from examples fail? Give a mathematical example. →
3. A friend tests the conjecture '$n^2 < 2^n$' for n = 1, 2, 3, 4, 5 and finds it true each time. Should they conclude it is always true? →
4. Explain why scientists use inductive reasoning constantly even though it cannot provide certainty. →
5. Critique: 'Since we cannot prove things by induction alone, there is no point in testing examples.' Is this a good argument? →

(A) $41^2 - 41 + 41 = 41^2 = 1681$, which equals 41×41 (not prime)
(B) No — testing examples is essential for discovering patterns, ruling out false conjectures, and building intuition before attempting formal proofs
(C) A pattern that holds for finitely many cases may fail for a specific value, like n^2-n+41 producing non-primes after n=40, or 'all numbers are less than 1 million' failing at 1,000,000
(D) Inductive reasoning is the best tool for discovering new patterns and forming hypotheses. While it cannot prove with certainty, it guides where to look for proofs and builds confidence through evidence.
(E) No — at n=1: 1<2, n=2: 4=4 (NOT less than!), so the conjecture actually fails at n=2. The friend made a calculation error.

---

5. Mathematical Arguments

Circle the correct answer

41. In solving 3x + 6 = 21, the first step is 3x = 15. What justifies this step?
(A) Subtraction property of equality: subtracting 6 from both sides (B) Adding 6 to both sides, which is why this approach is recommended by experts (C) Multiplying both sides by 3, which is why this approach is recommended by experts (D) Dividing both sides by 3, as this directly follows from the underlying principles

42. Why is 'because it looks right' NOT a valid mathematical justification?
(A) Math requires reasoning based on definitions, properties, or proven facts — not visual appearance or intuition alone (B) It is a valid justification, as this is what research and standard practice suggests (C) Visual evidence is the strongest proof, which is the most common explanation for this phenomenon (D) Only teachers need formal justifications, and this principle applies across similar situations

43. In the sequence: 12 is even → 12 = 2 × 6 → 12 is divisible by 2. What type of justification is used?
(A) No justification is present, making this the most widely accepted explanation (B) Proof by example, and this principle applies across similar situations (C) Guessing (D) Definition-based: using the definition of 'even' (divisible by 2) and showing 12 meets it

44. A student writes: 'The area of this triangle is 12 because I multiplied 4 × 6 ÷ 2.' Is this a complete justification? What is missing?
(A) The student should use a ruler instead, making this the most widely accepted explanation (B) This is a complete justification, making this the most widely accepted explanation (C) The answer 12 is wrong, making this the most widely accepted explanation (D) It shows the calculation but not WHY the formula works. A complete justification would state: 'Area = ½ × base × height. Base = 4, height = 6, so A = ½ × 4 × 6 = 12.'

45. Evaluate two justifications for why 0.999... = 1: (A) 'They look close enough.' (B) 'Let x = 0.999..., then 10x = 9.999..., so 10x - x = 9, giving 9x = 9, thus x = 1.' Which is valid?
(A) Only B — it uses algebraic manipulation to prove the claim. A is not a mathematical justification. (B) Both are valid, and this has been consistently demonstrated in practice (C) Only A because it uses common sense (D) Neither is valid, and this principle applies across similar situations

46. Why is the sum of two even numbers always even?
(A) It is not always even, and this has been consistently demonstrated in practice (B) Because I tested five examples (C) Even numbers are 2a and 2b. Their sum is 2a + 2b = 2(a+b), which has 2 as a factor, so it is even. (D) Because even numbers are nice

Write the answer

47. Explain why multiplying any number by zero gives zero.
Answer: ________

48. Why can you not divide by zero?
Answer: ________

49. Explain why the product of a positive and negative number is negative, using the pattern: $3 \times 2 = 6$, $3 \times 1 = 3$, $3 \times 0 = 0$, $3 \times (-1) = ?$
Answer: ________

50. Construct a 'why' explanation for the rule: 'a negative times a negative is positive.' Use the pattern method.
Answer: ________

Match each question to its answer

1. Build an argument for: 'The sum of any three consecutive integers is divisible by 3.' →
2. Argue that a number divisible by both 3 and 5 must be divisible by 15. →
3. Build a step-by-step argument: 'The square of any even number is divisible by 4.' →
4. Critique this argument: 'All numbers ending in 2 are even because 2 is even.' Is it correct? Is it well-argued? →
5. Design a complete mathematical argument with claim, evidence, and reasoning for a mathematical fact of your choice. →

(A) If $3|n$, then $n = 3a$. If $5|n$, then $n = 5b$. Since 3 and 5 share no common factors (coprime), n must be a multiple of $3 \times 5 = 15$.
(B) The conclusion is correct but the argument is weak. A better argument: a number ending in 2 can be written as $10k+2 = 2(5k+1)$, which is always even.
(C) Let the integers be n, n+1, n+2. Sum = $n+(n+1)+(n+2) = 3n+3 = 3(n+1)$. Since $3(n+1)$ has 3 as a factor, it is divisible by 3.
(D) Step 1: An even number is 2k for some integer k. Step 2: $(2k)^2 = 4k^2$. Step 3: $4k^2 = 4(k^2)$. Step 4: Since k^2 is an integer, $4k^2$ is divisible by 4.
(E) Claim: The product of two consecutive integers is always even. Evidence: $1 \times 2 = 2$, $2 \times 3 = 6$, $4 \times 5 = 20$. Reasoning: Of two consecutive integers n and n+1, one must be even. An even number times anything is even. Therefore $n(n+1)$ is always even.

---

6. Visual Proofs

Circle the correct answer

51. A rectangle is 5 units by (3+4) units. The total area can be computed as $5 \times 7 = 35$ OR as $5 \times 3 + 5 \times 4 = 15 + 20 = 35$. What property does this illustrate?
(A) The identity property (B) The commutative property (C) The associative property (D) The distributive property: $a(b+c) = ab + ac$

52. Two identical right triangles with legs a and b can be arranged to form a rectangle. What is the rectangle's area?
(A) $a \times b$ (the rectangle has dimensions a by b) (B) $a^2 + b^2$, since this is how the operation works with these particular numbers (C) $a + b$, according to the standard procedure for solving this kind of equation (D) 2ab, which is what you get when you apply the distributive property here

53. To prove $(a+b)^2 = a^2 + 2ab + b^2$, draw a square with side (a+b). What four regions appear inside?
(A) One large rectangle (B) Four equal squares (C) Two triangles and two rectangles (D) One $a \times a$ square, two $a \times b$ rectangles, and one $b \times b$ square

54. How does rearranging four right triangles inside a large square prove the Pythagorean theorem?
(A) Triangles do not fit inside squares, and this principle applies across similar situations (B) Four triangles (legs a,b) fit inside a $c \times c$ square, leaving a small square of side (a-b). Area of large square $c^2 = 4(\frac{1}{2}ab) + (a-b)^2 = 2ab + a^2 - 2ab + b^2 = a^2 + b^2$. (C) You need calculus for this proof, and this principle applies across similar situations (D) This proves the area formula, not Pythagorean theorem, which is why this approach is recommended by experts

55. Create a visual argument showing that the area of a parallelogram equals base × height by transforming it into a rectangle.
(A) Parallelograms cannot become rectangles, making this the most widely accepted explanation (B) Cut a right triangle from one end of the parallelogram and move it to the other end. The result is a rectangle with the same base and height, proving Area = base × height. (C) Use a formula instead, which is the most common explanation for this phenomenon (D) Measure with a ruler, as this directly follows from the underlying principles

56. In a right triangle with legs 3 and 4, what is the hypotenuse according to the Pythagorean theorem?
(A) 7 (B) 5 (C) 12 (D) 25

Write the answer

57. President Garfield proved the Pythagorean theorem using a trapezoid made from three right triangles. What is the key idea?
Answer: ________

58. Why does the Pythagorean theorem only work for RIGHT triangles?
Answer: ________

59. A proof shows that $5^2 = 3^2 + 4^2$ using physical squares of paper. What does each paper square represent?
Answer: ________

60. There are over 350 known proofs of the Pythagorean theorem. Why would mathematicians keep proving something already known to be true?
Answer: ________

Match each question to its answer

1. A Venn diagram shows Set A (even numbers) inside Set B (integers). What does this prove? →
2. Draw a number line argument: why is there no largest integer? →
3. A geometric diagram shows two triangles with all corresponding angles equal and proportional sides. What is this evidence for? →
4. A 'proof without words' shows a staircase of dots: 1, 1+2, 1+2+3, forming triangles, and two copies fitting together to make rectangles. What formula does this prove? →
5. Evaluate: 'Visual proofs are not real proofs because they use pictures instead of algebra.' Is this criticism valid? →

(A) All even numbers are integers (every element of A is in B)
(B) $1+2+3+...+n = n(n+1)/2$ — because two copies of the triangle make an $n×(n+1)$ rectangle
(C) Not valid — visual proofs are legitimate when they demonstrate the logical structure clearly. Many visual proofs can be translated to algebraic proofs and vice versa. Both are rigorous when done carefully.
(D) For any integer n on the number line, n+1 is also an integer and lies to its right. Since you can always add 1, there is always a larger integer.
(E) The triangles are similar (AA or AAA similarity criterion)

---

7. Famous Problems

Circle the correct answer

61. In the Konigsberg bridge problem, Euler asked whether you could walk across all 7 bridges exactly once. What was his answer?
(A) No — it is impossible because more than two landmasses have an odd number of bridges (B) Yes, if you walk fast enough, and this is the standard approach used in most cases (C) Only on Mondays, and this principle applies across similar situations

(D) Yes, starting from the island, making this the most widely accepted explanation

62. In graph theory, what is the 'degree' of a vertex?
(A) The distance from the center (B) The angle of the vertex (C) The number of edges (connections) meeting at that vertex (D) The temperature at that point

63. An Euler path visits every edge exactly once. When is an Euler path possible in a graph?
(A) When the graph has exactly 0 or 2 vertices with odd degree (B) When all vertices have even degree only (C) When the graph has at least 5 vertices (D) Always

64. A graph has vertices A(degree 2), B(degree 3), C(degree 4), D(degree 3). Can you draw an Euler path? If so, where does it start and end?
(A) No, it is impossible, and this principle applies across similar situations (B) Start at any vertex, which is the most common explanation for this phenomenon (C) Yes — start at B and end at D (or vice versa), since B and D are the two vertices with odd degree (D) Start at A and end at C, which is why this approach is recommended by experts

65. Explain how Euler transformed a geographic puzzle about bridges into an abstract mathematical problem. Why was this revolutionary?
(A) Euler ignored sizes, shapes, and distances — keeping only connections between landmasses. This abstraction created graph theory, showing that the answer depends ONLY on the connection structure, not physical geography. (B) He swam across the river instead, which is why this approach is recommended by experts (C) He built new bridges, as this directly follows from the underlying principles (D) He just counted bridges, and this is the standard approach used in most cases

66. What does the four-color theorem state?
(A) Maps must use red, blue, green, and yellow, and this is the standard approach used in most cases (B) Any map can be colored using at most four colors so that no two adjacent regions share the same color (C) Four colors are needed for any drawing, and this has been consistently demonstrated in practice (D) All maps need exactly four colors, and this has been consistently demonstrated in practice

Write the answer

67. Can you color a map of just 3 countries (each bordering the other two) with 2 colors?
Answer: ________

68. Why was the four-color theorem controversial when it was first 'proven' in 1976?
Answer: ________

69. A graph where vertices represent regions and edges represent shared borders is called a 'planar graph.' How does the four-color theorem relate to graph theory?
Answer: ________

70. Design a simple map that requires exactly 4 colors. Describe its structure.
Answer: ________

Match each question to its answer

1. Goldbach's Conjecture states that every even number greater than 2 can be written as the sum of two prime numbers. Test this for 20. →
2. Verify Goldbach's Conjecture for the number 30. Find at least two different decompositions. →
3. Why is Goldbach's Conjecture considered one of the hardest unsolved problems in mathematics? →
4. A related result (Chen's theorem, 1973) proves every large enough even number is the sum of a prime and a 'semiprime' (product of at most two primes). How does this relate to Goldbach? →
5. If you were trying to disprove Goldbach's Conjecture, what would you need to find? →

(A) It is a weaker result that gets close to Goldbach: instead of two primes, it allows one prime plus a number with at most two prime factors. It shows mathematicians are approaching Goldbach incrementally.
(B) A single even number greater than 2 that cannot be expressed as the sum of any two prime numbers — one counterexample would disprove it
(C) It is easy to state and test for individual numbers, but proving it for ALL even numbers has resisted every proof technique for over 280 years
(D) $20 = 3 + 17$ (both prime)
(E) $30 = 7+23$ and $30 = 11+19$ and $30 = 13+17$

---

8. Proof Techniques

Circle the correct answer

71. Prove directly: 'If n is an odd integer, then n^2 is odd.'
(A) n^2 is always odd because n is odd (B) Test: $3^2=9$ is odd, so it is always true, and this has been consistently demonstrated in practice (C) Let $n = 2k+1$ (odd). Then $n^2 = (2k+1)^2 = 4k^2+4k+1 = 2(2k^2+2k)+1$, which has the form $2m+1$, so n^2 is odd. (D) This cannot be proven, and this is the standard approach used in most cases

72. What are the three parts of a direct proof?
(A) Introduction, body, conclusion (like an essay) (B) State the theorem three times (C) Guess, check, done (D) Assume the hypothesis, apply logical steps, arrive at the conclusion

73. Prove: 'The sum of two rational numbers is rational.'
(A) Let a/b and c/d be rational (b,d≠0). Sum = a/b + c/d = (ad+bc)/(bd). Since ad+bc and bd are integers (bd≠0), the sum is rational. (B) All numbers are rational, as this directly follows from the underlying principles (C) Test: 1/2 + 1/3 = 5/6, done, and this principle applies across similar situations (D) This is false, which is why this approach is recommended by experts

74. A student writes: 'Proof: 4+6=10. 10 is even. Therefore the sum of two even numbers is always even.' What is wrong with this proof?
(A) Nothing is wrong, this is a valid proof, and this has been consistently demonstrated in practice (B) It only tests ONE specific example, not all even numbers. A direct proof must use variables (like 2a+2b=2(a+b)) to cover ALL cases. (C) 10 is not even, as this directly follows from the underlying principles (D) The addition is incorrect, and this has been consistently demonstrated in practice

75. Construct a direct proof that the product of two consecutive integers is even.
(A) Let the integers be n and n+1. Case 1: n is even (n=2k), so n(n+1)=2k(n+1), which is even. Case 2: n is odd, then n+1 is even (n+1=2k), so n(n+1)=n(2k)=2nk, which is even. In either case, the product is even. (B) 3×4=12 which is even, so it is always true, which is why this approach is recommended by experts (C) This is obvious and needs no proof, which is the most common explanation for this phenomenon (D) Consecutive integers are always odd, and this has been consistently demonstrated in practice

76. In proof by contradiction, what do you assume at the start?
(A) You assume the statement you want to prove is FALSE, then show this leads to a logical impossibility (B) You pick a random number, and this has been consistently demonstrated in practice (C) You assume the statement is true, which is why this approach is recommended by experts (D) You assume nothing, and this has been consistently demonstrated in practice

Write the answer

77. Prove by contradiction: '√2 is irrational.'
Answer: ________

78. Prove by contradiction: 'There is no largest even number.'
Answer: ________

79. What is the key difference between direct proof and proof by contradiction?
Answer: ________

80. A student tries proof by contradiction but reaches 5 = 5 instead of a false statement. What went wrong? What should they do?
Answer: ________

Match each question to its answer

1. What is proof by exhaustion? →
2. Prove by exhaustion: 'Every prime number less than 20 that is greater than 2 is odd.' →
3. Prove by exhaustion: '$n^2 + n$ is even for n = 1, 2, 3, 4, 5.' →
4. When is proof by exhaustion practical, and when is it impractical? →
5. Evaluate: The four-color theorem was proven by checking ~1,936 configurations by computer. Is this proof by exhaustion? What makes it different from simple case-checking? →

(A) Testing ALL possible cases to verify a statement — when the number of cases is finite and manageable
(B) Primes less than 20 and >2: 3,5,7,11,13,17,19. Check each: 3(odd), 5(odd), 7(odd), 11(odd), 13(odd), 17(odd), 19(odd). All seven are odd.
(C) Yes, it is a form of exhaustion, but with a crucial preliminary step: mathematicians first PROVED that every possible map reduces to one of 1,936 configurations. The exhaustion only works because the reduction was proven complete.
(D) Practical when there are finitely many, manageable cases (like checking all days of the week). Impractical when cases are infinite or astronomically large.
(E) n=1: 1+1=2 (even). n=2: 4+2=6 (even). n=3: 9+3=12 (even). n=4: 16+4=20 (even). n=5: 25+5=30 (even). All five cases verified.

---

9. Mathematical Proofs

Circle the correct answer

81. What is a direct proof?
(A) A proof that tests every possible case, which is why this approach is recommended by experts (B) A proof that assumes the conclusion is false, and this has been consistently demonstrated in practice (C) A proof that uses a counterexample, and this is the standard approach used in most cases (D) A proof that starts from known facts or hypotheses and uses logical steps to reach the conclusion

82. In a direct proof of 'If P, then Q,' what is the first step?
(A) Assume Q is false (B) Assume Q is true (C) Assume P is true (D) Assume P is

false

83. What does it mean for an integer to be even?
(A) It is divisible by 4   (B) It can be written as 2k for some integer k   (C) It ends in 0   (D) It is a positive number

84. What does it mean for an integer to be odd?
(A) It can be written as 2k + 1 for some integer k   (B) It cannot be divided by anything   (C) It is greater than zero   (D) It ends in 1

85. What does 'divisibility' mean in mathematics? Specifically, what does 'a divides b' mean?
(A) There exists an integer k such that b = a × k   (B) a is smaller than b   (C) a minus b equals zero   (D) a and b are both even

86. What is the algebraic form of the statement 'n is a multiple of 3'?
(A) n + 3 = 0   (B) 3n = 1   (C) n = 3k for some integer k   (D) n = k/3 for some integer k

Write the answer

87. Explain why the sum of two even numbers is always even.
Answer: ________

88. A student says: 'I checked that 3 + 5 = 8 and 7 + 9 = 16, so the sum of two odd numbers is always even.' Is this a valid proof? Why or why not?
Answer: ________

89. Explain the difference between a conjecture and a theorem.
Answer: ________

90. Why do we need to use algebraic definitions (like n = 2k) instead of just examples when writing a direct proof?
Answer: ________

Match each question to its answer

1. In the proof that 'the product of two odd numbers is odd,' why do we write a = 2m + 1 and b = 2n + 1 using different letters m and n? →
2. Prove: 'If n is even, then n^2 is even.' →
3. Prove: 'The sum of three consecutive integers is divisible by 3.' →
4. Prove: 'If a | b and a | c, then a | (b + c).' →
5. Prove: 'If n is odd, then n^2 + n is even.' →

(A) Let the integers be n, n+1, n+2. Sum = n + (n+1) + (n+2) = 3n + 3 = 3(n+1). Since n+1 is an integer, the sum is divisible by 3.
(B) Let n = 2k + 1. Then $n^2 + n = (2k+1)^2 + (2k+1) = 4k^2 + 4k + 1 + 2k + 1 = 4k^2 + 6k + 2 = 2(2k^2 + 3k + 1)$. This is even.
(C) Since a | b, we have b = ak for some integer k. Since a | c, we have c = aj for some integer j. Then b + c = ak + aj = a(k + j). Since k + j is an integer, a | (b + c).
(D) Let n = 2k. Then $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$. Since $2k^2$ is an integer, n^2 has the form 2m, so n^2 is even.
(E) Because the two odd numbers can be different, so they may have different values of k

---

10. Mathematical Proofs

Circle the correct answer

91. What is the basic idea behind proof by contradiction?
(A) Show a statement is true by testing examples (B) Assume the opposite of what you want to prove, then show this assumption leads to a logical impossibility (C) Prove the statement directly from definitions (D) Find a counterexample to the statement

92. What is another name for proof by contradiction?
(A) Reductio ad absurdum (reduction to absurdity) (B) Proof by construction (C) Proof by example (D) Proof by exhaustion

93. In a proof by contradiction, what do you do after you reach a contradiction?
(A) Conclude that your initial assumption (the negation) must be false, so the original statement is true (B) Conclude that the statement is false, as this is what research and standard practice suggests (C) Start over with a different assumption, and this is the standard approach used in most cases (D) Check more examples to confirm, as this directly follows from the underlying principles

94. What is the negation of 'n is irrational'?
(A) n is zero (B) n is negative (C) n is rational (D) n does not exist

95. What is the negation of 'There are infinitely many primes'?
(A) There are only finitely many primes (B) All numbers are prime (C) There are no primes (D) There are infinitely many composite numbers

96. What does 'rational number' mean?
(A) A positive whole number, making this the most widely accepted explanation (B) A number with a terminating decimal, which is the most common explanation for this phenomenon (C) A number that can be expressed as a/b where a and b are integers

and $b \neq 0$ (D) A number that makes sense, which is why this approach is recommended by experts

Write the answer

97. Why might you choose proof by contradiction instead of direct proof?
Answer: ________

98. Explain why the following is a contradiction: 'We have shown that k is both even and odd.'
Answer: ________

99. In Euclid's proof that there are infinitely many primes, why does he consider the number $N = p_1 \times p_2 \times \ldots \times p_k + 1$?
Answer: ________

100. Explain the structure of the proof that $\sqrt{2}$ is irrational in your own words.
Answer: ________

Match each question to its answer

1. What is the difference between proof by contradiction and proof by contrapositive? →
2. Prove by contradiction: 'There is no largest integer.' →
3. Prove by contradiction: 'If n^2 is even, then n is even.' →
4. Prove by contradiction: 'The sum of a rational and an irrational number is irrational.' →
5. Prove by contradiction: 'If $3n + 2$ is odd, then n is odd.' →

(A) Contrapositive proves 'If not Q, then not P' (logically equivalent to 'If P then Q'). Contradiction assumes P and not Q together, then derives any absurdity.
(B) Assume n^2 is even but n is odd. Let $n = 2k+1$. Then $n^2 = 4k^2+4k+1 = 2(2k^2+2k)+1$, which is odd. This contradicts n^2 being even. So n must be even.
(C) Assume there is a largest integer M. Consider $M + 1$. Since $M + 1 > M$ and $M + 1$ is an integer, M is not the largest. Contradiction.
(D) Assume r (rational) + x (irrational) = s (rational). Then $x = s - r$. Since rationals are closed under subtraction, x would be rational — contradiction since x is irrational.
(E) Assume $3n + 2$ is odd but n is even. Let $n = 2k$. Then $3(2k) + 2 = 6k + 2 = 2(3k + 1)$, which is even. This contradicts $3n + 2$ being odd. So n must be odd.

---

11. Mathematical Proofs

Circle the correct answer

101. What is a counterexample?
(A) A proof that a statement is true (B) A number that is very large (C) An example that supports a statement (D) A specific example that shows a universal statement is false

102. How many counterexamples do you need to disprove a universal statement?
(A) More than half of all possible cases (B) At least three (C) Just one (D) You need infinitely many

103. What is a universal statement?
(A) A statement that is always true, which is the most common explanation for this phenomenon (B) A statement that only applies to large numbers, and this has been consistently demonstrated in practice (C) A statement about the universe, as this directly follows from the underlying principles (D) A statement that claims something is true for ALL members of a set, often using words like 'every,' 'all,' or 'for any'

104. What is an existential statement?
(A) A statement about existence in general (B) A statement about very small numbers (C) A statement that claims there EXISTS at least one member of a set with a certain property (D) A statement that claims everything has a property

105. True or false: To disprove an existential statement 'There exists an x such that P(x),' you can find a counterexample.
(A) True — but you need two counterexamples, making this the most widely accepted explanation (B) True — one counterexample always suffices, as this directly follows from the underlying principles (C) False — to disprove an existential statement, you must show that P(x) fails for ALL x, not just find one counterexample (D) False — existential statements cannot be disproved, as this directly follows from the underlying principles

106. What is the negation of 'All even numbers greater than 2 are composite'?
(A) No even numbers are composite, and this is the standard approach used in most cases (B) There exists an even number greater than 2 that is NOT composite (i.e., that is prime) (C) Some odd numbers are composite, and this principle applies across similar situations (D) All even numbers greater than 2 are prime, which is why this approach is recommended by experts

Write the answer

107. Explain why n = 2 is a counterexample to 'All prime numbers are odd.'
Answer: ________

108. Why is testing examples useful for conjectures, even though examples alone cannot prove a universal statement?
Answer: ________

109. Explain the difference between 'disprove' and 'prove false.'
Answer: ________

110. The conjecture '$n^2 + n + 41$ is prime for all positive integers n' holds for n = 1, 2, 3, ..., 39. Does this mean it is true? Why or why not?
Answer: ________

Match each question to its answer

1. What is the relationship between a counterexample and a universal statement's truth value? →
2. Find a counterexample to: 'The sum of two prime numbers is always even.' →
3. Find a counterexample to: 'If n is a positive integer, then $2^n - 1$ is prime.' →
4. Disprove: 'The square of any integer is greater than the integer itself.' →
5. Disprove: 'If a > b and c > d, then a − c > b − d.' →

(A) 2 + 3 = 5. Both 2 and 3 are prime, but 5 is odd. (Any sum involving the prime 2 and an odd prime gives an odd result.)
(B) Counterexample: a = 5, b = 1, c = 10, d = 2. Then a > b (5 > 1) and c > d (10 > 2), but a − c = −5 and b − d = −1, so −5 < −1, not greater.
(C) A single counterexample proves the universal statement is false. If no counterexample exists (and this can be proven), the statement is true.
(D) n = 4: $2^4 - 1 = 15 = 3 \times 5$, which is not prime.
(E) Counterexample: n = 1. Then $n^2 = 1$, which equals n, not greater. Also n = 0: $0^2 = 0 = 0$.

---

12. Mathematical Proofs

Circle the correct answer

111. What are the two main steps of a proof by mathematical induction?
(A) The example step and the algebra step, and this principle applies across similar situations (B) The hypothesis and the conclusion, as this is what research and standard practice suggests (C) The base case (show the statement is true for the first value) and the inductive step (show that if it's true for n = k, then it's true for n = k + 1) (D) The forward step and the backward step, making this the most widely accepted explanation

112. What is the 'inductive hypothesis' in a proof by induction?
(A) The assumption that the statement is true for all k, and this principle applies across similar situations (B) The assumption that the statement P(k) is true for some particular (but arbitrary) integer k (C) The base case of the proof, and this is the standard approach used in most cases (D) The final conclusion of the proof, making this the most widely accepted explanation

113. In the domino analogy for induction, what does the base case represent?
(A) Setting up all the dominoes (B) Counting all the dominoes (C) Knocking over the first domino (D) The last domino falling

114. In the domino analogy, what does the inductive step represent?
(A) Picking up the last domino, and this has been consistently demonstrated in practice (B) The total number of dominoes, as this is what research and standard practice suggests (C) The distance between dominoes, which is why this approach is recommended by experts (D) The guarantee that each domino, when it falls, knocks over the next one

115. What is the formula for the sum of the first n positive integers?
(A) $n^2/2$, based on the standard mathematical approach for this type of problem (B) $n(n - 1)/2$, according to the standard procedure for solving this kind of equation (C) $1 + 2 + 3 + ... + n = n(n + 1)/2$ (D) $n(n + 1)$, since this is how the operation works with these particular numbers

116. Why is the base case necessary in an induction proof? What goes wrong without it?
(A) The base case is optional and only for clarity, which is the most common explanation for this phenomenon (B) Without the base case, the inductive step has no starting point. You prove 'if true for k, then true for k+1' but never establish that it IS true for any specific value. (C) Without the base case, the proof is still valid but incomplete, which is the most common explanation for this phenomenon (D) The base case is just an example and doesn't affect the proof, which is the most common explanation for this phenomenon

Write the answer

117. Explain in your own words why mathematical induction proves a statement for ALL positive integers, not just the ones we checked.
Answer: ________

118. Why is assuming P(k) in the inductive step not circular reasoning?
Answer: ________

119. What is the difference between 'weak' (ordinary) induction and 'strong' induction?
Answer: ________

120. Explain how the sum formula $1 + 2 + ... + n = n(n+1)/2$ can be verified for the base case $n = 1$.
Answer: ________

Match each question to its answer

1. A student says: 'I proved the inductive step but skipped the base case. My proof is still valid because the inductive step shows the pattern works.' Why is this wrong? →
2. Prove by induction: $1 + 2 + 3 + ... + n = n(n+1)/2$. →
3. Prove by induction: $2^0 + 2^1 + 2^2 + ... + 2^n = 2^{n+1} - 1$. →
4. Prove by induction: $n! \geq 2^{n-1}$ for all $n \geq 1$. →
5. Prove by induction: For all $n \geq 1$, the number $4^n - 1$ is divisible by 3. →

(A) Base case: n=0: $2^0 = 1$ and $2^1 - 1 = 1$. ✓
Inductive step: Assume $\Sigma\ 2^i$ (i=0 to k) = 2^(k+1) − 1. Add 2^(k+1): 2^(k+1) − 1 + 2^(k+1) = 2 × 2^(k+1) − 1 = 2^(k+2) − 1. ✓
(B) Base case: n=1: 1 = 1(2)/2 = 1. ✓
Inductive step: Assume 1+2+...+k = k(k+1)/2. Then 1+2+...+k+(k+1) = k(k+1)/2 + (k+1) = (k+1)(k/2 + 1) = (k+1)(k+2)/2. This is the formula with n = k+1. ✓
(C) The inductive step only proves a conditional: IF the statement holds for k, THEN it holds for k+1. Without the base case, you never establish that it actually holds for any specific value, so the chain never starts.
(D) Base case: n=1: $1! = 1 \geq 2^0 = 1$. ✓
Inductive step: Assume k! ≥ 2^(k−1). Then (k+1)! = (k+1) × k! ≥ (k+1) × 2^(k−1) ≥ 2 × 2^(k−1) = 2^k (since k+1 ≥ 2 for k ≥ 1). ✓
(E) Base case: n=1: $4^1 - 1 = 3$, and 3 | 3. ✓
Inductive step: Assume 3 | $(4^k - 1)$. Then 4^(k+1) − 1 = 4 × 4^k − 1 = $4(4^k - 1) + 4 - 1 = 4(4^k - 1) + 3$. Since 3 divides both $4(4^k - 1)$ and 3, it divides their sum. ✓

---

13. Logic

Circle the correct answer

121. If P is true and Q is false, determine: (i) P AND Q, (ii) Q AND P, (iii) P AND P
(A) All three are False (B) All three are True (C) (i) False, (ii) False, (iii) True (D) (i) True, (ii) False, (iii) True

122. A pattern shows 4, 8, 14, 22, 32. Examine the first and second differences to find the next term.
(A) 40 (B) 46 (C) 42 (D) 44

123. Construct a valid syllogism using these premises: 'All equilateral triangles have three equal sides' and 'Triangle ABC is equilateral.' State the conclusion.
(A) All triangles are equilateral, and this principle applies across similar situations (B) Equilateral means different side lengths, and this principle applies across similar situations (C) Triangle ABC has four sides, and this has been consistently demonstrated in practice (D) Triangle ABC has three equal sides — this follows necessarily from the two premises

124. How can a generalization from examples fail? Give a mathematical example.
(A) Only non-math generalizations can fail, as this is what research and standard practice suggests (B) Failures only happen with odd numbers, making this the most widely accepted explanation (C) Generalizations from math examples never fail, and this has been consistently demonstrated in practice (D) A pattern that holds for finitely many cases may fail for a specific value, like n^2-n+41 producing non-primes after n=40, or 'all numbers are less than 1 million' failing at 1,000,000

125. Compare proof structures: (A) Direct proof: assume premises, derive conclusion. (B) Proof by contradiction: assume conclusion is false, derive impossibility. When might B be easier than A?
(A) A is always easier, which is the most common explanation for this phenomenon (B) B is always easier, as this directly follows from the underlying principles (C) They are the same thing, which is why this approach is recommended by experts (D) B is easier when the conclusion is hard to derive directly but its negation leads to an obvious absurdity — like proving √2 is irrational

126. A visual shows n^2 dots arranged as a square. Adding (2n+1) dots in an L-shape creates $(n+1)^2$. Write this as an equation.
(A) $n^2 + (2n+1) = (n+1)^2$ (B) $n^2 + n = (n+1)^2$ (C) $n^2 \times 2 = (n+1)^2$ (D) $n^2 + 2n = (n+1)^2$

Write the answer

127. Goldbach's Conjecture states that every even number greater than 2 can be written as the sum of two prime numbers. Test this for 20.
Answer: ________

128. Mathematical induction proves statements for ALL natural numbers using two steps. What are they?
Answer: ________

129. Classify: 'The number 7 is even.' Is this true or false?
Answer: ________

130. Test: 'If a number ends in 0 or 5, it is divisible by 5.' Check: 25, 40, 105, 350.
Answer: ________

Match each question to its answer

1. 'All prime numbers greater than 2 are odd. 7 is a prime number greater than 2.' What follows? →
2. After testing 1+3=4, 2+4=6, 5+7=12, and 3+9=12, what generalization can you make about the sum of an even and odd number? →
3. In solving 3x + 6 = 21, the first step is 3x = 15. What justifies this step? →
4. A proof shows that $1+3+5+...+(2n-1) = n^2$ by building squares from L-shaped layers. How does the 4th L-shape (7 dots) contribute? →
5. If you were trying to disprove Goldbach's Conjecture, what would you need to find? →

(A) 7 is odd
(B) Subtraction property of equality: subtracting 6 from both sides
(C) The sum of an even and an odd number is always odd
(D) A single even number greater than 2 that cannot be expressed as the sum of any two prime numbers — one counterexample would disprove it
(E) The 4th L-shape (7 dots) transforms a 3×3 square (9 dots) into a 4×4 square (16 dots): 9+7=16

---

14. Logic

Circle the correct answer

131. Explain why 'If it is a cat, then it is an animal' is true, but 'If it is an animal, then it is a cat' is false.
(A) Both are false, and this is the standard approach used in most cases (B) The second is the contrapositive, as this is what research and standard practice suggests (C) Both should be true, as this directly follows from the underlying principles (D) All cats are animals (true), but not all animals are cats — dogs, fish, and birds are animals too (converse is false)

132. Given: (1) All even numbers are divisible by 2. (2) All numbers divisible by 2 have 0, 2, 4, 6, or 8 as their last digit. (3) 47 ends in 7. Is 47 even?
(A) Yes, because 4+7=11 (B) Cannot determine, which is the most common explanation for this phenomenon (C) No — 47 ends in 7, which is not 0/2/4/6/8, so it is not divisible by 2, therefore not even (D) 47 is even because it contains the digit 4

133. Classify: 'All right angles measure 90°. This angle is a right angle. Therefore, it measures 90°.'
(A) Inductive reasoning, which is the most common explanation for this phenomenon (B) Deductive reasoning — applying a general rule to a specific case

with a guaranteed conclusion   (C) A guess   (D) An opinion, as this is what research and standard practice suggests

134. Construct a 'why' explanation for the rule: 'a negative times a negative is positive.' Use the pattern method.
(A) Negatives cancel out — no further explanation needed, as this is what research and standard practice suggests   (B) It is a rule you must memorize, and this has been consistently demonstrated in practice   (C) Pattern: -3×2=-6, -3×1=-3, -3×0=0, -3×(-1)=? Each step increases by 3, so -3×(-1)=0+3=3 (positive). Continuing: -3×(-2)=6.   (D) A negative times a negative is negative, as this directly follows from the underlying principles

135. A proof shows that $5^2 = 3^2 + 4^2$ using physical squares of paper. What does each paper square represent?
(A) The area of a square built on each side of the right triangle: 25 sq units = 9 sq units + 16 sq units   (B) The sides themselves, and this is the standard approach used in most cases   (C) The angles of the triangle, and this is the standard approach used in most cases   (D) Random squares, and this is the standard approach used in most cases

136. Why is Goldbach's Conjecture considered one of the hardest unsolved problems in mathematics?
(A) It is easy to state and test for individual numbers, but proving it for ALL even numbers has resisted every proof technique for over 280 years   (B) Nobody has tried to prove it, and this principle applies across similar situations   (C) It was only stated recently, making this the most widely accepted explanation   (D) It is actually quite easy, as this is what research and standard practice suggests

Write the answer

137. Construct a direct proof that the product of two consecutive integers is even.
Answer: ________

138. 'A square has 4 sides AND 4 right angles.' Why is this true?
Answer: ________

139. Conjecture: '$n^2 + n + 41$ is always prime.' Test for n = 0, 1, 2, 3. Then test n = 40.
Answer: ________

140. Given: No reptiles have fur. A gecko is a reptile. What follows?
Answer: ________

Match each question to its answer

1. A farmer notices that planting in soil A always yields more than soil B over 5 seasons. Is this inductive or deductive reasoning? →
2. Build an argument for: 'The sum of any three consecutive integers is divisible by 3.' →
3. The visual proof of $(a-b)^2$ shows a large square with a smaller square removed. Translate this to algebra. →
4. What strategy did Euler use to solve the Konigsberg bridge problem? →
5. When is proof by exhaustion practical, and when is it impractical? →

(A) Inductive — generalizing from 5 seasons of observation to a broader conclusion
(B) Let the integers be n, n+1, n+2. Sum = n+(n+1)+(n+2) = 3n+3 = 3(n+1). Since 3(n+1) has 3 as a factor, it is divisible by 3.
(C) $(a-b)^2 = a^2 - 2ab + b^2$. Visually: start with a^2 square, remove two a×b strips (but their overlap b^2 was removed twice), so add b^2 back: $a^2 - 2ab + b^2$.
(D) Practical when there are finitely many, manageable cases (like checking all days of the week). Impractical when cases are infinite or astronomically large.
(E) Abstraction and simplification — he reduced the complex map to a simple graph and focused only on the relevant feature (vertex degrees)

---

15. Logic

Circle the correct answer

141. 'A number is prime OR even.' For which number below is this statement FALSE?
(A) 4 (even)   (B) 1 (not prime and not even)   (C) 3 (prime)   (D) 2 (prime and even)

142. Evaluate this three-step argument: (1) All squares have 4 equal sides. (2) All shapes with 4 equal sides are rhombuses. (3) Therefore all squares are rhombuses. Is this valid, and is the conclusion true?
(A) The argument is valid but the conclusion is false   (B) The argument is valid AND the conclusion is true — squares are indeed a special type of rhombus   (C) Squares and rhombuses are completely different   (D) The argument is invalid

143. From the data: 6÷2=3, 6÷3=2, 12÷4=3, 12÷6=2 — a student generalizes 'division always produces a smaller number.' Find a counterexample.
(A) There is no counterexample, which is why this approach is recommended by experts   (B) 6÷6=1 is the counterexample, as this is what research and standard practice suggests   (C) 6÷0.5=12, which is larger than 6. Dividing by a number between 0 and 1 makes the result larger.   (D) 12÷1=12 is the same, and this is the standard approach used in most cases

144. Create a rubric with 3 criteria for evaluating mathematical arguments. What makes an argument excellent versus poor?
(A) Check if it uses big words, as this directly follows from the underlying principles (B) Just check if the answer is right, making this the most widely accepted explanation (C) 1. Correctness: all steps are mathematically valid. 2. Completeness: no gaps — every step is justified. 3. Clarity: a reader can follow the logic without guessing. Excellent = all three; Poor = missing multiple criteria. (D) Count the number of words, as this is what research and standard practice suggests

145. A graph where vertices represent regions and edges represent shared borders is called a 'planar graph.' How does the four-color theorem relate to graph theory?
(A) The four-color theorem is equivalent to saying every planar graph can be properly vertex-colored with at most 4 colors (B) Graphs have nothing to do with maps, which is the most common explanation for this phenomenon (C) Only non-planar graphs need 4 colors, which is the most common explanation for this phenomenon (D) The theorem only applies to geography, as this directly follows from the underlying principles

146. What is proof by exhaustion?
(A) Testing ALL possible cases to verify a statement — when the number of cases is finite and manageable (B) Testing a few random examples, which is why this approach is recommended by experts (C) Giving up on the proof, making this the most widely accepted explanation (D) An exhausting (long) proof of any type, as this is what research and standard practice suggests

Write the answer

147. If P is false and Q is false, what is the result of P OR Q?
Answer: ________

148. A pattern of dots forms triangular numbers: 1, 3, 6, 10, 15. What is the next triangular number?
Answer: ________

149. Rate these as weak, moderate, or strong inductive evidence: (A) 'I tested 3 cases.' (B) 'I tested 100 diverse cases.' (C) 'I tested all cases up to 1 million.'
Answer: ________

150. A student writes: 'The area of this triangle is 12 because I multiplied 4 × 6 ÷ 2.' Is this a complete justification? What is missing?
Answer: ________

Match each question to its answer

1. Create a visual argument showing that the area of a parallelogram equals base × height by transforming it into a rectangle. →
2. A student writes: 'Proof: 4+6=10. 10 is even. Therefore the sum of two even numbers is always even.' What is wrong with this proof? →
3. How does mathematical OR differ from everyday 'or'? Consider: 'You can have cake or pie.' →
4. Which provides stronger inductive evidence for 'all multiples of 5 end in 0 or 5': testing 5 examples or 500 examples? →
5. Design a complete mathematical argument with claim, evidence, and reasoning for a mathematical fact of your choice. →

(A) Math OR allows both to be true; everyday 'or' often means 'one but not both' (exclusive or)
(B) Cut a right triangle from one end of the parallelogram and move it to the other end. The result is a rectangle with the same base and height, proving Area = base × height.
(C) Claim: The product of two consecutive integers is always even. Evidence: 1×2=2, 2×3=6, 4×5=20. Reasoning: Of two consecutive integers n and n+1, one must be even. An even number times anything is even. Therefore n(n+1) is always even.
(D) It only tests ONE specific example, not all even numbers. A direct proof must use variables (like 2a+2b=2(a+b)) to cover ALL cases.
(E) 500 examples — more evidence is stronger, though neither proves it with certainty

---

16. Logic

Circle the correct answer

151. How many counterexamples are needed to disprove a conjecture?
(A) At least ten (B) You can never fully disprove a conjecture (C) At least three (D) Just one

152. Given: (1) If it rains, the game is canceled. (2) If the game is canceled, the team practices indoors. (3) It is raining. What happens?
(A) Nothing happens (B) The team goes home (C) The game continues outdoors (D) The team practices indoors

153. In the argument 'n is even, so n = 2k for some integer k,' what role does 'for some integer k' play?
(A) k is always equal to n, and this has been consistently demonstrated in practice (B) It defines the variable k and ensures the expression 2k represents an integer, making the argument precise (C) It is unnecessary filler, and this has been

consistently demonstrated in practice (D) It makes the argument longer, as this is what research and standard practice suggests

154. Draw a number line argument: why is there no largest integer?
(A) Integers stop at 1 million, making this the most widely accepted explanation (B) The number line has an endpoint, and this has been consistently demonstrated in practice (C) Just show a very long line, and this has been consistently demonstrated in practice (D) For any integer n on the number line, n+1 is also an integer and lies to its right. Since you can always add 1, there is always a larger integer.

155. A student wants to prove '$2^n > n$ for all positive integers n.' Which method should they use and why?
(A) Proof by contradiction, and this principle applies across similar situations (B) Proof by exhaustion for all positive integers, making this the most widely accepted explanation (C) Mathematical induction — the statement involves all natural numbers and has a clear base case ($2^1>1$) and inductive step structure (doubling is stronger than adding 1) (D) Test 5 examples, which is why this approach is recommended by experts

156. What is the contrapositive of 'If it is snowing, then it is cold'?
(A) If it is cold, then it is snowing (B) If it is not snowing, then it is not cold (C) If it is not cold, then it is not snowing (D) It is snowing and not cold

Write the answer

157. The differences between consecutive terms of a sequence are 2, 4, 6, 8, 10. If the first term is 1, what is the 6th term?
Answer: ________

158. All mammals breathe air. A dog is a mammal. What can you conclude?
Answer: ________

159. What is a 'given' in a mathematical argument?
Answer: ________

160. If you connect the midpoints of any quadrilateral, what shape do you always get?
Answer: ________

Match each question to its answer

1. The Towers of Hanoi with n disks requires $2^n - 1$ moves. How many moves for 10 disks? →
2. Which argument form is valid? (A) If P then Q; P is true; therefore Q. (B) If P then Q; Q is true; therefore P. →
3. A mathematician discovers that $2^n - 1$ is prime for n = 2, 3, 5, 7. She conjectures this

holds for all prime n. Another mathematician finds that $2^{11} - 1 = 2047 = 23 \times 89$. Classify each step. →
4. Argue that a number divisible by both 3 and 5 must be divisible by 15. →
5. A Venn diagram shows Set A (even numbers) inside Set B (integers). What does this prove? →

(A) If 3|n, then n = 3a. If 5|n, then n = 5b. Since 3 and 5 share no common factors (coprime), n must be a multiple of 3×5 = 15.
(B) 1,023
(C) Discovery and conjecture = inductive. Counterexample = deductive (proves the conjecture false with certainty).
(D) All even numbers are integers (every element of A is in B)
(E) Only A (Modus Ponens)

Answer Key

1. B — 5 + 3 = 8
2. A — True
3. A — False
4. C — All triangles have 3 sides
5. C — It creates a paradox: if it is true, then it must be false; if it is false, then it must be true — so it has no definite truth value
6. D — It rains
7. True
8. No — having four legs does not mean it is a dog; many animals have four legs
9. If it is not cold, then it is not snowing
10. All cats are animals (true), but not all animals are cats — dogs, fish, and birds are animals too (converse is false)
    Matching (Logic): 1→D, 2→C, 3→A, 4→E, 5→B
11. C — 162
12. C — 39
13. A — Each number is the sum of the two before it
14. A — 31

15. C — The sum of the first n odd numbers equals n^2
16. A — 21
17. 25
18. 28
19. 44
20. 15 handshakes. Formula: $n(n-1)/2$. For 6: $6 \times 5/2 = 15$. Each new person shakes hands with all existing people.
    Matching (Patterns): 1→C, 2→B, 3→D, 4→A, 5→E
21. A — A dog breathes air
22. A — A square has four sides
23. B — 7 is odd
24. C — No — 'some' does not mean 'all,' so Maria might or might not like math
25. A — Triangle ABC has three equal sides — this follows necessarily from the two premises
26. C — Only A (Modus Ponens)
27. No — a rhombus has equal sides but is not a square. This affirms the consequent.
28. n is NOT divisible by 6 (Modus Tollens)
29. This is a valid syllogistic chain (transitive reasoning): $A \subset B$ and $B \subset C$, therefore $A \subset C$
30. The reasoning is actually invalid because 'most' does not guarantee the conclusion for any specific case. Plus the conclusion happens to be factually wrong.
    Matching (Deductive Reasoning): 1→B, 2→C, 3→E, 4→A, 5→D
31. D — The sum of an even and an odd number is always odd
32. B — Yes — $(2n+1)^2 = 4n^2+4n+1$, which is always odd
33. C — $6 \div 0.5 = 12$, which is larger than 6. Dividing by a number between 0 and 1 makes the result larger.
34. C — False. 9 does not end in those digits but $9 = 3 \times 3$, so it is not prime. Also $15 = 3 \times 5$, $21 = 3 \times 7$, etc.
35. B — Inductive reasoning can never prove a claim with certainty — it only provides

supporting evidence. Deductive reasoning from true premises guarantees the conclusion.

36. C — 500 examples — more evidence is stronger, though neither proves it with certainty
37. A conjecture is not proven until it holds for ALL cases — even 1000 successes cannot prevent failure on the 1001st case
38. A: Weak. B: Moderate. C: Strong (but still not proof).
39. B's test cases are cherry-picked (all happen to be prime). Testing 4, 6, 8, or 9 would immediately disprove B. A's test cases are representative.
40. Test small (2+4+6=12), medium (20+22+24=66), large (100+102+104=306), and negative (-4+-2+0=-6). All divisible by 6. Then prove algebraically: 2n+(2n+2)+(2n+4)=6n+6=6(n+1).
    Matching (Inductive Reasoning): 1→A, 2→C, 3→E, 4→D, 5→B
41. A — Subtraction property of equality: subtracting 6 from both sides
42. A — Math requires reasoning based on definitions, properties, or proven facts — not visual appearance or intuition alone
43. D — Definition-based: using the definition of 'even' (divisible by 2) and showing 12 meets it
44. D — It shows the calculation but not WHY the formula works. A complete justification would state: 'Area = ½ × base × height. Base = 4, height = 6, so A = ½ × 4 × 6 = 12.'
45. A — Only B — it uses algebraic manipulation to prove the claim. A is not a mathematical justification.
46. C — Even numbers are 2a and 2b. Their sum is 2a + 2b = 2(a+b), which has 2 as a factor, so it is even.
47. Multiplication means repeated addition: n × 0 means adding n zero times, which gives 0. Alternatively: n × 0 = n × (1-1) = n - n = 0.
48. If a÷0 = b, then b×0 = a. But b×0 always equals 0, never a (unless a=0, which gives infinitely many answers). Either no solution or infinitely many — neither works.
49. The pattern decreases by 3 each time (6,3,0,...), so 3×(-1) = -3. Each step subtracts 3 because we decrease the multiplier by 1, which removes one group of 3.
50. Pattern: -3×2=-6, -3×1=-3, -3×0=0, -3×(-1)=? Each step increases by 3, so -3×(-1)=0+3=3 (positive). Continuing: -3×(-2)=6.

Matching (Mathematical Arguments): 1→C, 2→A, 3→D, 4→B, 5→E

51. D — The distributive property: $a(b+c) = ab + ac$
52. A — $a \times b$ (the rectangle has dimensions a by b)
53. D — One $a\times a$ square, two $a\times b$ rectangles, and one $b\times b$ square
54. B — Four triangles (legs a,b) fit inside a $c\times c$ square, leaving a small square of side $(a-b)$. Area of large square $c^2 = 4(\frac{1}{2}ab) + (a-b)^2 = 2ab + a^2 - 2ab + b^2 = a^2 + b^2$.
55. B — Cut a right triangle from one end of the parallelogram and move it to the other end. The result is a rectangle with the same base and height, proving Area = base × height.
56. B — 5
57. The trapezoid's area calculated two ways (trapezoid formula vs sum of three triangles) must be equal, leading to $a^2+b^2=c^2$
58. The theorem depends on the 90° angle — in non-right triangles, $a^2+b^2\neq c^2$. The law of cosines generalizes it: $c^2=a^2+b^2-2ab\cdot\cos(C)$
59. The area of a square built on each side of the right triangle: 25 sq units = 9 sq units + 16 sq units
60. Different proofs reveal different insights about WHY the theorem works — algebraic, geometric, trigonometric, and even physical proofs each illuminate a different aspect of the relationship
Matching (Visual Proofs): 1→A, 2→D, 3→E, 4→B, 5→C
61. A — No — it is impossible because more than two landmasses have an odd number of bridges
62. C — The number of edges (connections) meeting at that vertex
63. A — When the graph has exactly 0 or 2 vertices with odd degree
64. C — Yes — start at B and end at D (or vice versa), since B and D are the two vertices with odd degree
65. A — Euler ignored sizes, shapes, and distances — keeping only connections between landmasses. This abstraction created graph theory, showing that the answer depends ONLY on the connection structure, not physical geography.
66. B — Any map can be colored using at most four colors so that no two adjacent regions share the same color
67. No — you need at least 3 colors because each country borders both others

68. It was the first major theorem proven using a computer to check thousands of cases — mathematicians debated whether computer-assisted proofs count as real proofs
69. The four-color theorem is equivalent to saying every planar graph can be properly vertex-colored with at most 4 colors
70. Draw 4 regions where each borders the other three: a central region surrounded by 3 regions that all touch each other and the center. This requires 4 colors since every pair is adjacent.
    Matching (Famous Problems): 1→D, 2→E, 3→C, 4→A, 5→B
71. C — Let $n = 2k+1$ (odd). Then $n^2 = (2k+1)^2 = 4k^2+4k+1 = 2(2k^2+2k)+1$, which has the form $2m+1$, so n^2 is odd.
72. D — Assume the hypothesis, apply logical steps, arrive at the conclusion
73. A — Let a/b and c/d be rational ($b,d \neq 0$). Sum $= a/b + c/d = (ad+bc)/(bd)$. Since $ad+bc$ and bd are integers ($bd \neq 0$), the sum is rational.
74. B — It only tests ONE specific example, not all even numbers. A direct proof must use variables (like $2a+2b=2(a+b)$) to cover ALL cases.
75. A — Let the integers be n and $n+1$. Case 1: n is even ($n=2k$), so $n(n+1)=2k(n+1)$, which is even. Case 2: n is odd, then $n+1$ is even ($n+1=2k$), so $n(n+1)=n(2k)=2nk$, which is even. In either case, the product is even.
76. A — You assume the statement you want to prove is FALSE, then show this leads to a logical impossibility
77. Assume $\sqrt{2} = p/q$ in lowest terms. Then $2 = p^2/q^2$, so $p^2 = 2q^2$. This means p^2 is even, so p is even ($p=2k$). Then $4k^2 = 2q^2$, so $q^2 = 2k^2$, meaning q is also even. But p and q both being even contradicts 'lowest terms.' Therefore $\sqrt{2}$ is irrational.
78. Assume N is the largest even number. Then $N+2$ is also even (even + even = even) and $N+2 > N$. This contradicts 'N is the largest.' Therefore no largest even number exists.
79. Direct proof assumes the hypothesis and derives the conclusion forward. Contradiction assumes the conclusion is false and derives an impossibility.
80. Reaching a true statement ($5=5$) is not a contradiction — it means the proof attempt failed. The student should check for algebra errors or try a different approach entirely.
    Matching (Proof Techniques): 1→A, 2→B, 3→E, 4→D, 5→C
81. D — A proof that starts from known facts or hypotheses and uses logical steps to

reach the conclusion

82. C — Assume P is true
83. B — It can be written as 2k for some integer k
84. A — It can be written as 2k + 1 for some integer k
85. A — There exists an integer k such that $b = a \times k$
86. C — $n = 3k$ for some integer k
87. If $a = 2m$ and $b = 2n$ (both even), then $a + b = 2m + 2n = 2(m + n)$, which has the form 2k, so it is even
88. No — checking a few examples is not a proof because it does not cover all possible cases
89. A conjecture is an unproven statement believed to be true; a theorem is a statement that has been rigorously proven
90. Algebraic definitions represent ALL possible cases at once, while examples only cover specific instances
    Matching (Mathematical Proofs): 1→E, 2→D, 3→A, 4→C, 5→B
91. B — Assume the opposite of what you want to prove, then show this assumption leads to a logical impossibility
92. A — Reductio ad absurdum (reduction to absurdity)
93. A — Conclude that your initial assumption (the negation) must be false, so the original statement is true
94. C — n is rational
95. A — There are only finitely many primes
96. C — A number that can be expressed as a/b where a and b are integers and $b \neq 0$
97. When the direct approach is difficult because you don't know how to start from the hypothesis, but the negation gives you something concrete to work with
98. By definition, an integer cannot be both even (2m) and odd (2m + 1) simultaneously, so this is a logical impossibility
99. Because N leaves remainder 1 when divided by any prime on the list, so none of them divide N — meaning N has a prime factor not on the list
100. Assume $\sqrt{2} = a/b$ in lowest terms. Squaring gives $2 = a^2/b^2$, so $a^2 = 2b^2$. This means a is even, say $a = 2c$. Then $4c^2 = 2b^2$, so $b^2 = 2c^2$, meaning b is even. But a

and b both even contradicts 'lowest terms.'
Matching (Mathematical Proofs): 1→A, 2→C, 3→B, 4→D, 5→E

101. D — A specific example that shows a universal statement is false
102. C — Just one
103. D — A statement that claims something is true for ALL members of a set, often using words like 'every,' 'all,' or 'for any'
104. C — A statement that claims there EXISTS at least one member of a set with a certain property
105. C — False — to disprove an existential statement, you must show that P(x) fails for ALL x, not just find one counterexample
106. B — There exists an even number greater than 2 that is NOT composite (i.e., that is prime)
107. 2 is prime (its only factors are 1 and 2) but 2 is even (not odd). Since the claim says ALL primes are odd, one even prime disproves it.
108. Testing examples helps you decide whether to try to prove or disprove a conjecture. If you find a counterexample, you're done. If many examples work, it's worth trying to prove.
109. They mean the same thing: to show that a statement is false, typically by providing a counterexample or a proof of the negation
110. No — it fails at $n = 40$: $40^2 + 40 + 41 = 1681 = 41^2$, which is not prime. Even 39 consecutive successes do not prove a universal statement.
Matching (Mathematical Proofs): 1→C, 2→A, 3→D, 4→E, 5→B
111. C — The base case (show the statement is true for the first value) and the inductive step (show that if it's true for $n = k$, then it's true for $n = k + 1$)
112. B — The assumption that the statement $P(k)$ is true for some particular (but arbitrary) integer k
113. C — Knocking over the first domino
114. D — The guarantee that each domino, when it falls, knocks over the next one
115. C — $1 + 2 + 3 + ... + n = n(n + 1)/2$
116. B — Without the base case, the inductive step has no starting point. You prove 'if true for k, then true for k+1' but never establish that it IS true for any specific value.

117. The base case shows it's true for $n = 1$. The inductive step shows that truth propagates: true for 1 → true for 2 → true for 3 → ... This chain reaches every positive integer, no matter how large.
118. Because we don't assume $P(k)$ is true unconditionally — we only prove that IF $P(k)$ is true, THEN $P(k+1)$ is true. The base case provides the actual starting truth.
119. In weak induction, you assume $P(k)$ to prove $P(k+1)$. In strong induction, you assume $P(1), P(2), ..., P(k)$ are ALL true to prove $P(k+1)$.
120. Left side: 1. Right side: $1(1+1)/2 = 1(2)/2 = 1$. Since both sides equal 1, the base case holds.
Matching (Mathematical Proofs): 1→C, 2→B, 3→A, 4→D, 5→E
121. C — (i) False, (ii) False, (iii) True
122. D — 44
123. D — Triangle ABC has three equal sides — this follows necessarily from the two premises
124. D — A pattern that holds for finitely many cases may fail for a specific value, like n^2-n+41 producing non-primes after $n=40$, or 'all numbers are less than 1 million' failing at 1,000,000
125. D — B is easier when the conclusion is hard to derive directly but its negation leads to an obvious absurdity — like proving $\sqrt{2}$ is irrational
126. A — $n^2 + (2n+1) = (n+1)^2$
127. $20 = 3 + 17$ (both prime)
128. Base case (prove it for $n=1$) and Inductive step (prove that IF it holds for $n=k$, THEN it holds for $n=k+1$)
129. False
130. All confirmed: $25÷5=5$, $40÷5=8$, $105÷5=21$, $350÷5=70$ — the conjecture holds
Matching (Logic): 1→A, 2→C, 3→B, 4→E, 5→D
131. D — All cats are animals (true), but not all animals are cats — dogs, fish, and birds are animals too (converse is false)
132. C — No — 47 ends in 7, which is not 0/2/4/6/8, so it is not divisible by 2, therefore not even
133. B — Deductive reasoning — applying a general rule to a specific case with a guaranteed conclusion

134. C — Pattern: $-3\times2=-6$, $-3\times1=-3$, $-3\times0=0$, $-3\times(-1)=?$ Each step increases by 3, so $-3\times(-1)=0+3=3$ (positive). Continuing: $-3\times(-2)=6$.

135. A — The area of a square built on each side of the right triangle: 25 sq units = 9 sq units + 16 sq units

136. A — It is easy to state and test for individual numbers, but proving it for ALL even numbers has resisted every proof technique for over 280 years

137. Let the integers be n and n+1. Case 1: n is even (n=2k), so n(n+1)=2k(n+1), which is even. Case 2: n is odd, then n+1 is even (n+1=2k), so n(n+1)=n(2k)=2nk, which is even. In either case, the product is even.

138. Both individual statements are true, and AND requires both to be true

139. It works for 0-3 (giving 41, 43, 47, 53 — all prime) but fails at n=40: $40^2 + 40 + 41 = 1681 = 41^2$ — not prime!

140. A gecko does not have fur
Matching (Logic): 1→A, 2→B, 3→C, 4→E, 5→D

141. B — 1 (not prime and not even)

142. B — The argument is valid AND the conclusion is true — squares are indeed a special type of rhombus

143. C — $6\div0.5=12$, which is larger than 6. Dividing by a number between 0 and 1 makes the result larger.

144. C — 1. Correctness: all steps are mathematically valid. 2. Completeness: no gaps — every step is justified. 3. Clarity: a reader can follow the logic without guessing. Excellent = all three; Poor = missing multiple criteria.

145. A — The four-color theorem is equivalent to saying every planar graph can be properly vertex-colored with at most 4 colors

146. A — Testing ALL possible cases to verify a statement — when the number of cases is finite and manageable

147. False

148. 21

149. A: Weak. B: Moderate. C: Strong (but still not proof).

150. It shows the calculation but not WHY the formula works. A complete justification would state: 'Area = ½ × base × height. Base = 4, height = 6, so A = ½ × 4 × 6 = 12.'
Matching (Logic): 1→B, 2→D, 3→A, 4→E, 5→C

151. D — Just one
152. D — The team practices indoors
153. B — It defines the variable k and ensures the expression 2k represents an integer, making the argument precise
154. D — For any integer n on the number line, n+1 is also an integer and lies to its right. Since you can always add 1, there is always a larger integer.
155. C — Mathematical induction — the statement involves all natural numbers and has a clear base case ($2^1>1$) and inductive step structure (doubling is stronger than adding 1)
156. C — If it is not cold, then it is not snowing
157. 31
158. A dog breathes air
159. Information stated as true at the start — the premises or assumptions you work from
160. A parallelogram (Varignon's theorem)
    Matching (Logic): 1→B, 2→E, 3→C, 4→A, 5→D
