Discretequest — Activity Book

A Spark & Anvil activity book — puzzles, questions, and fun from Discretequest.

Write in the book, circle your answers, and check the Answer Key at the back.

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1. Sets and Sorting

Circle the correct answer

1. What is a set in mathematics?
(A) A set is a well-defined collection of distinct objects or elements. (B) A set is a type of equation used to solve word problems. (C) A set is a group of objects that always contains exactly ten items. (D) A set is a list of numbers that must be in order from least to greatest.

2. If Set A = {1, 3, 5, 7} and Set B = {2, 4, 6, 8}, what do we call these two sets that share no elements?
(A) Sets that share no elements are called equal sets. (B) Sets that share no elements are called subset pairs. (C) Sets that share no elements are called disjoint sets. (D) Sets that share no elements are called empty sets.

3. How do we write the set of all even numbers between 1 and 11 using set notation with curly braces?
(A) The set of even numbers between 1 and 11 is written as {2, 4, 6, 8, 10}. (B) The set is written as (2, 4, 6, 8, 10) using parentheses. (C) The set is written as {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}. (D) The set is written as {2, 4, 6, 8, 10, 12}.

4. What is the symbol ∈ used for in set notation?
(A) The symbol ∈ means 'is greater than' all elements in a set. (B) The symbol ∈ means 'is equal to' another set. (C) The symbol ∈ means 'is empty' and contains no elements. (D) The symbol ∈ means 'is an element of' or 'belongs to' a set.

5. What is the empty set, and how is it written?
(A) The empty set is a set that contains infinite elements, written as {∞}. (B) The empty set is a set with no elements, written as { } or ∅. (C) The empty set is a set missing its curly braces, written as ( ). (D) The empty set is a set containing only the number zero, written as {0}.

6. If Set A = {apple, banana, cherry} and Set B = {banana, date, elderberry}, what is the union of A and B?
(A) The union of A and B is {apple, cherry, date, elderberry} without the shared

element. (B) The union of A and B is {apple, banana, cherry, date, elderberry}. (C) The union of A and B is {apple, banana, cherry, banana, date, elderberry} with banana listed twice. (D) The union of A and B is {banana} because it's the only shared element.

Write the answer

7. If Set A = {1, 2, 3, 4, 5} and Set B = {3, 4, 5, 6, 7}, what is the intersection of A and B?
Answer: _____

8. In a Venn diagram with two overlapping circles, where would you place an item that belongs to both Set A and Set B?
Answer: _____

9. What does it mean for Set A to be a subset of Set B?
Answer: _____

10. Why does the set {3, 1, 2} equal the set {1, 2, 3}?
Answer: _____

Match each question to its answer

1. Your class has 20 students who like pizza and 15 students who like tacos. 8 students like both. Use set operations to find how many students like pizza OR tacos. →
2. You have the set of colors {red, blue, green, yellow, purple}. Create a subset of primary colors from this set. →
3. You have 5 books to arrange on a shelf. If you sort them by title alphabetically, how many comparisons does it take using bubble sort in the worst case? →
4. Sort the list [5, 2, 8, 1, 9] using selection sort. What does the list look like after the first pass? →
5. In a card game, you pick up cards one at a time and slide each into its correct position among the cards already in your hand. Which sorting algorithm does this resemble? →

(A) The subset of primary colors is {red, blue, yellow}.
(B) Bubble sort needs at most 10 comparisons to sort 5 items, calculated as 4 + 3 + 2 + 1 = 10.
(C) There are 27 students who like pizza or tacos, calculated as 20 + 15 - 8 = 27.
(D) After the first pass, the list is [1, 2, 8, 5, 9] because 1 (the smallest) swaps with 5 (the first element).
(E) This resembles insertion sort, where each new element is inserted into its correct position in the sorted portion.

2. Counting and Patterns

Circle the correct answer

11. If you have 3 shirts and 4 pairs of pants, how many different outfits can you make?
(A) You can make 4 different outfits because you're limited by the pants. (B) You can make 12 different outfits, calculated as 3 × 4 = 12. (C) You can make 7 different outfits, calculated as 3 + 4. (D) You can make 24 different outfits, calculated as 3 × 4 × 2.

12. What does the word 'permutation' mean in mathematics?
(A) A permutation is a group of objects where order does not matter. (B) A permutation is the total count of all objects in a collection. (C) A permutation is a way of dividing objects into equal groups. (D) A permutation is an arrangement of objects where the order matters.

13. What does 5! (five factorial) equal?
(A) Five factorial equals 120, calculated as 5 × 4 × 3 × 2 × 1 = 120. (B) Five factorial equals 25, calculated as 5 × 5. (C) Five factorial equals 5, because the exclamation mark just means 'definitely 5.' (D) Five factorial equals 15, calculated as 5 + 4 + 3 + 2 + 1.

14. What is 0! (zero factorial)?
(A) Zero factorial is undefined because you can't have zero of something. (B) Zero factorial equals 1 by mathematical convention. (C) Zero factorial equals negative 1 because it's below 1 factorial. (D) Zero factorial equals 0 because there's nothing to multiply.

15. What is the difference between a permutation and a combination?
(A) There is no difference — permutation and combination mean the same thing. (B) Permutations are for numbers only, while combinations work with any objects. (C) Permutations always involve more objects than combinations. (D) In a permutation the order of selection matters, while in a combination the order does not matter.

16. Why does a 3-digit lock with digits 0-9 have 1,000 possible codes instead of 30?
(A) There are only 720 codes because you can't repeat digits on a real lock. (B) Each of the 3 positions can independently be any of 10 digits, so there are 10 × 10 × 10 = 1,000 codes. (C) There are 1,000 codes because 3 × 10 = 30 and then we add 970 more for repeats. (D) There are 1,000 codes because 10! ÷ 7! = 720, which rounds to 1,000.

Write the answer

17. Why is 4! (24) the number of ways to arrange 4 different books on a shelf?
Answer: _____

18. What pattern do you see in this sequence: 1, 1, 2, 3, 5, 8, 13, ...? What are the next two numbers?
Answer: _____

19. A pizza shop offers 3 sizes, 2 crust types, and 5 toppings. If you pick one of each, how many different single-topping pizzas are possible?
Answer: _____

20. How many 3-letter 'words' (real or nonsense) can you make from the letters A, B, C, D, E if no letter can be repeated?
Answer: _____

Match each question to its answer

1. In how many ways can 6 runners finish a race in first, second, and third place? →
2. You need a 4-digit PIN using digits 0-9. How many PINs are possible if repetition is allowed? →
3. Find the pattern and fill in the blank: 1, 4, 9, 16, 25, **, 49. → __**
4. A school lunch offers 4 main dishes, 3 side dishes, and 2 drinks. The school wants to know how many different meal combinations exist. If they add one more drink option, how does the total change? →
5. Look at this pattern of triangular numbers: 1, 3, 6, 10, 15, 21, 28. What is the rule for finding the next triangular number, and why are they called 'triangular'? →

(A) Each triangular number adds one more than the previous gap (add 2, add 3, add 4, etc.), and they're called triangular because they represent dots arranged in equilateral triangles.
(B) Originally there are 4 × 3 × 2 = 24 meals. Adding one drink makes it 4 × 3 × 3 = 36 meals — an increase of 12.
(C) The missing number is 36, because these are perfect squares: 1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2.
(D) There are 10,000 possible PINs, calculated as 10 × 10 × 10 × 10 = 10,000.
(E) There are 120 ways, calculated as 6 × 5 × 4 = 120.

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3. Logic and Truth Tables

Circle the correct answer

21. In logic, what does the AND operation mean?
(A) The AND operation flips a true statement to false. (B) The AND operation is always true regardless of the statements. (C) The AND operation is true only when both statements connected by it are true. (D) The AND operation is true when at least one of the statements is true.

22. In logic, what does the OR operation mean?
(A) The OR operation is true only when both statements are true. (B) The OR operation is true when at least one of the statements connected by it is true. (C) The OR operation reverses the truth value of a statement. (D) The OR operation is true only when exactly one statement is true but not both.

23. What does the NOT operation do to a logical statement?
(A) The NOT operation makes any statement automatically false. (B) The NOT operation reverses the truth value — it turns true into false and false into true. (C) The NOT operation keeps the same truth value but changes the wording. (D) The NOT operation combines two statements together.

24. What is a truth table?
(A) A truth table is a multiplication table for Boolean numbers. (B) A truth table is a list of facts that have been proven to be true. (C) A truth table is a database of correct answers to quiz questions. (D) A truth table is a chart that shows every possible combination of true/false inputs and the resulting output for a logical expression.

25. What is a conditional statement in logic?
(A) A conditional statement is an 'if-then' statement, written as 'if P, then Q' where P is the hypothesis and Q is the conclusion. (B) A conditional statement is a question that requires a yes or no answer. (C) A conditional statement is one that is always true under all conditions. (D) A conditional statement is any statement that contains the word 'and.'

26. Why is the statement 'If it's Tuesday, then I have math class' different from 'If I have math class, then it's Tuesday'?
(A) The second statement is the negation of the first. (B) They're different only if you don't like math class. (C) They mean the same thing because both connect Tuesday and math class. (D) They're different because the second is the converse of the first, and a statement and its converse can have different truth values.

Write the answer

27. Complete the truth table for P AND Q. When P = True and Q = False, what is the result?
Answer: _____

28. How does exclusive OR (XOR) differ from regular OR?
Answer: _____

29. What does it mean when two logical expressions are 'logically equivalent'?
Answer: _____

30. In a video game, a treasure chest opens when the player 'hasKey AND (level >= 5 OR isPremium).' If a player has a key and is level 3 but is a premium member, does the chest open?
Answer: _____

Match each question to its answer

1. Build a truth table for NOT (P OR Q). How many rows does it have, and when is the result true? →
2. A school rule says: 'Students can use the computer lab if they have finished homework AND it is not lunchtime.' Express this as a logical expression and evaluate it when homework is done and it IS lunchtime. →
3. Use a truth table to verify whether 'P AND (P OR Q)' always equals P. →
4. Your friend argues: 'All dogs are animals. My cat is an animal. Therefore, my cat is a dog.' What logical error is your friend making? →
5. Compare the truth tables for P OR Q and P AND Q. In how many of the 4 input combinations do they give different results? →

(A) They give different results in 2 of the 4 combinations: when one input is true and the other is false.
(B) The expression is 'finishedHomework AND NOT lunchtime.' When homework is done (true) and it is lunchtime (true), NOT lunchtime is false, so the result is false — the student cannot use the lab.
(C) Your friend is committing the fallacy of affirming the consequent — just because all dogs are animals doesn't mean all animals are dogs.
(D) Yes, P AND (P OR Q) always equals P. When P is true, the expression is true; when P is false, the expression is false — matching P exactly.
(E) The truth table has 4 rows (for TT, TF, FT, FF), and NOT (P OR Q) is true only when both P and Q are false.

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4. Graph Theory Basics

Circle the correct answer

31. In graph theory, what is a vertex (also called a node)?
(A) A vertex is the path from one end of a graph to the other. (B) A vertex is the total number of connections in a graph. (C) A vertex is a point in a graph that represents an object, person, or location. (D) A vertex is a line connecting two points in a graph.

32. What is an edge in graph theory?
(A) An edge is a connection between two vertices in a graph, representing a relationship or link. (B) An edge is the shortest path between any two vertices. (C) An edge is a vertex with only one connection. (D) An edge is the outer boundary of a graph.

33. What is the degree of a vertex in a graph?
(A) The degree is the distance from the vertex to the center of the graph. (B) The degree of a vertex is the number of edges connected to it. (C) The degree is the angle formed between two edges at a vertex. (D) The degree is the total number of vertices in the entire graph.

34. What is the difference between a directed graph and an undirected graph?
(A) A directed graph must be drawn left to right, while undirected can face any direction. (B) In a directed graph, edges have a direction (like one-way streets), while in an undirected graph, edges go both ways. (C) A directed graph has vertices and an undirected graph has only edges. (D) There is no difference — all graphs are the same.

35. What is a path in graph theory?
(A) A path is any edge in the graph. (B) A path is a circle of vertices that returns to the starting point. (C) A path must visit every vertex in the graph. (D) A path is a sequence of vertices connected by edges where no vertex is visited more than once.

36. Why can a graph with 5 vertices and 11 edges not exist if the graph is simple (no self-loops or parallel edges)?
(A) A simple graph with 5 vertices can have at most 10 edges, calculated as $5 \times 4 / 2 = 10$, so 11 edges is impossible. (B) It can't exist because 11 is an odd number and graphs need even edges. (C) It can exist because you can always add more edges between vertices. (D) It can exist if some edges connect three vertices at once.

Write the answer

37. What does it mean for a graph to be 'connected'?
Answer: _____

38. What is a cycle in a graph, and how is it different from a path?
Answer: _____

39. Why is a tree (in graph theory) always a connected graph with no cycles?
Answer: _____

40. What is a weighted graph, and when would you use one?
Answer: _____

Match each question to its answer

1. Draw a graph representing the friendships in a group: Amy is friends with Ben and Cal. Ben is friends with Amy and Dana. Cal is friends with Amy only. Dana is friends with Ben only. How many vertices and edges does this graph have? →
2. A graph has vertices A, B, C, D, E with edges A-B, B-C, C-D, D-E, and E-A. What is the shortest path from A to C? →
3. In a weighted graph of cities, the edges show distances in miles: A-B = 10, B-C = 15, A-C = 30, B-D = 20, C-D = 5. What is the shortest path from A to D? →
4. The Handshaking Lemma says the sum of all vertex degrees in a graph equals twice the number of edges. Use this to determine if a graph can exist with 4 vertices having degrees 3, 2, 2, and 1. →
5. Social media platforms show you 'People You May Know' suggestions. How might a graph algorithm find these suggestions by looking at friend-of-friend connections? →

(A) The graph has 4 vertices (Amy, Ben, Cal, Dana) and 3 edges (Amy-Ben, Amy-Cal, Ben-Dana).
(B) The shortest path from A to C is A → B → C, which has length 2 (two edges).
(C) The shortest path from A to D is A → B → C → D with a total distance of 30 miles (10 + 15 + 5).
(D) The algorithm looks for people who share many mutual friends with you (vertices two edges away) but aren't directly connected to you yet.
(E) No, this graph cannot exist because the sum of degrees is 3 + 2 + 2 + 1 = 8, and 8/2 = 4 edges, but a vertex of degree 3 must connect to 3 others, and the degree-1 vertex can only connect to one — the constraints conflict.

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5. Combinatorics and Probability

Circle the correct answer

41. What is the difference between a permutation and a combination?
(A) There is no difference; they are the same thing. (B) In permutations order matters, but in combinations order does not matter. (C) Permutations are always larger numbers than combinations. (D) Combinations involve repetition while permutations do not.

42. How many ways can you arrange 4 different books on a shelf?
(A) There are 24 ways to arrange 4 books, since 4! = 4 x 3 x 2 x 1 = 24. (B) There are 16 ways because 4 x 4 = 16. (C) There are 10 ways because 4 + 3 + 2 + 1 = 10. (D) There are 12 ways because 4 x 3 = 12.

43. You have 5 friends and want to invite exactly 2 to a sleepover. How many different pairs can you choose?
(A) You can choose 25 pairs because 5 x 5 = 25. (B) You can choose 5 pairs because you pick 2 from 5. (C) You can choose 20 pairs because 5 x 4 = 20. (D) You can choose 10 different pairs, calculated as 5! / (2! x 3!) = 10.

44. What is the probability of rolling a sum of 7 with two standard dice?
(A) The probability is 6/36 or 1/6, because there are 6 combinations that sum to 7 out of 36 total outcomes. (B) The probability is 7/36 because the sum is 7. (C) The probability is 1/12 because 7 is one of 12 possible sums. (D) The probability is 1/7 because we want a sum of 7.

45. A bag has 3 red marbles, 4 blue marbles, and 5 green marbles. If you draw one marble without looking, what is the probability of drawing a blue marble?
(A) The probability is 4/5 because blue has 4 and green has 5. (B) The probability is 1/4 because there are 4 blue marbles. (C) The probability is 4/12 or 1/3, because there are 4 blue marbles out of 12 total. (D) The probability is 1/12 because there are 12 marbles total.

46. What does it mean for two events to be independent in probability?
(A) Independent events must involve different objects. (B) Two events are independent when the outcome of one event does not affect the probability of the other event. (C) Independent events always have the same probability. (D) Independent events are events that cannot happen at the same time.

Write the answer

47. You flip a fair coin 3 times. What is the probability of getting exactly 2 heads?
Answer: _____

48. A pizza shop offers 3 sizes, 4 crust types, and 6 toppings. If you choose one of each, how many different pizza combinations are possible?
Answer: _____

49. What is the complement of an event in probability?
Answer: _____

50. How many 3-letter codes can be made from the letters A, B, C, D, E if no letter can be repeated?
Answer: _____

Match each question to its answer

1. A class has 10 students. How many ways can you choose a president, vice president, and secretary? →
2. You draw two cards from a standard 52-card deck without replacement. What is the probability that both are hearts? →
3. What is Pascal's Triangle, and how does it relate to combinations? →
4. In a game, you spin a spinner divided into 5 equal sections (red, blue, green, yellow, purple) and flip a coin. What is the probability of landing on blue AND getting heads? →
5. Why is P(A or B) = P(A) + P(B) - P(A and B) instead of just P(A) + P(B)? →

(A) Pascal's Triangle is a triangular array where each number is the sum of the two numbers above it, and the entries represent combination values C(n,r).
(B) The probability is (13/52) x (12/51) = 156/2652 = 1/17.
(C) The probability is 1/10 because (1/5) x (1/2) = 1/10.
(D) We subtract P(A and B) to avoid counting the overlap where both events happen, which would otherwise be counted twice.
(E) There are 720 ways because 10 x 9 x 8 = 720.

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6. Recursion and Sequences

Circle the correct answer

51. What does it mean for something to be defined recursively?
(A) Recursive means the definition uses only multiplication. (B) A recursive definition lists every possible value one by one. (C) A recursive definition is one that goes backwards from the answer. (D) A recursive definition defines something in terms of itself, with a base case that stops the self-reference.

52. What are the base case and recursive case for the factorial function n!?
(A) The base case is 1! = 0, and the recursive case is n! = n + (n-1)!. (B) The base case is 0! = 1, and the recursive case is n! = n x (n-1)! for n > 0. (C) There is no base case; factorial just keeps multiplying forever. (D) The base case is n! = n, and the recursive case is n! = n x n.

53. The Fibonacci sequence starts 1, 1, 2, 3, 5, 8, 13, ... What are the next two numbers?
(A) The next two numbers are 18 and 23. (B) The next two numbers are 20 and 33. (C) The next two numbers are 21 and 34, because each number is the sum of the previous two. (D) The next two numbers are 14 and 15.

54. Write a recursive rule for the sequence: 2, 6, 18, 54, 162, ...
(A) The rule is a(n) = a(n-1) + 4 because the differences increase by 4. (B) The recursive rule is a(1) = 2 and a(n) = 3 x a(n-1), because each term is 3 times the previous term. (C) The rule is a(n) = a(n-1) x a(n-2) because you multiply the last two terms. (D) The rule is a(n) = 2^n because the sequence doubles each time.

55. What happens if a recursive definition has no base case?
(A) The recursion automatically stops after 100 steps. (B) Without a base case, the answer is always zero. (C) Without a base case, the recursion never stops — it creates an infinite loop that keeps calling itself forever. (D) It works fine; the base case is optional.

56. The Tower of Hanoi puzzle with n disks requires a minimum of 2^n - 1 moves. How many moves does a 6-disk puzzle require?
(A) It requires 64 moves because 2^6 = 64. (B) A 6-disk Tower of Hanoi requires 63 moves, because 2^6 - 1 = 64 - 1 = 63. (C) It requires 12 moves because 6 x 2 = 12. (D) It requires 36 moves because 6 x 6 = 36.

Write the answer

57. Trace through the recursive calculation: fib(5), where fib(1) = 1, fib(2) = 1, and fib(n) = fib(n-1) + fib(n-2).
Answer: _____

58. What is the closed-form (explicit) formula for the arithmetic sequence 3, 7, 11, 15, 19, ...?
Answer: _____

59. Why is a recursive definition sometimes preferred over a closed-form formula?
Answer: _____

60. What is the sum of the first n natural numbers? (1 + 2 + 3 + ... + n)
Answer: _____

Match each question to its answer

1. Define the sequence T(n) recursively: T(1) = 3, T(n) = 2 x T(n-1) + 1. Find T(4). →
2. The Sierpinski Triangle starts with one triangle and recursively divides it. After 4 iterations, how many small triangles are shaded? →
3. What is mathematical induction, and why is it useful? →
4. Use the domino analogy to explain why both parts of mathematical induction are necessary. →
5. A recursive sequence is defined as: a(1) = 1, a(2) = 1, a(n) = a(n-1) + 2 x a(n-2). Find a(5). →

(A) T(4) = 31, found by computing T(2) = 7, T(3) = 15, T(4) = 2 x 15 + 1 = 31.
(B) The base case is like pushing the first domino. The inductive step ensures each domino knocks down the next. Without either part, the chain fails.
(C) Mathematical induction is a proof technique that proves a statement is true for all natural numbers by showing it holds for a base case and that if it holds for n, it also holds for n+1.
(D) a(5) = 11. Working out: a(3) = 1 + 2(1) = 3, a(4) = 3 + 2(1) = 5, a(5) = 5 + 2(3) = 11.
(E) After 4 iterations, there are 3^4 = 81 shaded triangles, because each iteration triples the number of shaded triangles.

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7. Number Theory and Cryptography

Circle the correct answer

61. What is modular arithmetic, and what does '17 mod 5' equal?
(A) Modular arithmetic is the math of remainders after division. 17 mod 5 = 2, because 17 divided by 5 is 3 remainder 2. (B) 17 mod 5 = 12 because 17 - 5 = 12. (C) 17 mod 5 = 3 because 17/5 = 3.4 and you round down. (D) Modular arithmetic means dividing numbers into modules of equal size.

62. What is the Fundamental Theorem of Arithmetic?
(A) Every number can be divided evenly by at least one prime. (B) Every number is either prime or composite, with no exceptions. (C) The sum of any two prime numbers is always even. (D) Every integer greater than 1 can be expressed as a unique product of prime numbers, regardless of the order of factors.

63. Find the greatest common divisor (GCD) of 48 and 36 using the Euclidean algorithm.
(A) GCD(48, 36) = 4 because 48/12 = 4, which is what you get when you apply the distributive property here (B) GCD(48, 36) = 36 because 36 divides into 48, since this is how the operation works with these particular numbers (C) GCD(48, 36) = 12. Steps: 48 = 1 x 36 + 12, then 36 = 3 x 12 + 0. When the remainder is 0, the last non-zero remainder (12) is the GCD. (D) GCD(48, 36) = 6 because both are divisible by 6, based on the standard mathematical approach for this type of problem

64. What is a Caesar cipher, and how would you encrypt the word 'MATH' with a shift of 3?
(A) MATH becomes HTAM because Caesar cipher reverses the word. (B) MATH becomes OCVJ because each letter shifts by its position. (C) A Caesar cipher shifts each letter by a fixed number. With shift 3, MATH becomes PDWK (M→P, A→D, T→W, H→K). (D) MATH becomes NBUS because you shift by 1 and add 2.

65. Why is the Caesar cipher considered weak by modern standards?
(A) It is weak because Julius Caesar's enemies already knew about it. (B) It is weak because shifted letters look different from normal letters. (C) It is weak because there are only 25 possible shifts, so an attacker can try all of them quickly (brute force attack). (D) It is weak because it only works with the English alphabet.

66. What is the difference between symmetric and asymmetric encryption?
(A) Symmetric encryption uses the same key to encrypt and decrypt, while asymmetric encryption uses a public key to encrypt and a different private key to decrypt. (B) Symmetric encryption is always faster but asymmetric is always more secure, as this is what research and standard practice suggests (C) Symmetric uses numbers and asymmetric uses letters, and this is the standard approach used in most cases (D) Symmetric is for text and asymmetric is for images, and this principle applies across similar situations

Write the answer

67. Why are prime numbers important for modern internet security?
Answer: _____

68. What is 23 mod 7? What is 100 mod 12?
Answer: _____

69. Two numbers are 'congruent modulo n' if they have the same remainder when divided by n. Are 38 and 14 congruent modulo 8?
Answer: _____

70. What is the Sieve of Eratosthenes, and how does it find prime numbers?
Answer: _____

Match each question to its answer

1. Using the Sieve of Eratosthenes, find all prime numbers between 1 and 30. →
2. What are twin primes? Give three examples of twin prime pairs. →
3. Decrypt the message 'KHOOR' using a Caesar cipher with shift 3. →
4. What is a one-way function, and why is it important for cryptography? →
5. What is Euler's totient function φ(n), and what is φ(12)? →

(A) A one-way function is easy to compute in one direction but extremely difficult to reverse. It's the basis of public-key cryptography — anyone can encrypt, but only the key holder can decrypt.
(B) Twin primes are pairs of primes that differ by exactly 2. Examples: (3, 5), (11, 13), and (17, 19).
(C) φ(n) counts the numbers from 1 to n that are coprime to n (share no common

factors other than 1). $\varphi(12) = 4$, because 1, 5, 7, and 11 are coprime to 12.

(D) The primes between 1 and 30 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 — there are 10 primes.

(E) KHOOR decrypts to HELLO by shifting each letter back by 3: K→H, H→E, O→L, O→L, R→O.

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8. Algorithm Design and Complexity

Circle the correct answer

71. What is an algorithm?

(A) An algorithm is any list of numbers arranged in order. (B) An algorithm is a mathematical equation that solves for x. (C) An algorithm is a step-by-step set of instructions for solving a problem or completing a task, where each step is clear and unambiguous. (D) An algorithm is a type of computer program written in Python or Java.

72. What does Big O notation measure?

(A) Big O rates how difficult an algorithm is to understand. (B) Big O measures how much memory a program uses in megabytes. (C) Big O notation describes how an algorithm's running time or space grows as the input size increases, focusing on the worst-case scenario. (D) Big O counts the exact number of steps an algorithm takes.

73. Rank these Big O complexities from fastest to slowest: $O(n^2)$, $O(1)$, $O(n \log n)$, $O(n)$, $O(2^n)$.

(A) $O(1)$, $O(n)$, $O(n^2)$, $O(n \log n)$, $O(2^n)$. (B) $O(1)$, $O(n \log n)$, $O(n)$, $O(n^2)$, $O(2^n)$. (C) From fastest to slowest: $O(1)$, $O(n)$, $O(n \log n)$, $O(n^2)$, $O(2^n)$. (D) $O(2^n)$, $O(n^2)$, $O(n \log n)$, $O(n)$, $O(1)$.

74. How does binary search work, and what is its time complexity?

(A) Binary search checks every element one by one in $O(n)$ time. (B) Binary search works on a sorted list by repeatedly checking the middle element and eliminating half the remaining elements. Its time complexity is $O(\log n)$. (C) Binary search only works on lists of binary numbers (0s and 1s). (D) Binary search splits the list into two equal halves and searches both in $O(n)$ time.

75. A linear search checks 1,000,000 items. A binary search checks the same sorted list. Approximately how many checks does each need in the worst case?

(A) Linear search needs 1,000,000 and binary search needs 1,000 (square root). (B) Linear search needs 1,000,000 and binary search needs 500,000 (half). (C) Linear search needs up to 1,000,000 checks. Binary search needs only about 20 checks, because $\log_2(1{,}000{,}000) \approx 20$. (D) Both need about the same number of checks for

large lists.

76. Describe how bubble sort works. What is its time complexity?
(A) Bubble sort repeatedly compares adjacent elements and swaps them if they're in the wrong order, making multiple passes until no swaps are needed. Its time complexity is $O(n^2)$. (B) Bubble sort divides the list in half and sorts each half in $O(n \log n)$ time. (C) Bubble sort randomly shuffles elements until they're sorted in $O(n \log n)$ time. (D) Bubble sort finds the smallest element and moves it to the front in $O(n)$ time.

Write the answer

77. What is the divide and conquer strategy in algorithm design?
Answer: _____

78. Why is merge sort ($O(n \log n)$) faster than bubble sort ($O(n^2)$) for large lists?
Answer: _____

79. What is a greedy algorithm? Give an example.
Answer: _____

80. You need to sort 5 cards: [8, 3, 1, 5, 2]. Show the steps using selection sort.
Answer: _____

Match each question to its answer

1. What is the difference between best case, worst case, and average case complexity? →
2. What is the Traveling Salesman Problem (TSP), and why is it computationally difficult? →
3. What is memoization, and how does it improve the recursive Fibonacci algorithm? →
4. An algorithm takes 2 seconds for 100 items. If it's $O(n^2)$, approximately how long will it take for 1000 items? →
5. What is the difference between a brute force approach and an optimized algorithm for finding if a number n is prime? →

(A) It will take approximately 200 seconds, because when n increases 10x with $O(n^2)$, the time increases 100x ($10^2 = 100$).
(B) TSP asks for the shortest route visiting every city exactly once and returning home. It's difficult because the number of possible routes grows factorially — checking all routes for 20 cities would take millions of years.
(C) Best case is the fastest an algorithm can run (most favorable input), worst case is the slowest (least favorable input), and average case is the typical performance across

all possible inputs.

(D) Brute force checks all numbers from 2 to n-1 (O(n) checks). An optimized approach only checks up to the square root of n (O(√n) checks), because if n has a factor larger than √n, it must also have one smaller than √n.

(E) Memoization stores previously computed results to avoid redundant calculations. It improves recursive Fibonacci from O(2^n) to O(n) by caching each Fibonacci value after computing it once.

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9. Elements & Principles Deep Dive

Circle the correct answer

81. What is a set in mathematics?

(A) A set is a type of equation used to solve word problems. (B) A set is a list of numbers that must be in order from least to greatest. (C) A set is a well-defined collection of distinct objects or elements. (D) A set is a group of objects that always contains exactly ten items.

82. In a class survey, Set M = students who like math, Set S = students who like science, and Set A = students who like art. If 5 students like all three subjects, where would they appear in a three-circle Venn diagram?

(A) They appear only in the overlap between math and science, the two academic subjects. (B) Students who like all three subjects appear in the center region where all three circles overlap. (C) They appear outside all three circles since they can't be in three places at once. (D) They appear in three different places, once in each circle's exclusive region.

83. A library sorts books by author last name. New books arrive in random order. Which sorting strategy would be most practical: re-sorting all books from scratch each time, or inserting each new book into its correct position?

(A) Re-sorting from scratch is better because it guarantees perfect alphabetical order. (B) Neither approach matters because libraries use computers to track books, not physical order. (C) Inserting each new book into its correct position (insertion approach) is more practical because most books are already sorted. (D) Re-sorting is faster because comparing everything at once is quicker than one at a time.

84. If Set A = {1, 3, 5} and Set B = {2, 3, 4}, what is A ∩ B (the intersection)?

(A) {1, 2, 3, 4, 5} (B) {1, 5} (C) {2, 4} (D) {3}

85. What does the word 'permutation' mean in mathematics?
(A) A permutation is an arrangement of objects where the order matters. (B) A permutation is the total count of all objects in a collection. (C) A permutation is a group of objects where order does not matter. (D) A permutation is a way of dividing objects into equal groups.

86. Your teacher says the Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, ...) appears everywhere in nature. A classmate says this is just a coincidence. Who do you think makes a stronger argument, and what evidence supports your view?
(A) The classmate is correct because math is invented by humans and has nothing to do with nature. (B) The teacher makes a stronger argument because Fibonacci numbers appear in sunflower seed spirals, pinecone scales, flower petals, and shell curves, suggesting a deep connection to natural growth patterns. (C) Neither is correct because the Fibonacci sequence only works with numbers, not physical objects. (D) The teacher is wrong because only the golden ratio appears in nature, not Fibonacci numbers.

Write the answer

87. If a password must be 2 digits long and each digit is 0-9, how many possible passwords are there?
Answer: _____

88. In logic, what does the AND operation mean?
Answer: _____

89. Your friend claims: 'If NOT (P AND Q) is true, then both P and Q must be false.' Build a truth table to evaluate this claim.
Answer: _____

90. What is a conditional statement in logic?
Answer: _____

Match each question to its answer

1. If P is true and Q is false, what is the truth value of P AND Q? →
2. Draw a graph representing the friendships in a group: Amy is friends with Ben and Cal. Ben is friends with Amy and Dana. Cal is friends with Amy only. Dana is friends with Ben only. How many vertices and edges does this graph have? →
3. In a weighted graph of cities, the edges show distances in miles: A-B = 10, B-C = 15, A-C = 30, B-D = 20, C-D = 5. What is the shortest path from A to D? →
4. The Handshaking Lemma says the sum of all vertex degrees in a graph equals twice the number of edges. Use this to determine if a graph can exist with 4 vertices having degrees 3, 2, 2, and 1. →

5. Why is a tree (in graph theory) always a connected graph with no cycles? →

(A) The shortest path from A to D is A → B → C → D with a total distance of 30 miles (10 + 15 + 5).
(B) A tree connects all vertices using the minimum number of edges (n-1 edges for n vertices) without forming any loops.
(C) The graph has 4 vertices (Amy, Ben, Cal, Dana) and 3 edges (Amy-Ben, Amy-Cal, Ben-Dana).
(D) No, this graph cannot exist because the sum of degrees is 3 + 2 + 2 + 1 = 8, and 8/2 = 4 edges, but a vertex of degree 3 must connect to 3 others, and the degree-1 vertex can only connect to one — the constraints conflict.
(E) False

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10. Cultural & Historical Context

Circle the correct answer

91. How do we write the set of all even numbers between 1 and 11 using set notation with curly braces?
(A) The set is written as (2, 4, 6, 8, 10) using parentheses. (B) The set is written as {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}. (C) The set is written as {2, 4, 6, 8, 10, 12}. (D) The set of even numbers between 1 and 11 is written as {2, 4, 6, 8, 10}.

92. If Set A = {1, 2, 3, 4, 5} and Set B = {3, 4, 5, 6, 7}, what is the intersection of A and B?
(A) The intersection is {1, 2, 3, 4, 5, 6, 7} because we combine all elements. (B) The intersection is {3} because only the middle number counts. (C) The intersection is {1, 2, 6, 7} because those are the unique elements. (D) The intersection of A and B is {3, 4, 5}.

93. Your friend says bubble sort is the best sorting algorithm because it's the easiest to understand. Do you agree? What factors besides simplicity should we consider when choosing a sorting algorithm?
(A) Bubble sort IS the best algorithm because understanding the code is the most important factor. (B) All sorting algorithms are equally good because they all produce the same sorted result. (C) While bubble sort is easy to understand, we should also consider speed, the size of the data, and whether the data is already partially sorted. (D) The only factor that matters is how fast the algorithm runs on the biggest possible dataset.

94. What is the sum of the first n natural numbers? (1 + 2 + 3 + ... + n)
(A) The sum is 2n + 1 because that's the formula for odd numbers. (B) The sum is n(n-1)/2 because you subtract 1 from n first. (C) The sum is n^2 because you multiply n by itself. (D) The sum is n(n+1)/2. For example, 1 + 2 + 3 + ... + 10 = 10(11)/2 = 55.

95. How many 3-letter 'words' (real or nonsense) can you make from the letters A, B, C, D, E if no letter can be repeated?
(A) You can make 60 three-letter words, calculated as 5 × 4 × 3 = 60. (B) You can make 125 words, calculated as 5 × 5 × 5. (C) You can make 15 words, calculated as 5 × 3. (D) You can make 10 words, calculated as 5 + 4 + 3 - 2.

96. What pattern do you see in this sequence: 1, 1, 2, 3, 5, 8, 13, ...? What are the next two numbers?
(A) The pattern alternates odd and even, so the next two are 14 and 21. (B) Each number doubles the previous one, so the next two are 26 and 52. (C) Each number is the sum of the two before it (the Fibonacci sequence), so the next two numbers are 21 and 34. (D) The pattern adds increasing amounts, so the next two are 18 and 25.

Write the answer

97. What comes next in the pattern: 2, 6, 18, 54, ...?
Answer: _____

98. In a video game, a treasure chest opens when the player 'hasKey AND (level >= 5 OR isPremium).' If a player has a key and is level 3 but is a premium member, does the chest open?
Answer: _____

99. What is a truth table?
Answer: _____

100. Why is the statement 'If it's Tuesday, then I have math class' different from 'If I have math class, then it's Tuesday'?
Answer: _____

Match each question to its answer

1. What is the negation of the statement 'All birds can fly'? →
2. Why can a graph with 5 vertices and 11 edges not exist if the graph is simple (no self-loops or parallel edges)? →
3. What is a path in graph theory? →
4. A network has 5 computers connected by cables. One cable fails, disconnecting the network. An engineer says adding just one more cable would prevent any single cable failure from disconnecting the network. Is this always possible? →

5. What is a weighted graph, and when would you use one? →

(A) A simple graph with 5 vertices can have at most 10 edges, calculated as 5 × 4 / 2 = 10, so 11 edges is impossible.
(B) A weighted graph assigns a numerical value to each edge, representing cost, distance, time, or another quantity.
(C) Some birds cannot fly
(D) Not always. If the original network is a tree (minimum connections), one extra cable creates one cycle, but there might still be bridges whose failure disconnects the network.
(E) A path is a sequence of vertices connected by edges where no vertex is visited more than once.

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11. Creative Process & Critique

Circle the correct answer

101. What is the empty set, and how is it written?
(A) The empty set is a set containing only the number zero, written as {0}. (B) The empty set is a set missing its curly braces, written as ( ). (C) The empty set is a set with no elements, written as { } or ∅. (D) The empty set is a set that contains infinite elements, written as {∞}.

102. Look at this Venn diagram: Circle A has {1, 3, 5}, the overlap has {7, 9}, and Circle B has {2, 4, 6}. What can you conclude about the odd numbers in this diagram?
(A) All odd numbers (1, 3, 5, 7, 9) are in Set A, while Set B contains only even numbers. (B) Only the numbers in the overlap (7, 9) are odd numbers. (C) There are no odd numbers in Set B, which means Set B is the odd set. (D) The odd numbers are split evenly between Set A and Set B.

103. In a card game, you pick up cards one at a time and slide each into its correct position among the cards already in your hand. Which sorting algorithm does this resemble?
(A) This resembles insertion sort, where each new element is inserted into its correct position in the sorted portion. (B) This resembles merge sort, where you split the cards into halves. (C) This resembles bubble sort, where adjacent elements are swapped repeatedly. (D) This resembles selection sort, where you always find the smallest card first.

104. Compare the truth tables for P OR Q and P AND Q. In how many of the 4 input combinations do they give different results?

(A) They differ in only 1 combination: when both are false. (B) They never differ because both are logical operations on the same inputs. (C) They differ in all 4 combinations because OR and AND are opposites. (D) They give different results in 2 of the 4 combinations: when one input is true and the other is false.

105. Create a counting problem about planning a class trip. Include at least three independent choices that must be made, and calculate the total number of possible trip plans.

(A) A good problem might be: Choose a destination (4 options), a bus type (3 options), and a lunch plan (5 options), giving $4 \times 3 \times 5 = 60$ different trip plans. (B) The multiplication principle only works with exactly two choices, not three or more. (C) Counting problems can only involve numbers, not real-world scenarios like field trips. (D) A class trip only has one choice (the destination), so there's nothing to count.

106. Find the pattern and fill in the blank: 1, 4, 9, 16, 25, __, 49.

(A) The missing number is 36, because these are perfect squares: 1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2. (B) The missing number is 35 because the pattern alternates adding 5 and 7. (C) The missing number is 32 because the pattern adds 7 each time. (D) The missing number is 30 because each gap increases by 2.

Write the answer

107. How many different ways can you arrange the letters A, B, and C in a row?

Answer: _____

108. Design a set of rules for a simple robot using AND, OR, and NOT. The robot should: move forward if the path is clear AND battery is charged, turn left if there's an obstacle AND the left path is clear, and stop if the battery is NOT charged.

Answer: _____

109. Build a truth table for NOT (P OR Q). How many rows does it have, and when is the result true?

Answer: _____

110. A weather app says: 'If temperature > 90° F, then display heat warning.' The contrapositive is: 'If no heat warning is displayed, then temperature ≤ 90° F.' Are both statements always true at the same time?

Answer: _____

Match each question to its answer

1. In a truth table, how many rows are needed to evaluate a compound statement with 3 variables? →
2. What is the difference between a directed graph and an undirected graph? →
3. A graph has vertices A, B, C, D, E with edges A-B, B-C, C-D, D-E, and E-A. What is the shortest path from A to C? →
4. What does it mean for a graph to be 'connected'? →
5. Why is a binary search faster than a linear search for sorted data? →

(A) It eliminates half the remaining elements each step
(B) A connected graph has a path between every pair of vertices — you can reach any vertex from any other vertex.
(C) The shortest path from A to C is A → B → C, which has length 2 (two edges).
(D) 8
(E) In a directed graph, edges have a direction (like one-way streets), while in an undirected graph, edges go both ways.

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12. Professional Practice & Careers

Circle the correct answer

111. Why does the set {3, 1, 2} equal the set {1, 2, 3}?
(A) They are not actually equal because the order of elements is different. (B) They are equal because both sets have three elements and that's all that matters. (C) The two sets are equal because sets only care about which elements they contain, not the order. (D) They are equal only because the numbers are consecutive.

112. What does it mean for Set A to be a subset of Set B?
(A) Set A is a subset of Set B when every element in A is also found in B. (B) Set A is a subset of Set B when A and B share no elements in common. (C) Set A is a subset of Set B when A and B have the exact same elements. (D) Set A is a subset of Set B when A has more elements than B.

113. Sort the list [5, 2, 8, 1, 9] using selection sort. What does the list look like after the first pass?
(A) After the first pass, the list is [2, 5, 8, 1, 9] because we swap the first two elements. (B) After the first pass, the list is [1, 2, 5, 8, 9] because selection sort sorts everything at once. (C) After the first pass, the list is [5, 2, 1, 8, 9] because we move 1 next to 2. (D) After the first pass, the list is [1, 2, 8, 5, 9] because 1 (the smallest) swaps with 5 (the first element).

114. What does the NOT operation do to a logical statement?

(A) The NOT operation reverses the truth value — it turns true into false and false into true. (B) The NOT operation combines two statements together. (C) The NOT operation makes any statement automatically false. (D) The NOT operation keeps the same truth value but changes the wording.

115. Your 4-digit bike lock allows digits 0-9 with repetition. Your friend's lock also has 4 positions but uses letters A-Z (26 options) without repetition. Whose lock has more possible combinations?

(A) Your friend's letter lock has more combinations: 26 × 25 × 24 × 23 = 358,800, compared to your 10 × 10 × 10 × 10 = 10,000. (B) Your number lock has more because numbers go on forever but the alphabet ends at Z. (C) Both locks have the same number of combinations because they both have 4 positions. (D) Your number lock has more because repetition gives you more choices per position.

116. Look at this pattern of triangular numbers: 1, 3, 6, 10, 15, 21, 28. What is the rule for finding the next triangular number, and why are they called 'triangular'?

(A) Each number is a multiple of 3, and they're named after the mathematician Triangle. (B) Each number adds 7, and they're triangular because they always come in groups of three. (C) Each triangular number adds one more than the previous gap (add 2, add 3, add 4, etc.), and they're called triangular because they represent dots arranged in equilateral triangles. (D) Each number doubles the previous one, and they're triangular because they have three digits.

Write the answer

117. You flip a fair coin 3 times. What is the probability of getting exactly 2 heads?

Answer: _____

118. De Morgan's Law says NOT (P AND Q) = (NOT P) OR (NOT Q). Can you explain why this makes intuitive sense using a real-world example?

Answer: _____

119. Complete the truth table for P AND Q. When P = True and Q = False, what is the result?

Answer: _____

120. Create a logic puzzle for your classmates. Write three if-then clues that, when combined, allow the solver to determine which student (Amy, Ben, or Cal) plays which instrument (drums, flute, or guitar).

Answer: _____

Match each question to its answer

1. Why does a 3-digit lock with digits 0-9 have 1,000 possible codes instead of 30? →
2. What is the degree of a vertex in a graph? →
3. What is a cycle in a graph, and how is it different from a path? →
4. A student claims that if a graph has 6 vertices and each vertex has degree 2, the graph must be a single hexagon (6-cycle). Is this true? →
5. Use a truth table to verify whether 'P AND (P OR Q)' always equals P. →

(A) A cycle is a path that starts and ends at the same vertex, forming a closed loop.
(B) Each of the 3 positions can independently be any of 10 digits, so there are 10 × 10 × 10 = 1,000 codes.
(C) Yes, P AND (P OR Q) always equals P. When P is true, the expression is true; when P is false, the expression is false — matching P exactly.
(D) The degree of a vertex is the number of edges connected to it.
(E) No, the graph could also be two separate triangles (3-cycles), since each vertex would still have degree 2.

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13. Introduction to Sets

Circle the correct answer

121. If Set A = {1, 3, 5, 7} and Set B = {2, 4, 6, 8}, what do we call these two sets that share no elements?
(A) Sets that share no elements are called empty sets. (B) Sets that share no elements are called disjoint sets. (C) Sets that share no elements are called subset pairs. (D) Sets that share no elements are called equal sets.

122. What is 0! (zero factorial)?
(A) Zero factorial equals negative 1 because it's below 1 factorial. (B) Zero factorial equals 0 because there's nothing to multiply. (C) Zero factorial equals 1 by mathematical convention. (D) Zero factorial is undefined because you can't have zero of something.

123. Design a number sequence that follows a rule you create. Write the first 6 terms and describe the rule so a classmate could continue the pattern.
(A) Sequences must always start with 1 and go up by the same amount, and this is the standard approach used in most cases (B) A good sequence might be: 2, 6, 12, 20, 30, 42 where the rule is 'each term is n × (n + 1)' or equivalently 'the differences increase by 2 each time: 4, 6, 8, 10, 12.' (C) You can only make sequences using addition, not multiplication, which is the most common explanation for this phenomenon (D) Any random list of 6 numbers counts as a valid sequence, and this is the standard approach used in most cases

124. In logic, what does the OR operation mean?
(A) The OR operation is true only when exactly one statement is true but not both. (B) The OR operation is true only when both statements are true. (C) The OR operation is true when at least one of the statements connected by it is true. (D) The OR operation reverses the truth value of a statement.

125. What is an edge in graph theory?
(A) An edge is the shortest path between any two vertices. (B) An edge is a vertex with only one connection. (C) An edge is the outer boundary of a graph. (D) An edge is a connection between two vertices in a graph, representing a relationship or link.

126. A game costs $2 to play. You roll a die: if you roll a 6, you win $10; otherwise you win nothing. Should you play this game?
(A) The expected value is -$1/3 per game (lose about 33 cents on average), so mathematically you should not play. (B) Yes, because you have an equal chance of winning or losing. (C) Yes, because $10 is much more than $2. (D) No, because rolling a 6 is impossible.

Write the answer

127. Design a recursive sequence inspired by a real-world scenario (like population growth, compound interest, or a game score). Define the base case and recursive rule.
Answer: _____

128. The Collatz conjecture says: if n is even, divide by 2; if n is odd, multiply by 3 and add 1. Starting from 6, trace the sequence until it reaches 1.
Answer: _____

129. An algorithm takes 2 seconds for 100 items. If it's $O(n^2)$, approximately how long will it take for 1000 items?
Answer: _____

130. What is the divide and conquer strategy in algorithm design?
Answer: _____

Match each question to its answer

1. Rank these Big O complexities from fastest to slowest: $O(n^2)$, $O(1)$, $O(n \log n)$, $O(n)$, $O(2^n)$. →
2. What is a greedy algorithm? Give an example. →
3. You need to sort 5 cards: [8, 3, 1, 5, 2]. Show the steps using selection sort. →
4. Write a recursive rule for the sequence: 2, 6, 18, 54, 162, ... →
5. What is the probability of rolling a sum of 7 with two standard dice? →

(A) The recursive rule is a(1) = 2 and a(n) = 3 x a(n-1), because each term is 3 times the previous term.
(B) The probability is 6/36 or 1/6, because there are 6 combinations that sum to 7 out of 36 total outcomes.
(C) A greedy algorithm makes the locally optimal choice at each step, hoping to find a global optimum. Example: making change with the fewest coins by always picking the largest coin that fits.
(D) Selection sort finds the minimum and places it at the front. Steps: [1,3,8,5,2] → [1,2,8,5,3] → [1,2,3,5,8] → [1,2,3,5,8] → done.
(E) From fastest to slowest: O(1), O(n), O(n log n), O(n^2), O(2^n).

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14. Introduction to Sets

Circle the correct answer

131. A teacher asks students to line up by height. She tells them: 'Compare yourself to the person next to you, and swap if the taller person is in front.' They repeat this until everyone is in order. What sorting algorithm is the teacher using, and why might it be slow for a class of 30?
(A) The teacher is using bubble sort, and it could be slow because it may need up to 435 comparisons (30 × 29 / 2) in the worst case. (B) The teacher is using merge sort, and it's slow because you need a second line of students. (C) The teacher is using quick sort, and it's slow because students can't split into groups. (D) The teacher is using random sort, and it's slow because students just shuffle around.

132. A student says there are 24 ways to arrange the letters in the word MATH. Another student says there are also 24 ways to arrange MEET. Who is correct, and why?
(A) The first student is correct (MATH has 4! = 24 arrangements), but MEET has only 12 because the two E's are identical, giving 4!/2! = 12. (B) Neither is correct because you can't rearrange words — they only have one correct spelling. (C) Both students are correct because both words have 4 letters. (D) The second student is correct because MEET has a repeated letter, which adds more options.

133. A school rule says: 'Students can use the computer lab if they have finished homework AND it is not lunchtime.' Express this as a logical expression and evaluate it when homework is done and it IS lunchtime.
(A) The expression is 'finishedHomework AND NOT lunchtime.' When homework is done (true) and it is lunchtime (true), NOT lunchtime is false, so the result is false — the student cannot use the lab. (B) The result is true because the homework is done, which is the most important condition. (C) The result is true because the student has finished homework AND it is lunchtime, so both are true. (D) The expression cannot

be evaluated because it mixes AND and NOT.

134. A class has 10 students. How many ways can you choose a president, vice president, and secretary?
(A) There are 720 ways because 10 x 9 x 8 = 720. (B) There are 30 ways because 10 x 3 = 30. (C) There are 120 ways because C(10,3) = 120. (D) There are 1000 ways because 10 x 10 x 10 = 1000.

135. What is expected value, and how do you calculate it?
(A) Expected value is the average outcome you'd expect over many trials, calculated by multiplying each outcome by its probability and adding them together. (B) Expected value is the highest possible outcome of an experiment. (C) Expected value is the total of all possible outcomes divided by 2. (D) Expected value is the most likely outcome of a single trial.

136. Trace through the recursive calculation: fib(5), where fib(1) = 1, fib(2) = 1, and fib(n) = fib(n-1) + fib(n-2).
(A) fib(5) = 4 because 5 - 1 = 4, since this is how the operation works with these particular numbers (B) fib(5) = fib(4) + fib(3) = (fib(3) + fib(2)) + (fib(2) + fib(1)) = ((fib(2) + fib(1)) + 1) + (1 + 1) = ((1 + 1) + 1) + 2 = 5. (C) fib(5) = 8 because the Fibonacci sequence goes 1, 1, 2, 3, 5, 8. (D) fib(5) = 10 because 5 x 2 = 10, according to the standard procedure for solving this kind of equation

Write the answer

137. What is the Sieve of Eratosthenes, and how does it find prime numbers?
Answer: _____

138. What does it mean for two events to be independent in probability?
Answer: _____

139. Why is merge sort (O(n log n)) faster than bubble sort ($O(n^2)$) for large lists?
Answer: _____

140. What is the Traveling Salesman Problem (TSP), and why is it computationally difficult?
Answer: _____

Match each question to its answer

1. What is memoization, and how does it improve the recursive Fibonacci algorithm? →
2. Explain what it means for a problem to be 'NP-complete' in simple terms. →
3. What is Pascal's Triangle, and how does it relate to combinations? →
4. Design a simple substitution cipher that is harder to break than the Caesar cipher.

Explain why it's more secure. →
5. In how many ways can 6 runners finish a race in first, second, and third place? →

(A) Use a random mapping where each letter maps to a different letter (not just a fixed shift). This has 26! (about 4 x 10^26) possible keys instead of just 25, making brute force impractical.
(B) There are 120 ways, calculated as 6 × 5 × 4 = 120.
(C) Pascal's Triangle is a triangular array where each number is the sum of the two numbers above it, and the entries represent combination values C(n,r).
(D) An NP-complete problem is one where no known fast algorithm exists, but if you're given a proposed solution, you can verify it quickly. If a fast algorithm is found for any NP-complete problem, it would solve all of them.
(E) Memoization stores previously computed results to avoid redundant calculations. It improves recursive Fibonacci from O(2^n) to O(n) by caching each Fibonacci value after computing it once.

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15. Introduction to Sets

Circle the correct answer

141. A pizza shop offers 3 sizes, 2 crust types, and 5 toppings. If you pick one of each, how many different single-topping pizzas are possible?
(A) There are 30 different single-topping pizzas, calculated as 3 × 2 × 5 = 30. (B) There are 10 different pizzas, calculated as 3 + 2 + 5. (C) There are 15 different pizzas, calculated as 3 × 5. (D) There are 60 different pizzas, calculated as 3 × 2 × 5 × 2.

142. An advertisement claims: 'Professional athletes use our shoes. If you use our shoes, you'll perform like a professional.' Is this a valid logical argument? Explain the flaw.
(A) It's valid because it's an if-then statement from a trusted brand. (B) It's invalid only because not all professionals use those shoes. (C) This is not valid — it's the fallacy of affirming the consequent. Professional athletes using the shoes doesn't mean the shoes cause professional performance. (D) It's valid because professionals use the shoes, so the shoes must help performance.

143. Define the sequence T(n) recursively: T(1) = 3, T(n) = 2 x T(n-1) + 1. Find T(4).
(A) T(4) = 31, found by computing T(2) = 7, T(3) = 15, T(4) = 2 x 15 + 1 = 31. (B) T(4) = 13 because 3 + 3 + 3 + 4 = 13. (C) T(4) = 9 because 3 x 4 - 3 = 9. (D) T(4) = 25 because 3 x 8 + 1 = 25.

144. How does exclusive OR (XOR) differ from regular OR?
(A) XOR only works with numbers, while OR works with true/false values. (B) XOR and regular OR are the same — both are true when at least one input is true. (C) XOR is always the opposite of OR. (D) XOR is true when exactly one input is true, but false when both are true, while regular OR is true when both are true.

145. What does Big O notation measure?
(A) Big O rates how difficult an algorithm is to understand. (B) Big O notation describes how an algorithm's running time or space grows as the input size increases, focusing on the worst-case scenario. (C) Big O measures how much memory a program uses in megabytes. (D) Big O counts the exact number of steps an algorithm takes.

146. A game has a 1/6 chance of winning each round. What is the probability of losing 3 rounds in a row?
(A) The probability is 5/18 because 5/6 x 3 = 15/18 = 5/6. (B) The probability is 1/216 because (1/6)^3 = 1/216. (C) The probability is (5/6)^3 = 125/216, which is about 57.9%. (D) The probability is 3/6 = 1/2 because you lose 3 times.

Write the answer

147. In a Venn diagram with two overlapping circles, where would you place an item that belongs to both Set A and Set B?
Answer: _____

148. A PE teacher has students line up by birthday (January first, December last). She can either use bubble sort (compare neighbors) or selection sort (find the earliest birthday and move them to the front). With 25 students, which algorithm would likely finish faster in real life, and why?
Answer: _____

149. Compare the efficiency of computing fib(20) recursively versus iteratively. Which is faster and why?
Answer: _____

150. In a class of 25 students, what is the approximate probability that at least two students share a birthday? (Assume 365 days in a year.)
Answer: _____

Match each question to its answer

1. Explain why there are infinitely many prime numbers, using Euclid's argument. →
2. Decrypt the message 'KHOOR' using a Caesar cipher with shift 3. →
3. What are twin primes? Give three examples of twin prime pairs. →
4. Why is P(A or B) = P(A) + P(B) - P(A and B) instead of just P(A) + P(B)? →
5. Design an efficient algorithm to determine if a word is a palindrome (reads the same forwards and backwards). What is its time complexity? →

(A) We subtract P(A and B) to avoid counting the overlap where both events happen, which would otherwise be counted twice.
(B) Assume there are finitely many primes. Multiply them all together and add 1. This new number isn't divisible by any known prime (remainder 1), so it must have a new prime factor — a contradiction.
(C) KHOOR decrypts to HELLO by shifting each letter back by 3: K→H, H→E, O→L, O→L, R→O.
(D) Compare the first character with the last, second with second-to-last, and so on, moving inward. Stop when the pointers meet in the middle. Time complexity is O(n/2) = O(n).
(E) Twin primes are pairs of primes that differ by exactly 2. Examples: (3, 5), (11, 13), and (17, 19).

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16. Introduction to Sets

Circle the correct answer

151. If Set A = {apple, banana, cherry} and Set B = {banana, date, elderberry}, what is the union of A and B?
(A) The union of A and B is {apple, cherry, date, elderberry} without the shared element. (B) The union of A and B is {banana} because it's the only shared element. (C) The union of A and B is {apple, banana, cherry, date, elderberry}. (D) The union of A and B is {apple, banana, cherry, banana, date, elderberry} with banana listed twice.

152. You have 5 books to arrange on a shelf. If you sort them by title alphabetically, how many comparisons does it take using bubble sort in the worst case?
(A) Bubble sort needs at most 10 comparisons to sort 5 items, calculated as 4 + 3 + 2 + 1 = 10. (B) Bubble sort needs 25 comparisons, calculated as 5 × 5. (C) Bubble sort needs exactly 5 comparisons, one for each book. (D) Bubble sort needs 20 comparisons, calculated as 5 × 4.

153. What does it mean when two logical expressions are 'logically equivalent'?
(A) They are equivalent if they both evaluate to true, which is the most common explanation for this phenomenon (B) They are equivalent if one is the opposite of the other, which is the most common explanation for this phenomenon (C) Two

expressions are logically equivalent when they have identical truth tables — they produce the same output for every possible input combination. (D) They are equivalent if they contain the same variables, as this is what research and standard practice suggests

154. A bag has 3 red marbles, 4 blue marbles, and 5 green marbles. If you draw one marble without looking, what is the probability of drawing a blue marble?
(A) The probability is 1/12 because there are 12 marbles total. (B) The probability is 4/5 because blue has 4 and green has 5. (C) The probability is 1/4 because there are 4 blue marbles. (D) The probability is 4/12 or 1/3, because there are 4 blue marbles out of 12 total.

155. Using the Sieve of Eratosthenes, find all prime numbers between 1 and 30.
(A) The primes are: 2, 3, 5, 7, 11, 13, 15, 17, 19, 23, 29. (B) The primes between 1 and 30 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 — there are 10 primes. (C) The primes are: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 (including 1). (D) The primes are: 2, 3, 5, 7, 9, 11, 13, 17, 19, 23, 29.

156. Two numbers are 'congruent modulo n' if they have the same remainder when divided by n. Are 38 and 14 congruent modulo 8?
(A) Yes, 38 and 14 are congruent modulo 8 because 38 mod 8 = 6 and 14 mod 8 = 6 — they have the same remainder. (B) Yes, because 38 - 14 = 24 and 24/8 = 3, and 3 is odd. (C) No, because 38 mod 8 = 4 and 14 mod 8 = 6. (D) No, because 38 and 14 are different numbers.

Write the answer

157. Create a simple number-based encryption scheme using modular arithmetic. Encrypt the number 15 using your scheme and show how to decrypt it.
Answer: _____

158. Why are prime numbers important for modern internet security?
Answer: _____

159. What is the golden ratio, and how does it relate to the Fibonacci sequence?
Answer: _____

160. Why is the Caesar cipher considered weak by modern standards?
Answer: _____

Match each question to its answer

1. In a game, you spin a spinner divided into 5 equal sections (red, blue, green, yellow, purple) and flip a coin. What is the probability of landing on blue AND getting heads? →
2. What is modular arithmetic, and what does '17 mod 5' equal? →

3. Describe how bubble sort works. What is its time complexity? →
4. Your friend argues: 'All dogs are animals. My cat is an animal. Therefore, my cat is a dog.' What logical error is your friend making? →
5. How does a hash function work, and why can't you reverse it? →

(A) Bubble sort repeatedly compares adjacent elements and swaps them if they're in the wrong order, making multiple passes until no swaps are needed. Its time complexity is $O(n^2)$.
(B) The probability is 1/10 because (1/5) x (1/2) = 1/10.
(C) Modular arithmetic is the math of remainders after division. 17 mod 5 = 2, because 17 divided by 5 is 3 remainder 2.
(D) A hash function converts any input into a fixed-size output (hash). It's a one-way function: many different inputs can produce the same hash, so you can't determine the original input from the hash alone.
(E) Your friend is committing the fallacy of affirming the consequent — just because all dogs are animals doesn't mean all animals are dogs.

Answer Key

1. A — A set is a well-defined collection of distinct objects or elements.
2. C — Sets that share no elements are called disjoint sets.
3. A — The set of even numbers between 1 and 11 is written as {2, 4, 6, 8, 10}.
4. D — The symbol ∈ means 'is an element of' or 'belongs to' a set.
5. B — The empty set is a set with no elements, written as { } or ∅.
6. B — The union of A and B is {apple, banana, cherry, date, elderberry}.
7. The intersection of A and B is {3, 4, 5}.
8. An item belonging to both sets goes in the overlapping region where the two circles intersect.
9. Set A is a subset of Set B when every element in A is also found in B.
10. The two sets are equal because sets only care about which elements they contain, not the order.
    Matching (Sets and Sorting): 1→C, 2→A, 3→B, 4→D, 5→E
11. B — You can make 12 different outfits, calculated as 3 × 4 = 12.
12. D — A permutation is an arrangement of objects where the order matters.

13. A — Five factorial equals 120, calculated as 5 × 4 × 3 × 2 × 1 = 120.
14. B — Zero factorial equals 1 by mathematical convention.
15. D — In a permutation the order of selection matters, while in a combination the order does not matter.
16. B — Each of the 3 positions can independently be any of 10 digits, so there are 10 × 10 × 10 = 1,000 codes.
17. For the first position you have 4 choices, then 3 for the second, 2 for the third, and 1 for the last, giving 4 × 3 × 2 × 1 = 24.
18. Each number is the sum of the two before it (the Fibonacci sequence), so the next two numbers are 21 and 34.
19. There are 30 different single-topping pizzas, calculated as 3 × 2 × 5 = 30.
20. You can make 60 three-letter words, calculated as 5 × 4 × 3 = 60.
    Matching (Counting and Patterns): 1→E, 2→D, 3→C, 4→B, 5→A
21. C — The AND operation is true only when both statements connected by it are true.
22. B — The OR operation is true when at least one of the statements connected by it is true.
23. B — The NOT operation reverses the truth value — it turns true into false and false into true.
24. D — A truth table is a chart that shows every possible combination of true/false inputs and the resulting output for a logical expression.
25. A — A conditional statement is an 'if-then' statement, written as 'if P, then Q' where P is the hypothesis and Q is the conclusion.
26. D — They're different because the second is the converse of the first, and a statement and its converse can have different truth values.
27. When P is true and Q is false, P AND Q is false because AND requires both inputs to be true.
28. XOR is true when exactly one input is true, but false when both are true, while regular OR is true when both are true.
29. Two expressions are logically equivalent when they have identical truth tables — they produce the same output for every possible input combination.
30. Yes, the chest opens because hasKey is true, and isPremium is true which makes

the OR clause true, so the whole AND expression is true.
Matching (Logic and Truth Tables): 1→E, 2→B, 3→D, 4→C, 5→A

31. C — A vertex is a point in a graph that represents an object, person, or location.
32. A — An edge is a connection between two vertices in a graph, representing a relationship or link.
33. B — The degree of a vertex is the number of edges connected to it.
34. B — In a directed graph, edges have a direction (like one-way streets), while in an undirected graph, edges go both ways.
35. D — A path is a sequence of vertices connected by edges where no vertex is visited more than once.
36. A — A simple graph with 5 vertices can have at most 10 edges, calculated as 5 × 4 / 2 = 10, so 11 edges is impossible.
37. A connected graph has a path between every pair of vertices — you can reach any vertex from any other vertex.
38. A cycle is a path that starts and ends at the same vertex, forming a closed loop.
39. A tree connects all vertices using the minimum number of edges (n-1 edges for n vertices) without forming any loops.
40. A weighted graph assigns a numerical value to each edge, representing cost, distance, time, or another quantity.
Matching (Graph Theory Basics): 1→A, 2→B, 3→C, 4→E, 5→D
41. B — In permutations order matters, but in combinations order does not matter.
42. A — There are 24 ways to arrange 4 books, since 4! = 4 x 3 x 2 x 1 = 24.
43. D — You can choose 10 different pairs, calculated as 5! / (2! x 3!) = 10.
44. A — The probability is 6/36 or 1/6, because there are 6 combinations that sum to 7 out of 36 total outcomes.
45. C — The probability is 4/12 or 1/3, because there are 4 blue marbles out of 12 total.
46. B — Two events are independent when the outcome of one event does not affect the probability of the other event.
47. The probability is 3/8 because there are 3 favorable outcomes (HHT, HTH, THH) out of 8 total outcomes.
48. There are 72 different combinations because 3 x 4 x 6 = 72.

49. The complement of an event is everything that is NOT that event, and their probabilities add up to 1.
50. There are 60 codes because 5 x 4 x 3 = 60.
    Matching (Combinatorics and Probability): 1→E, 2→B, 3→A, 4→C, 5→D
51. D — A recursive definition defines something in terms of itself, with a base case that stops the self-reference.
52. B — The base case is 0! = 1, and the recursive case is n! = n x (n-1)! for n > 0.
53. C — The next two numbers are 21 and 34, because each number is the sum of the previous two.
54. B — The recursive rule is a(1) = 2 and a(n) = 3 x a(n-1), because each term is 3 times the previous term.
55. C — Without a base case, the recursion never stops — it creates an infinite loop that keeps calling itself forever.
56. B — A 6-disk Tower of Hanoi requires 63 moves, because 2^6 - 1 = 64 - 1 = 63.
57. fib(5) = fib(4) + fib(3) = (fib(3) + fib(2)) + (fib(2) + fib(1)) = ((fib(2) + fib(1)) + 1) + (1 + 1) = ((1 + 1) + 1) + 2 = 5.
58. The explicit formula is a(n) = 4n - 1, where n starts at 1.
59. Recursive definitions are sometimes simpler to understand and write, especially when each step depends directly on the previous step.
60. The sum is n(n+1)/2. For example, 1 + 2 + 3 + ... + 10 = 10(11)/2 = 55.
    Matching (Recursion and Sequences): 1→A, 2→E, 3→C, 4→B, 5→D
61. A — Modular arithmetic is the math of remainders after division. 17 mod 5 = 2, because 17 divided by 5 is 3 remainder 2.
62. D — Every integer greater than 1 can be expressed as a unique product of prime numbers, regardless of the order of factors.
63. C — GCD(48, 36) = 12. Steps: 48 = 1 x 36 + 12, then 36 = 3 x 12 + 0. When the remainder is 0, the last non-zero remainder (12) is the GCD.
64. C — A Caesar cipher shifts each letter by a fixed number. With shift 3, MATH becomes PDWK (M→P, A→D, T→W, H→K).
65. C — It is weak because there are only 25 possible shifts, so an attacker can try all of them quickly (brute force attack).
66. A — Symmetric encryption uses the same key to encrypt and decrypt, while

asymmetric encryption uses a public key to encrypt and a different private key to decrypt.

67. Large prime numbers are the foundation of RSA encryption because multiplying two large primes is easy, but finding the original primes from their product is extremely difficult.
68. 23 mod 7 = 2 (because 23 = 3 x 7 + 2), and 100 mod 12 = 4 (because 100 = 8 x 12 + 4).
69. Yes, 38 and 14 are congruent modulo 8 because 38 mod 8 = 6 and 14 mod 8 = 6 — they have the same remainder.
70. The Sieve of Eratosthenes finds all primes up to a given number by systematically crossing out multiples of each prime, starting with 2.
Matching (Number Theory and Cryptography): 1→D, 2→B, 3→E, 4→A, 5→C
71. C — An algorithm is a step-by-step set of instructions for solving a problem or completing a task, where each step is clear and unambiguous.
72. C — Big O notation describes how an algorithm's running time or space grows as the input size increases, focusing on the worst-case scenario.
73. C — From fastest to slowest: O(1), O(n), O(n log n), O(n^2), O(2^n).
74. B — Binary search works on a sorted list by repeatedly checking the middle element and eliminating half the remaining elements. Its time complexity is O(log n).
75. C — Linear search needs up to 1,000,000 checks. Binary search needs only about 20 checks, because $\log_2(1{,}000{,}000) \approx 20$.
76. A — Bubble sort repeatedly compares adjacent elements and swaps them if they're in the wrong order, making multiple passes until no swaps are needed. Its time complexity is O(n^2).
77. Divide and conquer breaks a problem into smaller subproblems, solves each one independently, then combines the solutions to solve the original problem.
78. For large n, n log n grows much slower than n^2. With 1 million items, merge sort does about 20 million operations versus bubble sort's 1 trillion operations.
79. A greedy algorithm makes the locally optimal choice at each step, hoping to find a global optimum. Example: making change with the fewest coins by always picking the largest coin that fits.
80. Selection sort finds the minimum and places it at the front. Steps: [1,3,8,5,2] → [1,2,8,5,3] → [1,2,3,5,8] → [1,2,3,5,8] → done.

Matching (Algorithm Design and Complexity): 1→C, 2→B, 3→E, 4→A, 5→D

81. C — A set is a well-defined collection of distinct objects or elements.
82. B — Students who like all three subjects appear in the center region where all three circles overlap.
83. C — Inserting each new book into its correct position (insertion approach) is more practical because most books are already sorted.
84. D — {3}
85. A — A permutation is an arrangement of objects where the order matters.
86. B — The teacher makes a stronger argument because Fibonacci numbers appear in sunflower seed spirals, pinecone scales, flower petals, and shell curves, suggesting a deep connection to natural growth patterns.
87. 100
88. The AND operation is true only when both statements connected by it are true.
89. The claim is false. NOT (P AND Q) is true whenever at least one is false — it includes (T,F), (F,T), and (F,F), not just (F,F).
90. A conditional statement is an 'if-then' statement, written as 'if P, then Q' where P is the hypothesis and Q is the conclusion.
Matching (Elements & Principles Deep Dive): 1→E, 2→C, 3→A, 4→D, 5→B
91. D — The set of even numbers between 1 and 11 is written as {2, 4, 6, 8, 10}.
92. D — The intersection of A and B is {3, 4, 5}.
93. C — While bubble sort is easy to understand, we should also consider speed, the size of the data, and whether the data is already partially sorted.
94. D — The sum is n(n+1)/2. For example, 1 + 2 + 3 + ... + 10 = 10(11)/2 = 55.
95. A — You can make 60 three-letter words, calculated as 5 × 4 × 3 = 60.
96. C — Each number is the sum of the two before it (the Fibonacci sequence), so the next two numbers are 21 and 34.
97. 162
98. Yes, the chest opens because hasKey is true, and isPremium is true which makes the OR clause true, so the whole AND expression is true.
99. A truth table is a chart that shows every possible combination of true/false inputs and the resulting output for a logical expression.

100. They're different because the second is the converse of the first, and a statement and its converse can have different truth values.
Matching (Cultural & Historical Context): 1→C, 2→A, 3→E, 4→D, 5→B
101. C — The empty set is a set with no elements, written as { } or ∅.
102. A — All odd numbers (1, 3, 5, 7, 9) are in Set A, while Set B contains only even numbers.
103. A — This resembles insertion sort, where each new element is inserted into its correct position in the sorted portion.
104. D — They give different results in 2 of the 4 combinations: when one input is true and the other is false.
105. A — A good problem might be: Choose a destination (4 options), a bus type (3 options), and a lunch plan (5 options), giving 4 × 3 × 5 = 60 different trip plans.
106. A — The missing number is 36, because these are perfect squares: 1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2.
107. 6
108. The rules could be: moveForward = clearAhead AND charged; turnLeft = obstacleAhead AND clearLeft; stop = NOT charged. These three rules cover the basic scenarios using logical operations.
109. The truth table has 4 rows (for TT, TF, FT, FF), and NOT (P OR Q) is true only when both P and Q are false.
110. Yes, a conditional statement and its contrapositive are logically equivalent — they always have the same truth value.
Matching (Creative Process & Critique): 1→D, 2→E, 3→C, 4→B, 5→A
111. C — The two sets are equal because sets only care about which elements they contain, not the order.
112. A — Set A is a subset of Set B when every element in A is also found in B.
113. D — After the first pass, the list is [1, 2, 8, 5, 9] because 1 (the smallest) swaps with 5 (the first element).
114. A — The NOT operation reverses the truth value — it turns true into false and false into true.
115. A — Your friend's letter lock has more combinations: 26 × 25 × 24 × 23 = 358,800, compared to your 10 × 10 × 10 × 10 = 10,000.
116. C — Each triangular number adds one more than the previous gap (add 2, add 3,

add 4, etc.), and they're called triangular because they represent dots arranged in equilateral triangles.

117. The probability is 3/8 because there are 3 favorable outcomes (HHT, HTH, THH) out of 8 total outcomes.

118. If it's NOT the case that 'you passed the test AND did the homework,' that means you either didn't pass the test OR didn't do the homework (or both).

119. When P is true and Q is false, P AND Q is false because AND requires both inputs to be true.

120. A good puzzle might be: (1) If Amy plays drums, then Ben plays guitar. (2) Cal does NOT play flute. (3) If Ben does NOT play drums, then Amy plays flute. From clue 2, Cal plays drums or guitar. Testing possibilities leads to a unique solution. Matching (Professional Practice & Careers): 1→B, 2→D, 3→A, 4→E, 5→C

121. B — Sets that share no elements are called disjoint sets.

122. C — Zero factorial equals 1 by mathematical convention.

123. B — A good sequence might be: 2, 6, 12, 20, 30, 42 where the rule is 'each term is n × (n + 1)' or equivalently 'the differences increase by 2 each time: 4, 6, 8, 10, 12.'

124. C — The OR operation is true when at least one of the statements connected by it is true.

125. D — An edge is a connection between two vertices in a graph, representing a relationship or link.

126. A — The expected value is -$1/3 per game (lose about 33 cents on average), so mathematically you should not play.

127. Example — Savings account: S(0) = $100 (initial deposit), S(n) = 1.05 x S(n-1) + 10 (5% interest plus $10 monthly deposit). This models real compound growth with regular contributions.

128. Starting from 6: 6, 3, 10, 5, 16, 8, 4, 2, 1. It takes 8 steps to reach 1.

129. It will take approximately 200 seconds, because when n increases 10x with $O(n^2)$, the time increases 100x ($10^2 = 100$).

130. Divide and conquer breaks a problem into smaller subproblems, solves each one independently, then combines the solutions to solve the original problem. Matching (Introduction to Sets): 1→E, 2→C, 3→D, 4→A, 5→B

131. A — The teacher is using bubble sort, and it could be slow because it may need up to 435 comparisons (30 × 29 / 2) in the worst case.

132. A — The first student is correct (MATH has 4! = 24 arrangements), but MEET has only 12 because the two E's are identical, giving 4!/2! = 12.

133. A — The expression is 'finishedHomework AND NOT lunchtime.' When homework is done (true) and it is lunchtime (true), NOT lunchtime is false, so the result is false — the student cannot use the lab.

134. A — There are 720 ways because 10 x 9 x 8 = 720.

135. A — Expected value is the average outcome you'd expect over many trials, calculated by multiplying each outcome by its probability and adding them together.

136. B — fib(5) = fib(4) + fib(3) = (fib(3) + fib(2)) + (fib(2) + fib(1)) = ((fib(2) + fib(1)) + 1) + (1 + 1) = ((1 + 1) + 1) + 2 = 5.

137. The Sieve of Eratosthenes finds all primes up to a given number by systematically crossing out multiples of each prime, starting with 2.

138. Two events are independent when the outcome of one event does not affect the probability of the other event.

139. For large n, n log n grows much slower than n^2. With 1 million items, merge sort does about 20 million operations versus bubble sort's 1 trillion operations.

140. TSP asks for the shortest route visiting every city exactly once and returning home. It's difficult because the number of possible routes grows factorially — checking all routes for 20 cities would take millions of years.
Matching (Introduction to Sets): 1→E, 2→D, 3→C, 4→A, 5→B

141. A — There are 30 different single-topping pizzas, calculated as 3 × 2 × 5 = 30.

142. C — This is not valid — it's the fallacy of affirming the consequent. Professional athletes using the shoes doesn't mean the shoes cause professional performance.

143. A — T(4) = 31, found by computing T(2) = 7, T(3) = 15, T(4) = 2 x 15 + 1 = 31.

144. D — XOR is true when exactly one input is true, but false when both are true, while regular OR is true when both are true.

145. B — Big O notation describes how an algorithm's running time or space grows as the input size increases, focusing on the worst-case scenario.

146. C — The probability is (5/6)^3 = 125/216, which is about 57.9%.

147. An item belonging to both sets goes in the overlapping region where the two circles intersect.

148. Selection sort would likely be faster because students can scan the whole line visually to find the earliest birthday, rather than making many small swaps.
149. Iterative is much faster because naive recursion recalculates the same values many times, while iteration computes each value once from left to right.
150. The probability is about 57%, which is surprisingly high despite having far fewer students than days.
Matching (Introduction to Sets): 1→B, 2→C, 3→E, 4→A, 5→D
151. C — The union of A and B is {apple, banana, cherry, date, elderberry}.
152. A — Bubble sort needs at most 10 comparisons to sort 5 items, calculated as 4 + 3 + 2 + 1 = 10.
153. C — Two expressions are logically equivalent when they have identical truth tables — they produce the same output for every possible input combination.
154. D — The probability is 4/12 or 1/3, because there are 4 blue marbles out of 12 total.
155. B — The primes between 1 and 30 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 — there are 10 primes.
156. A — Yes, 38 and 14 are congruent modulo 8 because 38 mod 8 = 6 and 14 mod 8 = 6 — they have the same remainder.
157. Example scheme: Encrypt: C = (M x 3 + 7) mod 26. Encrypt 15: C = (15 x 3 + 7) mod 26 = 52 mod 26 = 0. Decrypt: find M where (M x 3 + 7) mod 26 = 0, giving M = 15.
158. Large prime numbers are the foundation of RSA encryption because multiplying two large primes is easy, but finding the original primes from their product is extremely difficult.
159. The golden ratio (approximately 1.618) is the value that the ratio of consecutive Fibonacci numbers approaches as the numbers get larger.
160. It is weak because there are only 25 possible shifts, so an attacker can try all of them quickly (brute force attack).
Matching (Introduction to Sets): 1→B, 2→C, 3→A, 4→E, 5→D
